BACKWATER CURVES IN RIVER CHANNELS

I. Purposes and Scope

There are many reasons why it is necessary to study water-surface profiles. For instance, the stage of a river may need to be determined for a given discharge; or it may be necessary to calculate a Manning's "n" for a channel cross section. No matter what the reason for studying backwater curves, it is time consuming to hand calculate all of the steps necessary to obtain a final solution to many of the problems dealing with flow profiles. Therefore, the purpose and scope of this paper is to review the theory of water surface profile computations and to develop a knowledge of the HEC-RAS (Hydrologic Engineering Center U.S. Army Corps of Engineers River Analysis System Computer Program.)

II. Background

"The term backwater curve was first applied to long smooth profiles of the water surface upstream from a dam or other obstruction in a river channel. Ultimately, however, twelve different types of profile curves were found to be possible in a river channel."[1] These profile curves are best illustrated for uniform flow, the simplest flow possible.

A. Establishment of Uniform Flow

When flow occurs in an open channel, resistance is encountered by the water as it flows downstream. This resistance is generally counteracted by the components of gravity forces acting on the body of the water in the direction of motion. Uniform flow will develop if the resistance is

[1]Engineering Manual 1110-2-1409, U.S. Army Corps of Engineers, "Backwater Curves in River Channels."

balanced by these gravity forces. Figure 1 depicts the establishment of uniform flow for a mild, critical. and steep slope.

Uniform flow has been established when: (1) the depth, water area, velocity, and discharge at every section of the channel reach are constant; and (2) the energy line, water surface, and channel bottom are all parallel (i.e., $S_f = S_w = S_0$). Figure 2 depicts a definition sketch of the above criteria. Uniform flow refers specifically to the spatial variation of parameters, $\frac{\partial}{\partial s} \equiv 0$.

Two uniform flow equations used extensively in hydraulics are worth noting; the first, the Chezy equation

$$V = C\sqrt{RS} \qquad \text{Eq. 1}$$

where V is the average velocity in fps, C is the Chezy's resistance coefficient and R is the hydraulic radius in ft, and S is the channel slope.

$$V \frac{1.49}{n} R^{2/3} S^{1/2} \qquad \text{Eq. 2}$$

where n is the Manning's n resistance coefficient.

However, under natural conditions the alignment and slope of a channel are irregular, the degree of roughness variable, the cross section irregular, unsymmetrical and continuously changing. Consequently, conditions one and two of the definition for uniform flow are never met and the flow in a river should really be classified as nonuniform or varied. To what degree the flow is varied is important. If the channel parameters vary only slightly with respect to longitudinal distance the flow is defined as gradually varied. At the same time, if the channel

