A PROCEDURE FOR COMBINING VERTICAL AND HORIZONTAL SEISMIC ACTION EFFECTS

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A PROCEDURE FOR COMBINING VERTICAL AND HORIZONTAL SEISMIC ACTION EFFECTS

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The vertical component of earthquake ground motion has generally been neglected in the earthquake-resistant design of structures. This is gradually changing due to the increase in near-source records obtained recently, coupled with field observations confirming the possible destructive effect of high vertical vibrations.

In this paper, simple procedures are suggested for assessing the significance of vertical ground motion, indicating when it should be included in the determination of seismic actions on buildings. Proposals are made for the calculation of elastic and inelastic vertical periods of vibration incorporating the effects of vertical and horizontal motion amplitude and the cross-coupling between the two vibration periods. Simplified analysis may then be used to evaluate realistic vertical forces by employing the vertical period of vibration with pertinent spectra without resorting to inelastic dynamic analysis.

Finally, a procedure is suggested for combining vertical and horizontal seismic action effects which accounts for the likelihood of coincidence, or otherwise, of peak response in the two directions.

Keywords: vertical component of ground motion, code-based procedure, vertical periods of vibration, vertical response spectrum, inelastic dynamic analyses.

1. Background

1.1. Vertical component of ground motion

The vertical component of ground motion is mainly associated with the arrival of vertically propagating compressive $P$-waves, whilst secondary, shear $S$-waves are the main cause of horizontal components. The wavelength of $P$-waves is shorter than that of $S$-waves, which means that the former are associated with higher frequencies.

Near the source of an earthquake, ground motion is characterised mainly by source spectra, only modified by rupture dynamics. The $P$-wave spectrum has a higher corner frequency than that of $S$-wave. $P$ and $S$ corner frequencies gradually shift to lower frequencies as waves propagate away from the source due to the
differentially stronger attenuation of higher frequencies. Consequently, the vertical motion will be modified at a faster rate. The behaviour of these two components of ground motion is often characterised by the $V/H$ peak ground acceleration ratio.

Normally, the vertical component of ground motion has a lower energy content than the horizontal component over the frequency range. However, it tends to have all its energy concentrated in a narrow, high frequency band, which can prove damaging to engineering structures with vertical periods within this range.

It has been reported by many including Bozorgnia and Niazi [1991; 1992] and Abrahamson and Litehiser [1989] that the $V/H$ ratio on many occasions has been greater than 1.0 near the source of an earthquake. The 1994 Northridge earthquake in California produced $V/H$ ratios as high as 1.79 and the Hyogo-ken Nanbu earthquake of 1995 also exhibited peak $V/H$ ratios of up to 1.63.

There is strong evidence that the vertical component (assuming it is due to $P$-waves) is not strongly influenced by nonlinear site effects in the way that horizontal $S$-waves are, which would provide a reasonable explanation for the following observation. During the 1995 Kobe earthquake, liquefaction at the vertical array at Port Island caused an abrupt reduction in the horizontal ground shaking, but the vertical motion continued to be amplified through the liquefied layer.

Vertically propagating dilational waves are clearly amplified in a manner identical to that of vertically propagating shear waves. Consequently, the vertical component of motion can be linearly amplified from bedrock to the surface to very high levels, leading to the widely observed high $V/H$ ratios near the source.

1.2. Consequences of high vertical acceleration

Damage consistent with a high level of vertical acceleration was observed in the 1994 Northridge earthquake. The report from EERI [1995] highlighted cases of brittle failure induced by direct compression, or by reduction in shear strength and ductility due to variation in axial forces arising from the vertical motion.

For first mode vertical response, the reduction in axial force in columns or walls is more significant for higher storeys, since it represents a larger relative change in the pre-existing static axial load. Interior columns were shown to be more vulnerable, and vertical oscillations of slabs at their natural period caused considerable damage. Steel showed evidence of low cycle fatigue, which would have been contributed to by the vertical beam vibrations [Broderick et al., 1994].

