An online optimization method for bridge dynamic hybrid simulations

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**Article info**

**Article history:**
Received 7 February 2012
Received in revised form 20 June 2012
Accepted 21 June 2012

**Keywords:**
Bridge experiments
Hybrid simulation
Online optimization
Seismic response

**Abstract**

An online optimization method was proposed in this paper to improve the accuracy of numerical–experimental hybrid simulations of bridges subjected to earthquake forces. The online optimization method aims to solve the problem induced by the inconsistency of the multiple identical bridge piers where only one or a few of them are simulated experimentally by physical specimens, while the others are numerically simulated within a hybrid framework. This method adopts multi-variable nonlinear optimization in order to find the optimal material parameters of the numerical models, and updates the numerical models instantaneously during the hybrid simulation. A numerical verification of a bridge hybrid simulation was conducted using the proposed online optimization method. The numerical results showed that the online optimization method is capable of obtaining better material parameters during the first few seconds of a hybrid simulation, which results in more accurate nonlinear dynamic response under earthquake loading.

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1. Introduction

The numerical–experimental hybrid simulation method, sometimes called pseudo-dynamic testing, was originally published in 1980s [1]. The dynamic response of a new type of bridge subjected to earthquake forces can be simulated by running numerical simulation and experiments concurrently taking into account any interactions between them. Bridge decks and other devices having well established mechanical properties are simulated by numerical sub-structures, while new types of piers or bearings without reliable numerical models are simulated by experimental sub-structures. Interaction forces between numerical and experimental sub-structures are measured and analyzed, and instantly applied to these sub-structures in every time step of the process. The hybrid simulation method has been improved and modified for various purposes. Several numerical time integration algorithms have been proposed and evaluated to reduce the numerical errors and to improve numerical stability [2,3]. Facility controlling methods have been developed for real-time hybrid simulations to compensate for control and measurement errors induced by facility control and measurement time lags [4,5]. Some of the past research has conducted hybrid simulations with nonlinear and/or multiple experimental sub-structures [6,7], while software platforms and communication protocols have been implemented for network-based, geographically distributed, multi-laboratory collaborative hybrid simulations [8–13].

Unfortunately, researchers generally encounter difficulties when running a multi-pier bridge hybrid simulation. Due to the high cost of constructing an experimental environment of a bridge pier specimen, a hybrid simulation normally has only...
one or a few experimental piers simulated by physical specimens, as shown in Fig. 1. Hybrid simulations with nonlinear sub-structures are generally a challenging task and require advanced models [6]. They are even more difficult to handle if there are multiple identical sub-structures and one of them is a physical specimen. It is very likely that the physical specimens and numerical sub-structures possess different structural nonlinear behavior, despite their identical structural design, leading to inconsistent sub-structure response or improper system behavior such as overestimated torsion effects.

This paper proposes an online optimization method for hybrid simulations with multiple identical sub-structures. During a hybrid simulation, a set of parameters that matches the experimental data measured from the physical specimen are optimized. The models of the numerical sub-structures are then updated online according to the optimized parameters, ensuring that their response matches the experimental behavior.

2. Theory and methodology

This section briefly describes the concepts of a conventional hybrid simulation, elaborating on the concepts and formulations of the proposed online optimization method that can be used to perform a hybrid simulation with multiple identical or similar sub-structures. In order to distinguish hybrid simulation methods, a hybrid simulation without online optimization is referred to as a conventional hybrid simulation herein.

2.1. Conventional hybrid simulation

A conventional hybrid simulation is much alike running a numerical time integration dynamic structural analysis in which a part of the structure (i.e., experimental sub-structure) is simulated by a real physical specimen. Data of analyzed displacements and measured reacting forces are instantly exchanged between the numerical program and the experimental controllers. Fig. 2 presents the flowchart of a conventional hybrid simulation. The time integration estimates the required displacements to assign to the numerical and experimental sub-structures at time $t + \Delta t$. The $\{u\}^N$ and $\{u\}^E$ denote displacements of numerical and experimental sub-structure(s) at time $t$, respectively.

