Module 1, Lesson 2: Antenna Fundamentals and Definitions

Spherical Coordinate Systems

![Figure 1.2.1 Geometry for Computing the Antenna Parameters.](image)

**Definitions**

Antenna Radiation Pattern: The distribution of radiated energy from an antenna over a surface of constant radius centered upon the antenna.

Far-Field: Field region of the antenna where $r > 2D^2/\lambda$, provided $D > \lambda$ or, more generally, the antenna radiation pattern at $r = \infty$.

Near-Field: Field region of antenna where $r < 2D^2/\lambda$.


In this lecture, we are going to begin to study antenna fundamentals and some very important definitions.
Figure 1.2.1 shows a spherical coordinate system. At the origin there is an antenna. On the spherical coordinate system we have a sphere of radius, \( r \). Imagine this as a hypothetical spherical surface enclosing the antenna. We can identify a point on the sphere by radius \( r \), angle \( \theta \), and angle \( \phi \). At the indicated point, \( f \), there is some electric and magnetic field intensities created by the antenna. At the point of interest, marked as \( f \), we want to measure the electric field intensity or the magnetic field intensity as a function of position on the sphere, that is, as we vary \( \theta \) and \( \phi \) keeping the distance, \( r \), to the center of this antenna constant. A plot of the magnitude of the electric field intensity or the magnetic field intensity gives what we call the radiation pattern. The radiation pattern is a measure of how the electric or magnetic field intensities vary with angular positions \( \theta \) and \( \phi \) for a fixed range, \( r \). This will be a mathematical calculation, as we will show when we study a dipole. We will be able to compute exactly how the field varies over the sphere.

In a practical system, one can actually measure the electric field intensity or magnetic field intensity as a function of position. This would be done in an antenna test range. If we vary \( \theta \) by keeping \( \phi \) constant, we are moving along a line which would look like the longitudinal lines on a globe. If we are keeping \( \theta \) constant and vary \( \phi \) we are moving along what looks like latitudinal lines on the globe. So we have an option of varying \( \theta \) by keeping \( \phi \) constant or varying \( \phi \) by keeping \( \theta \) constant.

When we study a simple antenna, like a dipole, we will see that the electric field is made up of several terms. One is called the far-field; the other is called the near-field. When you are close to the antenna, the near-field term will dominate the far-field term. When you are far away from the antenna, the far-field term will dominate the near-field. In other words, the near-field will “die off” very rapidly as \( r \) is made large, leaving primarily the far-field component. But close to the antenna, the near-field is much stronger than the far-field and it dominates. Now, what is the dividing line when the near-field ends and the far-field begins?

We need to define a far-field region and we need to define a near-field region. This is important when we measure antenna patterns; we measure the antenna pattern in the far-field. Now it can be shown that the far-field of an antenna is where the range \( r \) exceeds twice \( D^2 \) divided by the operating wavelength. As the formula states here, \( r \) must be greater than two times \( D^2 \) divided by \( \lambda \). When we make some mathematical calculations of the antenna radiation pattern in the far-field, we will actually mathematically let \( r \) become very large, approaching \( 4 \). It turns out that our calculation becomes very easy there, and this will be evident later when we study an actual antenna.

But suppose you want to make a measurement of the radiation pattern of this antenna. You have to make sure that you are in the far-field. Let us take a practical example. In the previous lesson you were asked to click on the ICON which showed a view of the Arecibo spherical reflector antenna. That antenna was approximately 1,000 feet in diameter, or exactly 300 meters. The antenna is used for radio astronomy purposes and its beam is essentially upward. If you were
going to fly an aircraft over some imaginary sphere enclosing this antenna, how far should we be
to make sure we are in the far-field? Well, $D$ would be the largest dimension of the antenna
structure which is given as 300 meters. For radio astronomy purposes that facility sometimes
operates at 430 Mhz, therefore, the free space wavelength is .698 meters. You can check that out
yourself. Plugging some numbers into the formula tells us that to be in the far-field $r$ must
exceed $257,879$ meters or approximately 160 miles. 160 miles is the dividing region between far-
field and near-field; actually we should be much further than 160 miles to be considered in the
far-field. There is no aircraft that we can fly over that antenna that can reach the altitude of 160
miles, so we are perplexed on how to measure the radiation pattern of this antenna.
Unfortunately, we are confined to make the measurements in the near-field, that is, when $r$ is less
than 2 times $D^2$ divided by 8. Fortunately, we have mathematics which allows us to transform
near-field data into far-field data, and that is exactly how it can be done.

There are many instances where antennas are tested in antenna test ranges in the near-field
because one can always convert near-field data to far-field data. With the Arecibo antenna there
is another possibility. Consider a radio star as a source. Radio stars emit vast amounts of
electromagnetic energy in certain radio bands. So instead of thinking of the Arecibo antenna as a
transmitting antenna, and trying to measure the field intensity over a sphere, let’s view it as a
receiving antenna. The energy from the distant star is in the far-field. We can steer this antenna.
You notice from the picture of the Arecibo Reflector that there are feeds above the reflector dish
and they are moveable, therefore we can steer the antenna to scan through the radio star and
make a plot of the measurement of the received antenna signal as a function of $\theta$ and $\phi$. So we
do have a way, at least with the Arecibo observatory, of measuring truly a far-field.
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Polar Plot

Figure 1.2.2a Polar Plot of a Radiation Pattern.