During the Hyogo-ken Nanbu earthquake of 1995 buildings exhibited an amplification of the vertical acceleration of more than 2 times [AIJ, 1995]. This can be attributed to low damping in the vertical direction due to the absence of an efficient energy dissipating mechanism. A quasi-resonant response was observed because the buildings are very stiff in the vertical direction, corresponding to the high frequency pulses from vertical motion, coupled with low damping.
1.3. Current design philosophies

Many codes suggest scaling a single spectral shape, originally derived for horizontal components to deal with vertical earthquake motion. This implies that both components of motion have the same frequency content, which is clearly not the case.

This procedure was originally proposed by Newmark et al. [1973] and has since been widely used. It was suggested that the average peak vertical-to-horizontal spectral ratio could be taken as 2/3. This implies that the vertical-to-horizontal ratio is also 2/3 assuming constant amplification. Recent studies by Abrahamson and Litchiner [1989] and Ambraseys and Simpson [1996] confirm that the 2/3 rule is unreasonable. Evidence from the Loma Prieta earthquake of 1989, the Northridge earthquake of 1994 and the Kobe earthquake of 1995 confirm this, all with \( \frac{V}{H} \) ratios well in exceedence of 1.0 in the near-field. It is clear that this ratio is magnitude, distance and frequency dependent and should be a variable in code design. The new version of EC8 (prEN) has recognised this and European practice will now have independent vertical and horizontal spectral shapes.

2. Calculation Procedure

To advance the inclusion of the vertical component in seismic force calculation a new procedure has been developed, as shown in the flow-chart of Fig. 1. The vertical peak ground acceleration is established in Sec. 3, through vertical attenuation relationships or from consideration of the \( \frac{V}{H} \) ratio at a distance from the source. This paper demonstrates the relationship between the time interval between peak vertical and peak horizontal acceleration and the distance from source. In Sec. 5, these two parameters are used to establish multiplication factors to account for the amplitude and interaction between the two ground motions.

In parallel, simple methods for estimating the elastic vertical period of vibration are suggested in Sec. 4. Estimation of the inelastic vertical period is achieved by considering the above-mentioned amplitudes and interactions between the vertical and horizontal motions. The design forces due to vertical motion may be calculated by use of a vertical response spectrum in Sec. 6.1. Finally, methods are suggested for combining the vertical and horizontal seismic action effects in Sec. 6.2.

3. Vertical Ground Motion Parameters

Information for parts 1(a) and 2(a) of the procedure outlined in Fig. 1 is addressed in this section. Initially an engineering seismology, site-specific study is required. The time interval between peak horizontal and peak vertical accelerations, \( t_p \), is a function of distance from source and can be obtained from Sec. 3.1 below. The value of vertical peak ground acceleration can be estimated by use of the \( \frac{V}{H} \) ratio graph (Fig. 2) or an attenuation relationship similar to Eq. (1).
3.1. Phasing of peak vertical and peak horizontal ground motion

The arrival time (and hence coincidence, or otherwise) of peak vertical and peak horizontal ground motion is an important parameter which will have a significant effect on the level of forces to which a structure is subjected. However, it is a parameter that has not been studied in the context of vertical and horizontal attenuation.

It is likely that this parameter is magnitude- and distance-dependent, similar to the $V/H$ ratio. However, site conditions, travel path and depth of source must play an important role too. Knowledge of this parameter would be useful to the structural engineer since it enables an assessment of the extent of interaction between
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Fig. 2. Minimum time interval between peak vertical and horizontal accelerations plotted as a function of magnitude and distance for the 1979 Imperial Valley earthquake ($M_w = 6.5$) and the 1984 Morgan Hill earthquake ($M_w = 6.3$).

horizontal and vertical motion. Obtaining data for this parameter is difficult since it requires a number of records from the same event, at various distances, with the same site conditions, but not affected by azimuth.