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Fig. 1. Conventional hybrid simulation of a bridge subjected to earthquake forces.

Fig. 2. Flowchart of a conventional hybrid simulation.
2.2. Assumptions and procedure of an online optimization hybrid simulation

The proposed online optimization method is based on the following assumptions:

(1) The structural system has multiple identical or similar experimental sub-structures. One of them, for example the index sub-structure expressed as \( \Omega^I \), is simulated by a physical specimen. The other sub-structures are numerically simulated based on a numerical model with a set of unknown or uncertain material parameters.

(2) The numerical model used to simulate the ith experimental sub-structure is expressed as:

\[ \{n\}_{i0-t}^E = N\left( \{u\}_{i0-t}^E, \{\text{par}\}_{i}^{\text{shared}}, \{\text{par}\}_i^f \right) \]  

where \( \{n\}_{i0-t}^E \) is the numerical resisting force history of the ith experimental sub-structure from the initial state (time zero) to time \( t \). In addition, note that \( \{n\}_{i0-t}^E \) is a sequence of vectors, i.e., \( \{n\}_{i0}, \{n\}_{i1}, \{n\}_{i2}, \ldots, \{n\}_{it-\Delta t}, \{n\}_{it}^E \). Similarly, \( \{u\}_{i0-t}^E \) is likewise a sequence of vectors. Eq. (5) indicates that \( \{n\}_{i0-t}^E \) is a function of its displacement history, \( \{u\}_{i0-t}^E \). The numerical model requires a given set of material parameters \( \{\text{par}\}_i^{\text{shared}} \), which can be applied on all experimental sub-structures. The online optimization module determines a reasonably optimized set of parameters so that the numerical model approaches the experimental result of the physically simulated experimental sub-structure. The optimization formulation requires a given set of material parameters \( \{\text{par}\}_i^{\text{shared}} \), which can be applied on all the experimental sub-structures. Given that all experimental sub-structures are identical, all material parameters can be shared, and Eq. (1) can be simplified to:

\[ \{n\}_{i0-t}^E = N\left( \{u\}_{i0-t}^E, \{\text{par}\}_{i}^{\text{shared}} \right) \]  

The procedure for an online optimization hybrid simulation is mainly based on the conventional process. The numerical sub-structure does not include multiple identical or similar experimental sub-structures. One of the experimental sub-structures is identical, all material parameters can be shared, and Eq. (1) can be simplified to:

The online optimization module performs the optimization based on the ongoing experimental data of the physically simulated experimental sub-structures. The \( \{\text{par}\}_{i}^{\text{shared}} \) in Fig. 3 is an optimized set of material parameters at time \( t \), and is optimized to approach the experimental data of the physically simulated experimental sub-structure from time 0 to time \( t \). The numerically simulated experimental sub-structures update their material parameters according to \( \{\text{par}\}_{i}^{\text{shared}} \) and subsequently calculate their resisting forces \( \{n\}_{i}^E \) using Eq. (2).

The online optimization module determines a reasonably optimized set of parameters so that the numerical model approaches the experimental result of the physically simulated experimental sub-structure. The optimization formulation of time \( t \) is to minimize the error \( e_f^t \) defined by the following expressions:

\[ e_f^t = \sum_{\text{for each DOF}} \left( \int_{t-\Delta t}^{t} \left( \{r\}_{iz}^E \right) - \left( \{n\}_{iz}^E \right) d\left( \{u\}_{iz}^E \right) \right) \]  

where \( e_f^t \) is the cumulative difference of strain energy between the experimental and numerical results. The inconsistent energy factor can be seen as a simplified version of an error indicator used to estimate numerical and experimental errors in a
real-time hybrid simulation [14] and only a strain energy term is used. The subscript \( j \) in Eq. (3) denotes the \( j \)th DOF of a force vector or a displacement vector. In order to better understand the physical meaning, the \( e^f_t \) expressed in Eq. (3) is portrayed graphically as the shaded area in Fig. 4. In numerical operations, \( e^f_t \) can be estimated using the trapezoidal rule of numerical integration:

\[
e^f_t \approx \sum_{\text{for each DOF}} \left( \sum_{z=1}^{T-1/M} \frac{1}{2} \left( \left( r^{E_1}_{f_z,1} \right)_j + \left( r^{E_1}_{f_z} \right)_j \right) \right)
\]

