Applications of Engineering Economics:
• Selecting one or more projects for investment from a given set, using one or more criteria, based on the time value of money.

Interest and Time Value of Money:
• The cost of money is measured by the interest rate. For example, if a person takes a loan from a bank at 6% annual interest rate, then the cost of that loan to the borrower is 6 cents per dollar per year.
• Interest is the cost of money.

• Because of interest, money has a time value. Owning one dollar today is not same as owning one dollar, one year from now. In fact, a dollar today is more valuable than a dollar one year from now, because of the interest it earns during the period of one year.
Simple and Compound Interests

There are two ways for calculating the interest earned; these are simple and compound interests.

Simple Interest:  In this method, interest is computed only on the principal and not on the interest earned. This implies that the interest earned during any period does not earn interest in the future periods.
Example

A bank pays interest to its savings account customers at a rate of 10% every year, using simple interest method. Assume that a customer deposits $10,000 now. What would be the balance in this person’s savings account three years from now?
<table>
<thead>
<tr>
<th>Period</th>
<th>Balance at the beginning of period</th>
<th>Interest earned</th>
<th>Balance at the end of period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year 3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It can be seen that the interest earned every year remains constant and is equal to the principal * the interest rate. In this example, it is equal to
It is easy to develop a formula for the balance at the end of a specified time period.

Let

\[ P = \text{Initial deposit} \quad (\text{In this example, } P = 10,000) \]
\[ i = \text{Interest rate per period} \quad (\text{In this example, } i = 0.10) \]
\[ n = \text{The number of periods} \quad (\text{In this example, } n = 3) \]

\[ F = \text{Amount available in the account after } n \text{ periods} \]

\[ F = \]
Compound Interest: In this method, interest is computed not only on the principal but also on the interest earned during the previous periods. This implies that the interest earned during any period earns interest in the future periods.
Example A bank pays interest to its savings account customers at a rate of 10 % every year, using compound interest method. Assume that a customer deposits $10,000 now. What would be the balance in this person’s savings account three years from now?
<table>
<thead>
<tr>
<th>Period</th>
<th>Balance at the beginning of period</th>
<th>Interest earned</th>
<th>Balance at the end of period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year 3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A formula can be developed for the balance at the end of a specified time period, if the compound interest method is used.
Let

\[ P = \text{Initial deposit (In this example, } P = 10,000) \]

\[ i = \text{Interest rate per period (In this example, } i = 0.10 \text{ and the length of each period is one year.} \]

\[ n = \text{The number of periods (In this example, } n = 3) \]

\[ F = \text{Amount available in the account after } n \text{ periods} \]
If $n = 1$, then $F =$

$n = 2$, then $F =$

$n = 3$, then $F =$

In general, $F =$

= 

- Compound interest is the most commonly used method in real life applications and will be used in this course.
Specifying the Interest Rate and Frequency of Compounding

• Two pieces of information are required regarding interest computation. These are
  – The annual nominal interest rate, denoted by \( r \).
  – The frequency of compounding or interest calculations per year. The number of interest calculations per year is denoted by \( c \).
Example: Credit Card Company, A, charges a nominal interest (annual) rate of 9 % on the loans of its customers. The interest is compounded monthly.
• Here, the annual nominal interest rate = \( r = \)

• As the compounding is done every month, the number of interest calculations per year or the number of compounding per year = \( c = \)

• Hence, the interest rate per month, which is the compounding period =

• The Length of Interest Period (LIP) =
• Let us assume that a person charges $1000.00 to his/her credit card issued by this company, on January 1, 2009. Then, the amounts this person owes to the company at the end of each of the next 12 months, are as follows, assuming that this person does not make any payment towards the loan during this period:
<table>
<thead>
<tr>
<th>End of</th>
<th>Loan</th>
<th>Interest</th>
<th>Total amount owed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Feb.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>March</td>
<td>1015.26</td>
<td>1015.26 * 0.0075 = 7.61</td>
<td>1015.26 + 7.61 = 1022.87</td>
</tr>
<tr>
<td>April</td>
<td>1022.87</td>
<td>1022.87 * 0.0075 = 7.67</td>
<td>1022.87 + 7.67 = 1030.54</td>
</tr>
<tr>
<td>May</td>
<td>1030.54</td>
<td>1030.54 * 0.0075 = 7.73</td>
<td>1030.54 + 7.73 = 1038.27</td>
</tr>
<tr>
<td>June</td>
<td>1038.27</td>
<td>1038.27 * 0.0075 = 7.79</td>
<td>1038.27 + 7.79 = 1046.06</td>
</tr>
</tbody>
</table>
In the above table, the amounts were rounded to two digits after the decimal point.
• Observations:

(i) The total amount the borrower owes to the company at the end of December, 2009, (after a period of twelve months from the time of borrowing) equal to $1094.01, consists of the following:

Original amount of loan (principal) =

Interest for a period of twelve months, at the nominal rate of 9%, compounded monthly =
(ii) The time value of $1000.00 after twelve months is

(iii) It is really not necessary to perform monthly computations to obtain the total amount owed at the end of twelve months. A compact formula will be derived to achieve this later.
• Assume that the company changes its policy regarding compounding by changing it to semi-annual compounding keeping the annual nominal interest rate at 9%.

Now the number of compounding per year = $c =$

and the interest rate per compounding period now is =

Length of Interest Period (LIP) =
Now, the amounts this person owes to the company at the end of each of the next 2 compounding periods, are as follows, assuming that this person does not make any payment towards the loan during this period:

<table>
<thead>
<tr>
<th>End of</th>
<th>Loan</th>
<th>Interest</th>
<th>Total amount owed</th>
</tr>
</thead>
<tbody>
<tr>
<td>June</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dec.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- It can be seen that this amount is less than the amount with monthly compounding (1094.01).
• Now let us assume that the company changes its policy regarding compounding by changing it to annual compounding keeping the annual nominal interest rate at 9%.

    Now the number of compounding per year =

    \[ c = \]

    and the interest rate per compounding period now is =

    which is the same as the nominal rate.

    \[ \text{LIP} = \]
Now, the amounts this person owes to the company at the end of the year is as follows, assuming that this person does not make any payment towards the loan during this period:

<table>
<thead>
<tr>
<th>End of Dec.</th>
<th>Loan</th>
<th>Interest</th>
<th>Total amount owed</th>
</tr>
</thead>
</table>

- It can be seen that this amount is less than the amount with monthly compounding (1094.01).
• For the same nominal interest rate, the amount of interest earned increases, as the number of compounding per year increases.

• It will be useful to compute the equivalent annual interest rate, which assumes annual compounding for any given number of compounding periods per year and the same nominal rate. This will be done later.
Cash Flow Diagrams:

• These represent cash flows over a period of time. It consists of a horizontal line, which is a time scale, starting with ‘0’ representing ‘NOW’. End of periods are marked along the horizontal line. Arrows signify cash flows; downward arrows representing disbursements or negative cash flows and upward arrows representing receipts or positive cash flows.

0—--------------------------------------------------
Example 1: Mr. Adams deposits $5000 in a bank now and withdraws $1000 every month for the next three months.

Here the length of every period is one month.

Length of Cash flow Period (LCP) = one month.
Example 2: Ms. Jones plans to withdraw $3000 from her savings account, one year from now and to make three monthly deposits of $1000 each in the account, starting two months after her withdrawal of $3000.
Here the logical length of each period in the cash flow diagram is one month; that is LCP = one month.

It is always preferable to keep LIP = LCP.