Considering the above, investigations were conducted with records from the Imperial Valley earthquake of 1979 with a magnitude $M_w = 6.5$ and the Morgan Hill earthquake of 1984 with magnitude $M_w = 6.3$. The main purpose of the study was to observe the variation in time between peak vertical and peak horizontal accelerations as a function of distance. A plot of minimum time between peaks (positive or negative) versus distance is shown in Fig. 2. This was plotted removing records with peak accelerations less than 0.10 g. The results indicate that the time interval between peak horizontal and peak vertical acceleration increases with distance from source and is influenced by earthquake magnitude. It also indicates that horizontal and vertical peak ground motion can be coincident when the distance from source is less than 5 km.

3.2. Vertical-to-horizontal peak ground acceleration ratio

Elnashai and Papazoglu [1997] produced the graphs in Fig. 3 indicating vertical-to-horizontal peak ground acceleration ratios for varying magnitude and distance. Part of the data, from the Imperial College data bank, was for near-field earthquakes with a magnitude $M_s > 5.0$, whilst the remainder was from Borsognia and Niazi [1993]. This graph may be consulted to give values of vertical acceleration if the horizontal acceleration, magnitude and the distance from the source are known.

Alternatively, the vertical peak ground acceleration can be estimated by use of an attenuation relationship, such as that from Ambraseys and Simpsons [1996]
European study in Eq. (1).

\[ \log(a_v) = -1.74 + 0.273M_s - 0.954 \log(r) \]
\[ + 0.076S_A + 0.058S_S \]  

(1)

The standard deviation in the above equation is 0.26 and \( r^2 = d^2 + 4.7^2 \). For rock the constants \( S_A \) and \( S_S \) are 0, for stiff soil \( S_A \) is 1.0 and \( S_S \) is 0, and for soft soil \( S_S \) is 1.0 and \( S_A \) is 0.

The writers' study of the Imperial Valley and Morgan Hill earthquake records confirmed that the \( V/H \) ratio is greater than 1.0 in the very near field, but diminishes to less than half with distance, as shown in Fig. 4.

Greater \( V/H \) ratios were observed for the higher magnitude (Imperial Valley) records near the source, which agrees with the results of studies by Borzognia and Niazi [1993], Abrahamson and Litehiser [1989] and Elnashai and Papazoglou [1997]. However, these ratios are also shown to decrease more rapidly than those of lower magnitude. This penalty on structures close to the source is mitigated by the bonus of reduced \( V/H \) to much less than 2/3 further away.

3.3. Remarks

Whilst the data under investigation is limited, both sets of records do provide mutual confirmation. The investigations highlight the importance of considering the vertical component of ground motion and the interaction between the vertical and horizontal motion, particularly for distances less than 25 km. It also emphasises the significant effect of magnitude and distance on time interval between peak vertical and horizontal ground accelerations and the \( V/H \) ratio.
4. Estimation of Vertical Response Periods

4.1. Test structures

The computer models used for the initial analyses were based on an actual 10.8 m high, 4-storey reinforced concrete frame of typical 1960s European construction. The frame consists of two 5-metre bays and one 2.5-metre bay investigated by two European collaborative networks (Innovative Concepts for Seismic Design of New and Existing Structures “ICONS” and European Consortium of Earthquake Shaking Tables “ECOEST 11”). The test models were constructed at full-scale, one a bare-frame model, the other with masonry infills. Both were tested at the EU Joint Research Centre in Italy.

4.2. Rayleigh’s method

Buildings are generally stiffer in the vertical than the horizontal direction. While periods of vibration can be determined by the solution of the eigenvalue problem, it is desirable to have the ability to estimate quickly using approximate formulae. Rayleigh’s method is recommended by many codes for horizontal period calculation, and is given by Eq. (2).

$$T = 2\pi \sqrt{\frac{\sum_{i=1}^{n} m_i u_i^2}{\sum_{i=1}^{n} F_i u_i}}$$

where $F_i$ is the seismic lateral force at level $i$; $m_i$ is the mass assigned to level $i$; $u_i$ is the static lateral deflection at level $i$ due to $F_i$.