The mixture of the numerical and experimental sub-structures is implemented by adding a new element type into the finite-element based numerical engine. The numerical engine requires each element to report its resisting forces at the current time step so that the dynamic response of the next time step can be predicted. To carry out a hybrid simulation, a finite element program which allows researchers to add a user-defined element type is typically adopted, such as ABAQUS [15], OpenSees [16], or PISA3D [17].

The online optimization module requires a multi-variable nonlinear optimization algorithm as the numerical model in Eq. (2) is normally nonlinear and has multiple material parameters. A typical nonlinear optimization iteratively substitutes different trial parameter sets \( \{ \text{par}^{\text{trial}} \}_j^{\text{shared}} \), retrieves different \( n_{E_1}^{t} \) curves in Fig. 4, and finally acquires a trial parameter set \( \{ \text{par}^{\text{trial}} \}_j^{\text{shared}} \) that minimizes the corresponding \( e^f_t \). This work adopts the Simplex method [18] implemented in MATLAB to perform the online optimization. The process may be time consuming due to the requirement of performing a nonlinear static structural analysis to determine the corresponding \( n_{E_1}^{t} \) curve for each trial parameter \( \{ \text{par}^{\text{trial}} \}_j^{\text{shared}} \). Therefore at present this method is only suitable for low-speed hybrid simulations, which the material strain-rate effect is neglected. However, the strain-rate issue may be crucial in which the concrete and steel deform violently [19]. Investigation on the online optimization method that considers the strain-rate effect is needed in the future. Further details of the implementation of the method are not presented here due to space limitations and can be found in [20].

The Simplex method was chosen not because it was the best method in this case, but because it is one of the most used in engineering and is a built-in multi-variable optimization solver in MATLAB. Jung and Kim [21] employed the Simplex method for updating a numerical model of a bridge. Begambre and Laier [22] combined the particle swarm optimization (PSO) and the Simplex method to identify the damage location of bridge structures. Marwala [23] recently tested several optimization methods on updating the structural finite element models, including the Simplex method, the BFGS method, genetic algorithm, the PSO method, simulated annealing, etc. Future extensions of this work may include the usage of different optimization methods in terms of computational cost and the usage of the parallel computing [24].

It should be noted that some additional difficulties may be imposed when the proposed method is applied to a real experiment. The measurement error of displacements and resisting forces not only leads to an inaccurate response of the following numerical simulation, but also affects the optimization result. To minimize error accumulation, experimental apparatus that is used to run a hybrid simulation should be well calibrated. In addition, while this method takes only displacement resisting forces as input arguments for the optimization process, other measurement data such as strain fields and crack distributions are important in most of the structural experiments [25,26]. However, strain fields and cracks are not considered in this method because numerical models for strain fields and cracks are rarely adopted in laboratories.

3. Implementation and numerical verification

A verification example, which simulates an experiment in plan, was tested to investigate the performance of the proposed online optimization hybrid simulation method. The hypothetic structure in this example is a two-pier bridge. The structural design of the bridge was modified from the four-pier bridge structural design used in a Taiwan–Canada collaborative hybrid simulation carried out in 2006 [27]. Each pier of the bridge was a double-skinned concrete-filled tubular (DSCFT) pier. The
first and the second spans of the bridge are both 40 m long, while the third span is 45 m, resulting in an asymmetric structure. Due to the asymmetric nature, the responses of the two piers are not identical. The numerical simulation was conducted using OpenSees, an open source object-oriented structural analysis system designed for earthquake engineering research [16]. The numerical model of this bridge was built using a graphical user interface of OpenSees, the so-called OpenSees Navigator [28], as shown in Fig. 5.