Usually the normalized $E^*$ field pattern is plotted for the radiation pattern.

- $P'' = E^*\text{ power pattern}$
- $E^* \text{ dB} = 20 \log E^*$
- $P_{\text{dB}} = 10 \log P$

Now that we will be able to determine the electric field intensity over the sphere and that we are in the far-field, how do we present this data? Antenna engineers plot data in polar plots or in rectangular plots. So shown on Figure 1.2.2a is a polar plot and we see that we have plotted the magnitude of the field intensity, as you would in a polar plot, along the radial direction as a function of $\theta$. This is how we construct a polar diagram. We note a general characteristic of antennas; they have a main beam, some side lobes, minor lobes, possibly a back lobe, and there are several nulls.
Figure 1.2.2b is a rectangular plot of the pattern instead of a polar plot. We plot the field intensity along the $z$ axis vertically, and we plot the angle $\angle$ along the horizontal axis. Both plots display the same information. We see a very clear cut main beam, diminishing side lobes, in this case, a back lobe and several distinct nulls. We have a choice of plotting the magnitude of the electric field, which was measured, or as some engineers do, the square of the magnitude which is proportional to the Poynting vector, and that we would call a power pattern. You must be very careful when you see a radiation pattern. Is it a plot of the electric field or a plot of the electric field squared? Is it a field pattern or is it a power pattern? Usually we normalize the pattern. You can see the main beam has a maximum, in this case a $\angle = 0$. Whatever the value of the electric field intensity was at that particular point, we divide all measurements by the maximum, so now the maximum would have strength unity. This is a normalized plot. Antenna engineers prefer to plot not only the magnitude of the electric field, but the decibel equivalent. In other words, 20 times the log of the magnitude of the electric field intensity. Or 10 times the log of the power pattern. Both are acceptable forms. Now one important parameter on a radiation pattern is the half-power beamwidth, identified by the symbol HPBW. This is where the electric field intensity equals 0.707 of the mainbeam maximum field intensity. Also shown is the angular difference between the nulls on each side on the main beam giving the beamwidth measured between the first nulls, FNBW.
Look at Figure 1.2.2c which shows two points on the radiation pattern identified as having strength one over the square root of two. These are the half-power points. If the electric field is one over the square root of two, the power is ½. We have shown the pattern as normalized, when \( \mathbf{Z} \) is 0; the main beam is identified as unity, that is full power density. There are two angles on either side of the main beam at which the electric field will diminish to one over the square root of two of the maximum and the maximum was one. These are called the half-power points because power density is proportional to the square of the magnitude of the electric field intensity. The angular spread between those two marked half-power points, defines the half-power beamwidth. This convenient measure tells us how narrow the beam is. Is the beamwidth 40 degrees wide or is it 1/10th of a degree wide? If we made some calculations on the Arecibo spherical reflector we would find that the beamwidth is about a few tenths of a degree. It is like a search light which could have a very narrow beam of energy, or the average flash light which could have a wider beam of energy. The beamwidth is then a measure of how well the energy has been concentrated in a fixed region, that is the half-power region. Within the half-power beamwidth, the power density varies between one and down to ½. Outside the half-power beamwidth, the power density will be ½ or less, so half-power is a convenient dividing point. Sometimes it is easier to calculate the beamwidth between the first nulls, identified at FNBW. There is a null on each side of the main beam. Obviously, FNBW exceeds HPBW but sometimes it is more convenient to calculate FNBW, as you will see later on.
Power Density and Radiated Power

The Poynting Vector $\vec{P}$ is defined as:

$$\vec{P} = \frac{1}{2} \vec{E} \times \vec{H}^*$$  \hspace{1cm} (1.2.1)

which is a power density with units of W/m².

The total complex power flowing out through a closed surface $S$ is:

$$P_c = \iiint_S \vec{P} \cdot d\vec{S} = \frac{1}{2} \iiint_S \left( \vec{E} \times \vec{H}^* \right) \cdot d\vec{S}$$  \hspace{1cm} (1.2.2)

where $d\vec{S} = \hat{n}dS$

$\hat{n} = \text{unit normal directed outward from the surface.}$
per square meter that happens to be at a given point and the direction of the vector is the
direction of the power flow. We show in Figure 1.2.3, that the antenna is surrounded by this
imaginary or hypothetical enclosure, a spherical surface, with an element of area, $d\hat{S}$, on the
sphere. We want to compute the total complex power which flows out through the entire
enclosed surface, $S$. As we have already discussed on Figure 1.1.13, this requires one to
integrate the dot product of the complex power density, $\hat{P}$ a vector, with the element of area as
shown by Equation (1.2.2). The element of area $d\hat{S}$, a vector, is directed outward from the
sphere. The unit normal is outward.
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Power Density and Radiated Power