Application of Rayleigh’s formula for vertical vibrations for the ICONS frame yields a value of 0.07 seconds, compared to 0.25 seconds for the horizontal period.
However, due to the pre-load in the vertical direction, the characteristics of vertical vibrations are not symmetric about the unloaded position. This implies that for vertical motion there are two distinct periods for the "up" and "down" directions. The above provides an estimate of the vertical "down" period. In most cases it is acceptable to assume this period, however for situations when the input excitation matches the "up" period or the structure has exceptionally low gravity loads the "up" period would be significant.

4.3. Eigenvalue analysis

The next step in the refining process is to conduct an eigenvalue analysis. This was carried out to determine the elastic periods of the structure chosen for use in this study. It was found that the fundamental horizontal period of vibration was 0.7 s, which is particularly long considering the height of the structure, illustrating that it is quite flexible. The first four vertical periods of vibration were concentrated between 0.052 s and 0.069 s, thus agreeing with the Rayleigh-derived value. The variation in column stiffness across the width of the structure accounts for the uneven modal displacement and close spacing of the modes observed in Fig. 5.

4.4. Inelastic dynamic analyses

Inelastic dynamic analyses were subsequently undertaken using INDYAS [Elnashai, Pinho and Antoniou, 2000]. The program uses the Hilbert–Hughes–Taylor integration algorithm, which exhibits low numerical damping for lower modes and high damping for higher modes. It has been verified in numerous studies on this and other structures.

![Fig. 5](https://via.placeholder.com/150)

Fig. 5. Four highest elastic vertical modes of vibration from eigenvalue analysis.
4.4.1. Earthquake records

The criteria for selection of the analysis records included high vertical component, near source, with a frequency range to excite the periods of vibration of the structure (both horizontally and vertically). The response spectra of each record were examined carefully. Of the few records that matched these criteria, two were chosen; a record from the 1983 Coalinga earthquake and a record from the 1979 Imperial Valley earthquake. The former exhibits a large duration vertical component (Fig. 6)

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![Graphs showing earthquake recordings](image-url)

Fig. 6. The three components of the 1983 Coalinga earthquake record. (Note: Transverse and vertical components used in analysis.)
and the latter, in contrast, has energy concentrated in one or two sharp peaks. An exploratory study was conducted considering the two structural models and the two earthquake records. However, this was reduced to only the Coalinga record and the bare-frame model for the main part of the investigation.

4.4.2. Vertical periods of vibration

Inelastic response periods were calculated from a moving window Fourier analysis of the most significant 6-seconds of the INDYAS response. The analysis conducted with the vertical record alone gave vertical periods of vibration of 0.071, 0.082, 0.084 and 0.088 s, an average increase from the elastic periods of 1.32 times. This would be expected since the analysis accounts for cracking, which would elongate the structural periods. The horizontal periods also exhibited a considerable increase, to a maximum value of 1.5 s.

5. Results of Inelastic Dynamic Analyses

5.1. Inelastic dynamic analysis procedure

The horizontal and vertical records were applied both separately and concurrently in order to isolate the effect of each component. The horizontal records were manipulated by moving the time axis to the left or right. This was performed to study the effect of coincidence, or otherwise, of the peaks of the two component and to gauge the effect of high transverse motion on vertical period.

Coinciding the vertical and horizontal peaks to produce a severe case proved difficult, particularly due to the long duration of vertical motion exhibited by the Coalinga record (Fig. 6). The transverse record was moved in increments of 0.5 s. This gave an initial indication of the location of maxima, which were further investigated using increments of 0.1 s (approximate vertical period of vibration). This yielded results where the combination of horizontal and vertical ground motions were additive, although not necessarily a maximum.