The nonlinear response of the aforementioned bridge is heavily dependent on the structural behavior of the bridge piers, while the section design of the DSCFT piers is shown in Fig. 6. The foundations of the piers are assumed to be rigid and are perfectly fixed into the ground. It is assumed that the bridge deck remains in its linear elastic stage throughout the ground motion. It is simulated by linear–elastic beam elements, with corresponding sectional properties listed in Table 1.

The actual dynamic interaction between the deck and piers is complicated, thus, it has been simplified in this work. Each connection between the bridge deck and a pier is simplified as a bilateral hinge. As shown in Fig. 7a, a pier cap may transfer moments to the top of a pier due to deck bending and torsion. However, in a structural laboratory, it is not easy to apply moments to the top of a pier specimen without special experimental devices. At present, only a few large-scale laboratories possess these types of special experimental devices, such as the Multi-Axial Subassemblage Testing (MAST) [29] at the University of Minnesota, USA, and the Loading and Boundary Condition Boxes (LBCBs) [30] at the University of Illinois at Urbana-Champaign, USA. To simplify the verification example in this research, it was assumed that the main distress of the pier is due to the axial and lateral forces applied at the top of each pier. The distress induced by the top moments and torsion are ignored. In an experiment, the lateral forces are applied by hydraulic actuators, while the axial forces are typically applied through vertical pre-tensioned bars, as shown in Fig. 7b. The numerical model of the pier is hinged at the top of the pier, as shown in Fig. 7c. Based on the aforementioned simplifications, a preliminary modal analysis estimated that the first two natural periods of the verification example bridge structure are 1.4 s and 0.9 s, respectively.

While the dynamic interactions between the deck and piers, soil-structure interaction, and spatial variability of ground motions for long span bridges are neglected due to the limitation of facility capability, these issues can be crucial for the

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**Fig. 5.** Bridge structure as tested in this paper.

**Fig. 6.** Section design of the DSCFT pier.

**Table 1**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area ($A$)</td>
<td>2.573 m²</td>
</tr>
<tr>
<td>Modulus of elasticity ($E$)</td>
<td>200 GPa</td>
</tr>
<tr>
<td>Shear modulus ($G$)</td>
<td>74.4 GPa</td>
</tr>
<tr>
<td>Torsion of area ($J$)</td>
<td>5.96 m⁴</td>
</tr>
<tr>
<td>Moment of area ($I_y$)</td>
<td>0.3267 m⁴</td>
</tr>
<tr>
<td>Moment of area ($I_z$)</td>
<td>3.70 m⁴</td>
</tr>
</tbody>
</table>
dynamic response of bridges [31] and further investigation is needed. When these issues need to be considered, it is suggested to adopt experimental facilities that are capable of applying multi-axial forces and moments on the foundations and the top of each pier. In addition, a numerical engine that supports multiple point constraints (MPCs) (e.g., OpenSees or ABAQUS) is also required for multiple ground motions analysis.

The ground motion measured by the seismic station known as CHY024 during the Chi-Chi earthquake, one of the best recorded major earthquakes in history [32], was selected for this test. In this verification example, the ground motion is an amplified version of the CHY024 ground motion, and its peak ground acceleration is 8.8 m/s². A hybrid simulation platform, ISEEdb [33], which uses OpenSees [16] as the numerical simulation module, was employed in the tests. The duration of the aforementioned tri-axial ground motion was 40 s. The time increment of the hybrid simulation was 0.02 s, with a total of 2000 time steps in each simulation (40/0.02 = 2000). The classical Rayleigh's method was adopted based on the first two natural modes of the structure with a damping ratio of 0.03.

The verification example is solely computational, i.e., with no physical experiments were conducted. In order to minimize the budget used during the early stages of this research, the physically simulated experimental sub-structure is simulated by a sophisticated numerical model, which is a fiber-sectioned beam-column element. The numerically simulated experimental sub-structure of the bridge structure is simulated by a simple numerical model, which is composed of a rigid beam-column element with a bi-lateral rotational spring. The rotational spring is based on a modified Giuffré–Menegotto–Pinto (GMP) model implemented in OpenSees, which can be seen as a bilinear material model with a smooth transition stage between elasticity and plasticity. The sophisticated model was used to simulate the physically simulated experimental sub-structure, since the physical specimen behavior observed in laboratories is usually more complicated than any numerical model’s response.