Note that

\[
\text{Re} \left\{ \vec{P} \right\} = \text{Average real power density}
\]

\[
\text{Im} \left\{ \vec{P} \right\} = \text{Average reactive (stored) power density.}
\]

In the far-field of an antenna the power density is mostly real. Hence, the average power radiated by an antenna is

\[
P_{\text{rad}} = \frac{1}{2} \oint \text{Re} \left\{ \vec{P} \right\} \cdot d\vec{S}
\]

(1.2.3a)

\[
= \frac{1}{2} \oint \text{Re} \left\{ \vec{E} \times \vec{H}^* \right\} \cdot d\vec{S}
\]

(1.2.3b)

For a spherical surface \( d\vec{S} = \hat{n} dS \)

Where \( \hat{n} = \hat{a}_r \)

Figure 1.2.4 Element of Area for Power Calculations.
We continue with what we have already discussed that the real part of the Poynting vector is the average real power density and the imaginary part of the Poynting vector is the average reactive power density. We will show, and you will have to work on faith at this particular time, that when you are in the far-field of an antenna, the power density is mostly real. Hence, the average total power radiated by an antenna is found by integrating the real part of the power density over this large sphere because we are in the far-field.
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Assignment #1

On the spherical surface of Figure 1.2.4 show that

\[
\left(\vec{E} \times \vec{H}^\ast\right) \cdot \hat{a}_r = \frac{1}{\eta_0} \left( |E_\theta|^2 + |E_\phi|^2 \right)
\]

Where \( \eta_0 \) is the intrinsic impedance of free space.
Power Density and Radiated Power

Hence, \( P_{\text{rad}} = \frac{1}{2} \text{Re} \left\{ \iiint_{0}^{2\pi} \left( \mathbf{E} \times \mathbf{H}^* \right) \cdot \hat{r}, r^2 \sin \theta d\theta d\phi \right\} \) \hspace{1cm} (1.2.4)

The results of lesson assignment #1 may be used to show that

\[
P_{\text{rad}} = \frac{r^2}{2\eta_0} \iiint_{0}^{2\pi} \left( |E_\theta|^2 + |E_\phi|^2 \right) \sin \theta d\theta d\phi
\]

\[
= (P_{\text{rad}})_\theta + (P_{\text{rad}})_\phi
\]

where

\[
(P_{\text{rad}})_\theta = \frac{r^2}{2\eta_0} \iiint_{0}^{2\pi} |E_\theta|^2 \sin \theta d\theta d\phi
\]

\[
(P_{\text{rad}})_\phi = \frac{r^2}{2\eta_0} \iiint_{0}^{2\pi} |E_\phi|^2 \sin \theta d\theta d\phi
\]

Now suppose we define the radiation intensity for a given antenna to be

\[
U(\theta, \phi) = \frac{r^2}{2\eta_0} \left( |E_\theta|^2 + |E_\phi|^2 \right) \text{ Watts}
\]

\[
= U_\theta + U_\phi
\]

Then we may write: \( P_{\text{rad}} = \iiint_{0}^{2\pi} U(\theta, \phi) \sin \theta d\theta d\phi \) \hspace{1cm} (1.2.5)

We wish to perform the integration shown in Equation (1.2.4). Your first assignment was to show that the \( \mathbf{E} \times \mathbf{H}^* \) dotted into the unit vector, which is in the radial direction, is made up of two parts, involving the terms \( *E_2^* \) and \( *E_\phi^* \). Remember, \( \mathbf{E} \) in general can have \( r \) and \( \phi \) components and \( \mathbf{H} \) can have \( \phi \) and \( 2 \) components because the \( \mathbf{E} \) and \( \mathbf{H} \) fields are orthogonal in the far-field. \( \eta_0 \) is the intrinsic impedance of free space. Assuming that you have shown what the assignment requested, we will use that information in Equation (1.2.4) to obtain
the expression for the power radiated by the antenna, which is written as two terms \((P_{rad})_2\) and \((P_{rad})_\phi\). The power radiated due to the \(Z\) component of the field and the power radiated due to the \(\phi\) component of the field can be computed as shown in the expression for \((P_{rad})_2\) and \((P_{rad})_\phi\).

We make a change of variable here. Instead of working directly with the Poynting vector, some authors prefer to work with the radiation intensity, \(U\). The radiation intensity as shown is found by taking the Poynting vector terms and multiplying by the square of the radius. Therefore, there are two terms to the radiation intensity, a \(Z\) value and a \(\phi\) value, \(U_\theta\) and \(U_\phi\). The radiation intensity is a convenient way of writing some of the terms which appear in the integrals.

Therefore, we can write the total power radiated in terms of the radiation intensity as shown in Equation (1.2.5). We are now to a point that if we know the electric field components, that is \(E_Z\) and \(E_\phi\) radiated by a given antenna structure, and express the radiation intensity, the integral in Equation (1.2.5) gives us the total radiated power through a hypothetical sphere enclosing an antenna. Where did that power come from? It came from the antenna. Therefore, we have an expression for the total power radiated by an antenna.