The above was conducted to determine the effect of the horizontal motion on the vertical period of vibration with respect to time interval between the occurrence of horizontal and vertical peak amplitudes. In addition, the amplitude of the horizontal and vertical motion is also a significant factor. Hence, these were altered independently to gauge the effect of each. The vertical amplitude was factored in order to give $V/H$ ratios of 0.75, 1.0 and 1.25, which corresponded to an increase in the vertical acceleration of 1.5, 2.0 and 2.5 times the original record amplitude. Subsequently, further cases were considered for lower values of acceleration, corresponding to 0.25 and 0.5 times the original amplitude. The horizontal record was factored in a number of positions by values of 0.25, 0.5, 0.75, 1.25, and 1.50. This was to provide further verification for the results and to assess the combined effect of amplitude and time between peak horizontal and peak vertical accelerations.
5.2. Effect of vertical amplitude on vertical period of vibration

The effect of varying the amplitude of the vertical component was assessed by comparing the vertical elastic period obtained through eigenvalue analysis to the results from the original and scaled records. The results showed that a large increase in peak ground acceleration was required before significant period elongation occurred. It is likely that this was influenced by the frequency of input motion relative to the structural vertical periods of vibration. The input frequencies of the Coalinga record peak at 10 Hz (0.1 s), but the fundamental vertical period of the structure is 0.088 (inelastic). The period elongated quickly once it approached 0.1 s (approximate acceleration of 0.55 g), because the response increased. However, difficulties in identifying the vertical periods at 0.741 g make the results less conclusive than otherwise. Therefore, further refinement is required in the future. Table 1 gives the results of this analysis, whilst Fig. 7 depicts the vertical amplitude factor to be applied to the elastic vertical periods to account for the level of vertical acceleration.

5.3. Effect of horizontal motion on vertical period of vibration

The vertical and horizontal components of the earthquake were applied concurrently. The horizontal component was moved along the time axis in order to achieve varying time intervals between peak vertical and peak horizontal accelerations. The Fourier spectrum of the response under vertical motion was compared to each of the spectra from the combined cases. This was achieved by tracing the initial spectrum and methodically comparing this to the others obtained to gauge any shift in the dominant vertical periods. Four periods were found which corresponded to the elastic analysis. These were monitored throughout and were averaged, then normalised to the inelastic periods achieved under the vertical record alone to provide the interaction factor $E_v$. This factor depends on the position of the horizontal record with respect to the vertical record, which can be characterised by considering the time between peaks in the vertical and horizontal accelerations (the lesser of

<table>
<thead>
<tr>
<th>Vertical Acceleration (%g)</th>
<th>$T_v1$</th>
<th>$T_v2$</th>
<th>$T_v3$</th>
<th>$T_v4$</th>
<th>Vertical Amplitude Factor $E_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.150</td>
<td>0.088</td>
<td>0.084</td>
<td>0.082</td>
<td>0.071</td>
<td>1.32</td>
</tr>
<tr>
<td>0.300</td>
<td>0.088</td>
<td>0.084</td>
<td>0.082</td>
<td>0.071</td>
<td>1.32</td>
</tr>
<tr>
<td>0.444</td>
<td>0.088</td>
<td>0.084</td>
<td>0.082</td>
<td>0.071</td>
<td>1.32</td>
</tr>
<tr>
<td>0.591</td>
<td>0.089</td>
<td>0.086</td>
<td>0.083</td>
<td>0.072</td>
<td>1.34</td>
</tr>
<tr>
<td>0.741</td>
<td>0.101</td>
<td></td>
<td></td>
<td></td>
<td>1.60</td>
</tr>
</tbody>
</table>
positive and negative, or the most significant of the two when considered in conjunction with the amplitude).

It was observed that the horizontal ground motion considerably affected the vertical period of vibration, as indicated in Table 2.