The numerical test includes the following analysis cases (as shown in Fig. 8):

1. Case A: This case simulates the verification example using a conventional hybrid simulation. Pier 1 is simulated by a sophisticated numerical model, representing the physically simulated experimental sub-structure. Pier 2 is simulated by a simple numerical model, representing the numerically simulated experimental sub-structure.

2. Case B: This case simulates the response of the verification example using an online optimization hybrid simulation. As Case A, Pier 1 represents the physically simulated “experimental” sub-structure which is simulated by a sophisticated model and Pier 2 is simulated by a simple numerical model in which the material parameters are updated during the online optimization hybrid simulation. This analysis case represents the usage of the proposed online optimization hybrid simulation method.
Case C: Case C is referred to as a complete physically simulated hybrid simulation, which simulates the response of the verification example that both piers were physically simulated experimental sub-structures in laboratories, and were simulated by the sophisticated model. This case represents a case where the budget, experimental facilities and resources allow both piers to be physically constructed and simulated in laboratories. The response obtained in this case was regarded as the most accurate of the verification example.

In the verification example, the sophisticated model was a fiber-sectioned beam-column element. The DSCFT section of the sophisticated model was divided into patches, each called a ‘fiber.’ Each fiber has its own stress/strain status, simulated by a uni-axial material object. In the fiber-section model used in the verification example, each tube in a section was divided into 18 patches along the circular axis and one patch along the radius axis, while the in-filled concrete was divided into 18 and 6 patches along the circular and radius axes respectively, as shown in Fig. 9. A DSCFT pier model had five integration points; thus, there were a total of 720 patches \((18 \times 1 + 18 \times 6 + 18 \times 1) \times 5 = 720\). The tangent stiffness and the resisting forces of the numerical piers were calculated based on the integration of the behavior of the 720 uni-axial material objects. The material properties of the fiber-section model were based on the concrete and steel material tests completed in the aforementioned Taiwan–Canada transnational experiment and empirical estimation, in which some of the basic material properties of the steel and concrete were tested. These results are listed in Table 2. Other parameters used in the numerical models of the materials, listed in Table 3, were estimated based on suggestions in empirical formulations [34]. The sophisticated model was applied on Pier 1 in cases A and B, and on both piers in Case C. The sophisticated model in Case C basically ignores sectional warping in a pier. For highly nonlinear confined concrete pier simulation, numerical models that considered sectional warping should be adopted.

In the verification example, the simple model was simulated by a rigid bar with a rotational spring at the bottom of the pier, as shown in Fig. 10a. The bilateral rotational spring was composed of two independent uni-axial material objects of the GMP model. The GMP model is similar to a bilinear model [35] with a smooth transition stage between elasticity and plasticity. Therefore, the structural behavior of the simple model was represented by only two uni-axial material objects. This model was much simpler than the fiber-section model aforementioned, which consisted of 720 material objects. It is believed that the nonlinear structural behavior of a real specimen is generally more sophisticated compared to that of any numerical model, which is commonly controlled by a small number of material parameters. Hence, this is the main reason that the fiber-sectioned beam-column element (i.e., a sophisticated model) is used to simulate a physically simulated experimental sub-structure, while the rotational spring model (i.e., a simple model) is used to simulate the numerically simulated experimental sub-structure. An example of the GMP model is shown in Fig. 10b. The material parameters of the rotational model are based on empirical material parameters [33] and a previous experimental study of the DSCFT pier [34].

![Fiber section model of a DSCFT pier.](image)

<table>
<thead>
<tr>
<th>Steel</th>
<th>Concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yield stress of exterior tube steel</td>
<td>283 MPa</td>
</tr>
<tr>
<td>Yield stress of interior tube steel</td>
<td>328 MPa</td>
</tr>
</tbody>
</table>
material parameters of the rotational spring model are listed in Table 4. The rotational spring is used on Pier 2 in Cases A and B, while Table 5 presents the numerical models used for the piers in these testing cases.