A maximum interaction effect between horizontal and vertical motion occurred at time interval between peaks of $\leq 0.5$ s, and showed no interaction effect at intervals $> 4.0$ s. This was characterised by an increase in the vertical period of up to $1.26$ times. However, values have been averaged across the bands above as this gave the most appropriate representation of the results.

5.4. Effect of horizontal amplitude on vertical period of vibration

The amplitude of the horizontal motion, as shown in Table 3, also affected the vertical period. However, its effect is further dictated by the position of the

Table 2. Coupling factor $E_I$ as a function of time interval between peak vertical and peak horizontal accelerations.

<table>
<thead>
<tr>
<th>Time interval ($t_p$) between peaks (seconds)</th>
<th>Interaction factor $E_I$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_p \leq 2.0$</td>
<td>1.21</td>
</tr>
<tr>
<td>$2.0 &lt; t_p \leq 4.0$</td>
<td>1.06</td>
</tr>
<tr>
<td>$t_p &gt; 4.0$</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Table 3. Main vertical periods of vibration for various amplitudes of horizontal acceleration (missing entries indicate difficulties in identifying reliable periods in the Fourier amplitude spectrum).

<table>
<thead>
<tr>
<th>Horizontal Acceleration (%g)</th>
<th>V/H Ratio</th>
<th>T_v1</th>
<th>T_v2</th>
<th>T_v3</th>
<th>T_v4</th>
<th>E_h</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.148</td>
<td>0.25</td>
<td>0.096</td>
<td>0.091</td>
<td>0.087</td>
<td>0.079</td>
<td>0.93</td>
</tr>
<tr>
<td>0.297</td>
<td>0.50</td>
<td>0.097</td>
<td>0.093</td>
<td>0.081</td>
<td>0.074</td>
<td>0.97</td>
</tr>
<tr>
<td>0.445</td>
<td>0.75</td>
<td>0.101</td>
<td>0.095</td>
<td>0.086</td>
<td>0.078</td>
<td>0.98</td>
</tr>
<tr>
<td>0.593</td>
<td>1.00</td>
<td>0.103</td>
<td>0.100</td>
<td>0.085</td>
<td>0.100</td>
<td>1.00</td>
</tr>
<tr>
<td>0.741</td>
<td>1.25</td>
<td>0.100</td>
<td>0.097</td>
<td>0.088</td>
<td>0.098</td>
<td>1.02</td>
</tr>
<tr>
<td>0.890</td>
<td>1.50</td>
<td>0.107</td>
<td>0.103</td>
<td>0.101</td>
<td>0.096</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Fig. 8. Coupling factor \( E_h \) to account for effect of amplitude of horizontal acceleration on vertical period of vibration.

horizontal peak with reference to the vertical peak. It has already been shown that the horizontal record has no influence when the peaks are more than 4.0 s apart, therefore the amplitude of acceleration would not be relevant in these locations. Notwithstanding, it is likely that the amplitude of horizontal motion would affect the maximum time interval at which it would influence the vertical period of vibration. However, this latter hypothesis was out of the scope of this work due to time constraints, although it is thought that the effect would be quite small, as the time interval bands are wide, and the difference in amplification factors small.

Figure 8 gives a graphical representation of the values of the horizontal amplitude factor, \( E_h \). This is to be applied to the value of vertical period obtained
after the interaction effect from the horizontal motion, \( E_t \) has been considered and provided the time between peaks is less than 4.0 s.

5.5. **Effect of vertical motion on horizontal period of vibration**

The results show that there was little or no effect on the horizontal period of vibration from the vertical component of ground motion. However, this is not conclusive. In most cases considered, the vertical component did not significantly affect the horizontal response, but it is possible that a worst-case combination of vertical and horizontal motion was not achieved. The results suggest that the peak amplitude of vertical acceleration would need to be significant (in excess of \( \sim 0.6 \, g \)) before a small change is evident and still the vertical motion would need to be synchronous with the horizontal.

6. **Combination of Vertical and Horizontal Seismic Actions**

6.1. **Vertical response spectrum**

Earthquake design codes that do consider the effects of vertical ground motion generally derive the vertical response spectrum from the horizontal spectral shape, with the sole exception of EC8 prEN. Usually this is achieved by scaling the horizontal spectrum by \( 2/3 \). The current EC-8 does go a step further, but the vertical spectrum \( S_{sv} \) is still tied to the horizontal spectrum \( S_{ah} \), both in terms of shape and values.

Elnashai and Papazoglou [1997] proposed the inclusion of vertical motion in reinforced concrete design and suggested the vertical spectrum (normalised to peak vertical acceleration) of Fig. 9. It was argued that it was not justifiable to use an equivalent viscous damping of greater than 2%, and also that the spectral amplification coefficient would be higher for vertical spectral values than for horizontal. This is because firstly, vertical spectra are associated with higher frequency oscillations and hence lower damping. Secondly, energy dissipating mechanisms in the inelastic range under vertical motion are associated with localised inelasticity in limited locations, i.e. columns of one or two storeys. Therefore, hysteretic energy dissipation for vertical inelastic response is inherently lower than in the case of transverse response.

The spectral amplification \( \beta_0 \) was calculated as 3.48 corresponding to 2% damping. An expression for the damping factor \( \eta \) was proposed, as follows:

\[
\eta = \sqrt{2.72/(0.72 + \xi)}, \quad \text{where } \xi \text{ is the value of damping in } \%.
\]

The corner periods \( T_B \) and \( T_C \) of the spectrum were fixed to 0.05 s and 0.15 s.

In contrast with horizontal motion, it was found that it was unnecessary to use modified spectral shapes for varying soil classes since the effect of local geology on the predominant periods of vertical motion is minimal.
6.2. Combination procedure

Currently, for modal analysis procedures, the horizontal modal forces are summed using either the SRSS or CQC method.

The horizontal forces may be based on either the fundamental mode or a number of mode contributions. By including vertical motion, there are two sets of modal forces to be considered. A valid simplification would be to consider that all mass acts in the fundamental vertical period of vibration only, as the vertical modes tend to be very closely spaced.

It has been shown that the coincidence of peak vertical and peak horizontal motion is likely to occur very near to the source of an earthquake (< 5 km), and that there is interaction between the two until peaks separate at distances of approximately 25 km from the source. Outside this range the vertical and horizontal actions may be considered independent of each other. Indeed the vertical motion may not a significant consideration at all.

Therefore there are two boundaries; one for complete interaction, i.e. direct addition of the horizontal and vertical forces, and another for complete independence, i.e. horizontal and vertical forces treated independently, or vertical forces are ignored. The behaviour can be expressed by the stepping function in Fig. 10, emulating the interaction coupling factor $E_I$. This interaction is converted into a fraction of vertical force, $P$ that should be added to the total force due to transverse motion, as shown in Eq. (3).

$$PV + 1.0H$$  \hspace{1cm} (3)

where $P$ is the fraction of force due to vertical motion from Fig. 10.
Alternatively, an approach in line with the SRSS and CQC methods based on probability and correlation may be considered for $t_p < 4.0$ s.

\[ F = \sqrt{H^2 + V^2 + 2P(HV)} \]  

(4)

where again, $P$ is the coefficient from Fig. 10. This corresponds to part 8 of the procedure outlined in Fig. 1.

7. Verification and Application

7.1. Verification of results

The results of this study need to be verified by investigation with further records and structures. However, results from the preliminary dynamic investigation [Collier, 1999] using the Imperial Valley record and the infilled-frame structural model can be used to provide minimal validation.