The online optimization hybrid simulation of the bridge structure (i.e., Case B) was completed based on a modified version of ISEEdb [32]. There were five software modules in the test: Analysis Engine, Data Center, Physically simulated experimental sub-structure (Pier 1), Online Optimization and Numerically simulated experimental sub-structure (Pier 2). Fig. 11 presents the time sequence diagram of the online optimization verification test. The five modules are described as follows:

(1) Analysis engine: The analysis engine is based on a modified version of the OpenSees program. In this test, a 40-s triaxial ground motion (CHY024 of Chi-Chi earthquake) with a time increment of 0.02 s is used as the ground motion input, and no iteration is done within each time step. During each time increment, for example time \( t \), the Analysis Engine sends the boundary nodal displacements of the physically simulated experimental sub-structure (Pier 1) \( \{u\}^{E,1}_t \), and the numerically simulated experimental sub-structure (Pier 2) \( \{u\}^{E,2}_t \) to the Data Center. The Analysis Engine then obtains their resisting forces, \( \{f\}^{E,1}_t \) and \( \{f\}^{E,2}_t \) respectively, as soon as they are available in the Data Center, completing the time integration calculation of the time step.

(2) Data Center: The Data Center is exactly the same as that used in the previous Internet-based hybrid simulation [32]. To access the Data Center through MATLAB, which is used in the physically simulated experimental sub-structure module, the online optimization model, and the numerically simulated experimental sub-structure module as described below, a set of MATLAB-based Data Center access toolkits were implemented in this work.

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### Table 3

<table>
<thead>
<tr>
<th>Material model</th>
<th>Concrete Material model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity</td>
<td>Strain at ultimate strength (fc')</td>
</tr>
<tr>
<td>Hardening ratio</td>
<td>Crushing stress</td>
</tr>
<tr>
<td>Transition R0</td>
<td>Crushing strain</td>
</tr>
<tr>
<td>Transition R1</td>
<td>Unloading slope ratio</td>
</tr>
<tr>
<td>Transition R2</td>
<td>Tensile strength(^a)</td>
</tr>
<tr>
<td>Hardening parameters</td>
<td>Tensile softening slope</td>
</tr>
</tbody>
</table>

\( a \) Estimated based on empirical formulations [26].

---

### Table 4

<table>
<thead>
<tr>
<th>Material property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotational stiffness ( (K_r) )(^a)</td>
<td>18500 MN m/rad</td>
</tr>
<tr>
<td>Ultimate moment ( (M_u) )(^a)</td>
<td>146.6 MN m</td>
</tr>
<tr>
<td>Hardening ratio(^b)</td>
<td>0.0</td>
</tr>
<tr>
<td>Transition parameter ( R_0 )(^b)</td>
<td>10.0</td>
</tr>
<tr>
<td>Transition parameter ( R_1 )(^b)</td>
<td>0.925</td>
</tr>
<tr>
<td>Transition parameter ( R_2 )(^b)</td>
<td>0.15</td>
</tr>
</tbody>
</table>

\( a \) Estimated based on experimental empirical formulations [26].

\( b \) Estimated based on suggestions by OpenSees.
Physically simulated experimental sub-structure (Pier 1): as aforementioned, Pier 1 was simulated by a fiber-section model. This module performed a displacement controlled nonlinear static analysis of a single displacement-based beam-column element with the fiber section using OpenSees. It received $f_{\text{u}}^E$ from the Data Center, combined it with the displacement history $f_{\text{u}}^E$ from the Data Center, applied the displacement history onto the single-element sub-structure, analyzed the resisting forces $f_{\text{r}}^E$, and sent it back to the Data Center. To simulate the measured noises of resisting forces, random numbers up to 1 kN were added into the resisting forces. In a real experiment, the measured forces include noises or measurement error that may be induced by specimen imperfection, control inaccuracy, and experimental facilities deficiencies. Note that the error may even be amplified in a small scale experiment. For example, in a 1/5th-scale experiment, the noise or error of the measured force would be amplified by 25 times when it is converted back to the full-scale model. To prevent significant error propagation in a hybrid simulation, most researchers choose experimental apparatus in good condition to carry out hybrid simulations.