For the bare-frame building subjected to the Imperial Valley record, the initial elastic values of vertical periods are applicable, but these must be increased by factor $E_v$ to account for the increased level of vertical acceleration. The vertical peak ground acceleration for this record is 0.624 g, therefore from Fig. 7 the vertical amplitude factor $E_v$ is 1.38. The time interval between peaks is 0.42 s, which, from Table 2, yields an interaction coupling factor $E_I$ of 1.21. Finally, the maximum horizontal peak ground acceleration of 0.299 g yields a value for $E_h$ of 0.957, from Fig. 8. The resulting inelastic vertical periods are 0.110, 0.101, 0.099 and 0.083 s, which are 5% in excess of the values computed from the non-linear dynamic analysis.

Examination of the results for the Coalinga record and the infilled-frame building show lower fundamental elastic periods of vibration due to the increased stiffness.
Table 4. Comparison between estimated and computed values of vertical inelastic period from preliminary analysis results.

<table>
<thead>
<tr>
<th>Record</th>
<th>Mean Elastic Vert. Period</th>
<th>Mean Vertical Period</th>
<th>Ratio (E/C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bare Frame + Imperial Valley</td>
<td>0.062 0.624 g/1.38 0.42/1.21 0.299 g/0.957</td>
<td>0.098 0.093</td>
<td>1.05</td>
</tr>
<tr>
<td>Infill Frame + Coalinga</td>
<td>0.060 0.355 g/1.32 0/1.21 0.593 g/1.0</td>
<td>0.095 0.083</td>
<td>1.14</td>
</tr>
</tbody>
</table>

of the frame from the infill panels (0.067, 0.063, 0.060 and 0.048 s). After calculating the relevant factors and multiplying the above elastic periods, the estimated inelastic vertical periods are 0.107, 0.101, 0.096 and 0.077 s. These also are higher than those recorded in analysis (0.090, 0.087, 0.084 and 0.069 s). The data and results for this and the previous calculation and summarised in Table 4.

As the figures calculated to date slightly overestimate the elongation, the elastic period should be used in circumstances where elongating the period produces lower spectral values.

7.2. Application

In terms of current code application, it is suggested that for assessment purposes the elongated inelastic period be used. For design purposes the elastic period should be used until further studies can provide an elongation parameter that is guaranteed to be conservative.

It is recognised that results from one frame under one earthquake are inadequate for general conclusions. The numerical result values given are therefore less significant than the trends observed and are subject to further verification. More importantly, the investigations highlight the significance of vertical motion and the interaction effect with the horizontal component of ground motion.

8. Closing Summary

- The vertical component of ground motion was shown to be significant and should be considered in analysis when the proposed structure is sited within approximately 25 km of an earthquake source.
- The time interval between peak vertical and peak horizontal accelerations was shown to be zero (i.e. coincident) within a radius of 5 km of an earthquake source. The interaction between has some significance within a radius of 25 km. This is magnitude-dependent.
The \( V/H \) ratio was confirmed to be > 1.0 within a 5 km radius of earthquake source, > 2/3 within 25 km radius and dependent on earthquake magnitude. This agrees with the results of numerous other studies.

- Inelastic dynamic analyses confirm the significance of amplitude of both vertical and horizontal accelerations on the vertical period of vibration. It also identifies the interaction effect of the horizontal motion as a function of the time interval between peak vertical and peak horizontal accelerations.
- A procedure for calculating inelastic vertical periods of vibration based on magnitude, distance and time interval between peaks was outlined.
- A method for calculating forces due to vertical motion was recommended, using a vertical response spectrum and the calculated vertical period.
- A procedure was suggested for the combination of forces due to vertical and horizontal motion based on the observed interaction.

The procedure in Fig. 1 is proposed for assessing the effect of vertical ground motion, from the point of assessing the significance of the vertical component to providing a simple code-type method for calculating and combining the vertical and horizontal seismic forces. This procedure provides a viable framework that can be solidified by further investigation and verification.

References