In the online optimization module, the rotational stiffness ($K_r$), ultimate moment ($M_u$), and the first transition parameter ($R_0$) were analyzed during the online optimization hybrid simulation. It should be noted that the online optimization process was not active during the entire online optimization hybrid simulation. Instead, the online optimization was activated only when all the threshold criteria of the material parameters (see Table 6) are satisfied. In addition, a limit on the maximum and a minimum values for each parameter. If any optimized parameter exceeds the upper limit, it is set to the maximum value, while similarly small values are bounded by the lower limits. The criteria and the boundary limits of each parameter are listed in Table 6.

An unconstrained nonlinear optimization based on the Simplex method [18] implemented in MATLAB was adopted in the online optimization module. The optimization module utilizes an iterative process that finds a set of parameters which re-
results in a relatively minimal difference between the hysteresis loops of the physically simulated experimental sub-structure (Pier 1) and its numerical model (see Eq. (2)). Each iteration triggers a displacement controlled static nonlinear analysis, and an optimization process requires many iterations in order to obtain an optimal solution. Therefore, the online optimization process is time consuming. In order to reduce the computational cost, the maximum number of iterations for each optimization process is set to 30. The online optimization module then returns a set of parameters \( f_{\text{par}} \), \( g_{\text{shared}} \), and \( t_{\text{t}} \) (5). The numerically simulated experimental sub-structure module simulates the behavior of Pier 2. In each time step, it retrieves the optimized parameters \( f_{\text{par}} \), \( g_{\text{shared}} \), and \( t_{\text{t}} \) from the Data Center, updates its numerical model, and re-runs a displacement-controlled nonlinear static analysis.

In the examined verification example, it was found that the optimization process provided material parameters \( K_r \), \( M_u \), and \( R_0 \) that led to better matching between the physically simulated “experimental” sub-structure loop (i.e., \( \{ u \}_t^{E,1} \) and \( \{ r \}_t^{E,1} \)) and its numerical model loop (i.e., \( \{ u \}_t^{n,1} \) and \( \{ n \}_t^{n,1} \)). Fig. 12 presents an example of the development of the hysteresis loop \( \{ u \}_t^{n,1} \) and \( \{ n \}_t^{n,1} \) (the solid curve) that approached very closely the \( \{ u \}_t^{E,1} \) and \( \{ r \}_t^{E,1} \) loop (the dashed curve) after running 15 online optimization iterations at a time step of \( t = 9.10 \) s, resulting in better consistency among the results of the numerical and “experimental” bridge piers.

The results of the three cases in the verification example showed that, a hybrid simulation using a conventional method (Case A), did not match the displacement response of the complete physically simulated test (Case C) as shown in Fig. 13a and b that depict the displacement along the \( X \) axis (\( U_x \)) and the \( Y \) axis (\( U_y \)). In contrast, \( U_x \) and \( U_y \) displacement time histories when the online optimization method was used (Case B) are significantly closer to those of the complete physically simulated test (Case C), as shown in Fig. 13c and d. The comparison of the hysteresis loops present similar results, as shown in Fig. 14a–d, and verify that Case B results are closer to Case C. As mentioned above, the complete physically simulated test, or Case C, used a sophisticated numerical model to simulate a hybrid simulation with two physically simulated piers, and is

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**Table 6**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Threshold criteria</th>
<th>Upper bounds</th>
<th>Lower bounds</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotational stiffness ((K_r))</td>
<td>Max. drift ratio &gt;0.1%, Max. drift ratio &lt; 2%, and ( c_t &lt; 0.01 )</td>
<td>( 2 \times (\text{initial } K_r)^b )</td>
<td>( 0.5 \times (\text{initial } K_r) )</td>
</tr>
<tr>
<td>Ultimate moment ((M_u))</td>
<td>Max. drift ratio &gt; 1%, Max. drift ratio &lt; 2%, and ( c_t &lt; 0.01 )</td>
<td>( 2 \times (\text{initial } M_u)^b )</td>
<td>( 0.5 \times (\text{initial } M_u) )</td>
</tr>
<tr>
<td>First transition parameter ((R_0))</td>
<td>Max. drift ratio &gt; 1%, Max. drift ratio &lt; 2%, and ( c_t &lt; 0.01 )</td>
<td>( 20^c )</td>
<td>( 10^c )</td>
</tr>
</tbody>
</table>

\(^a\) Drift ratio is the ratio of the displacement to the pier height.

\(^b\) The estimated values for \( K_r \) and \( M_u \) are based on Lin and Tsai [34].

\(^c\) The maximum and minimum values of \( R_0 \) are based on OpenSees’s suggestions.

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![Fig. 12](image-url) Development of hysteresis loops at \( t = 9.10 \) s showing consistency between numerical (labeled Optmz.) and “experimental” (labeled “Exp.”) models after optimization process.
thus is regarded as the “correct” results. The numerical model used in Case A overestimated the lateral stiffness (dominated by the material parameter $K_r$) and the ultimate strength of Pier 2 (dominated by the material parameter $M_u$). The displacements obtained in Case A are generally smaller, while the maximal resisting forces are larger than those of Case C. The response of Case A represent the results that one would obtain if a conventional hybrid simulation of the verification example.

Fig. 13. Pier 2 displacement time histories in the verification example.

Fig. 14. Pier 2 hysteresis loops of the verification example.
A structural test was carried out. The error of the absolute peak displacement in Case A is approximately equal to 30% (i.e., (1.83–1.28%)/1.83%).

In Case B, the proposed online optimization method generally matched the response of Case C, despite the fact that the initial material parameters being poorly estimated (as in Case A). The rotational stiffness \( K_r \) was updated to match the response of Pier 1 as soon as the online optimization was activated (when the drift ratio reached 0.1%, see the threshold criteria of the optimization of each parameter in Table 6), as shown in Fig. 15a. The ultimate moment \( M_u \) and the transition parameter \( R_0 \) were updated later on. Fig. 15b presents the change of \( M_u \). It is obvious that \( R_0 \) changed slightly between 10.0 and 10.1. The online optimization was deactivated when the drift ratio of displacements reached the threshold of 2%. The authors believe that the online optimization process should only be applied at the early stages of a hybrid simulation, since changing numerical models when highly non-linear simulations have been performed may lead to significantly inconsistent strain energy. The error of the absolute peak displacement in Case B is 2% (i.e., (1.83–1.79%)/1.83%), much lower than the error in Case A.

4. Conclusions

An online optimization hybrid simulation method is proposed and tested in this work. This method aims to improve the accuracy of a hybrid simulation of a bridge with multiple identical or similar piers, where only one or a few of them are simulated physically. Through a multi-variable nonlinear optimization process, material parameters of the numerically simulated piers can be updated to match those that are experimentally simulated within a hybrid framework.

A numerically simulated verification example was also performed, and three different hybrid simulation methods were implemented: the conventional method, the online optimization method, and the complete physically simulated test method. The physically simulated “experimental” sub-structure was numerically simulated by a sophisticated fiber-section model, while the numerically simulated experimental sub-structure was simulated by a simple rotational spring model. The results of this verification example confirmed that the online optimization method is capable of improving the accuracy of a hybrid simulation with multiple identical sub-structures. The error of absolute peak displacement in this example was reduced from 30% to 2%. As compared to a complete physically simulated test, which is normally impractical due to limited research funding and experimental resources, an online optimization hybrid simulation method is an alternative and cost-efficient option.

Acknowledgement

The financial supports from the National Center for Research on Earthquake Engineering in Taiwan through research project NCREE-06098A5313 and National Science Council in Taiwan through project NSC-99-2211-E-027-042-MY3 are gratefully acknowledged.

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