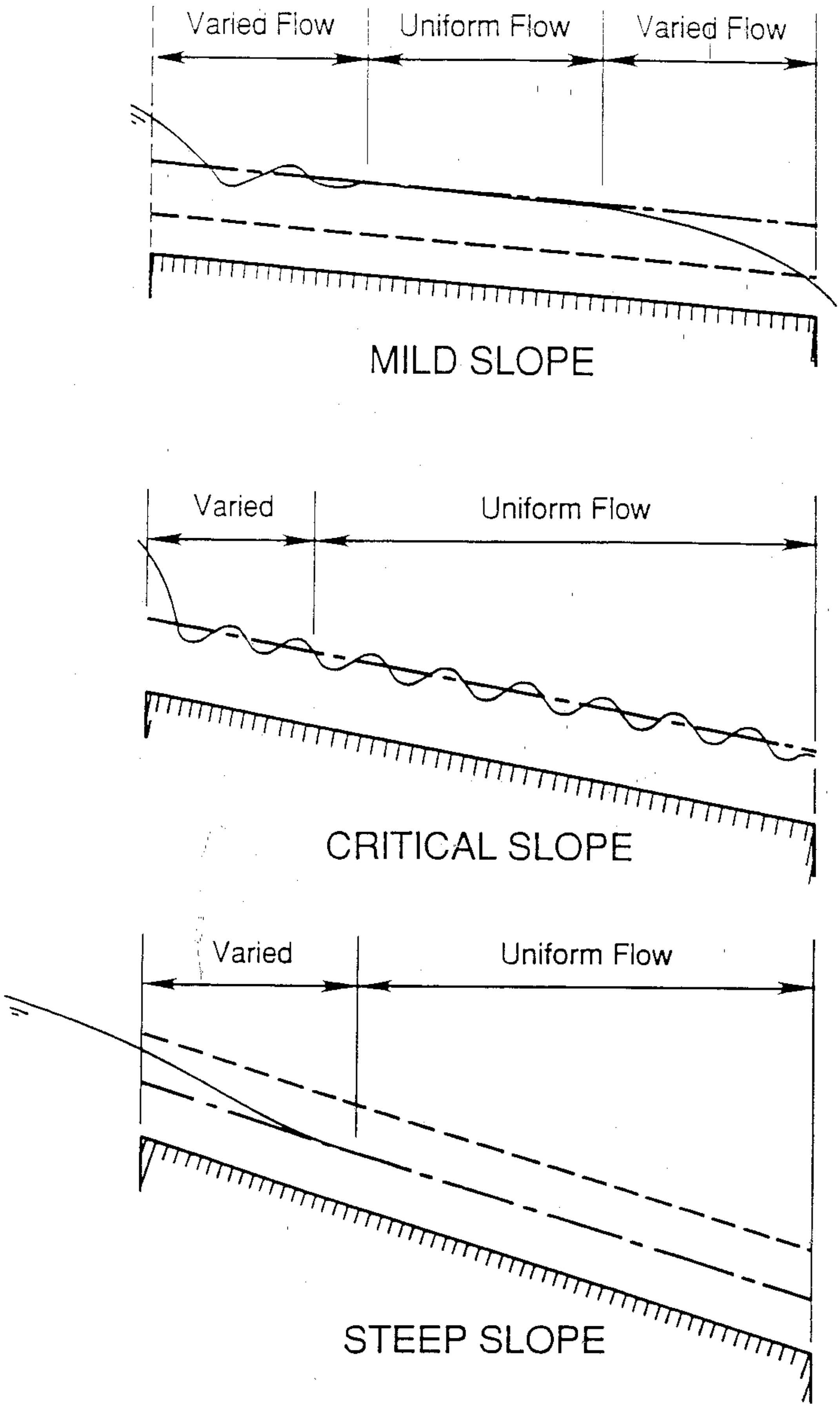
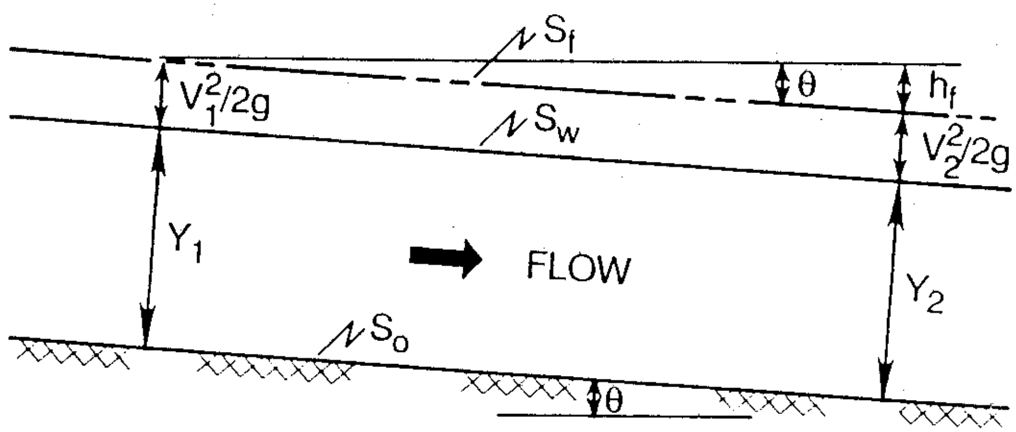
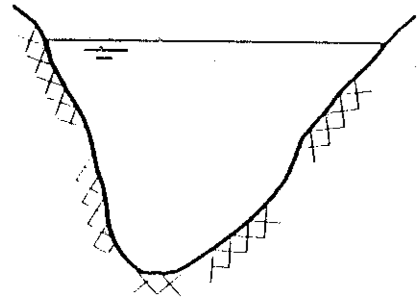

Varied Flow
Uniform Flow
Varied Flow
MILD SLOPE
Varied
Uniform Flow
CRITICAL SLOPE
Varied
Uniform Flow
STEEP SLOPE

FIGURE 1



PROFILE VIEW



CROSS SECTION

LIST OF SYMBOLS

e = Slope of the channel bottom
Y = Depth of channel bottom, ft
V = Velocity (average), fps
g = Acceleration of gravity, ft/sec^2
h = Head loss, ft

S_o = Slope of channel bottom
S_w = Slope of channel water surface
S_f = Slope of energy grade line
L = Length of reach, ft

FIGURE 2 DEFINITION SKETCH - UNIFORM FLOW parameters vary greatly with longitudinal distance the flow is defined as rapidly varied. The significance of gradually varied flow (Figure 3) is that, as an approximation, the basic equations developed for uniform flow are applied as an approximation. Rapidly varied flow is more complicated and is not discussed here.

B. Gradually Varied Flow

Gradually varied flow is steady flow whose depth varies gradually along the length of the channel. This definition requires that two conditions be met: (1) the flow is steady (i.e. all of the variables remain constant for the time interval under consideration), and (2) the streamlines are approximately parallel (i.e. the pressure distribution over the cross section can be considered hydrostatic).

The theory of gradually varied flow really requires the headloss through a river reach can be approximated using uniform flow equations with the same velocity and hydraulic radius of the section. The Manning's "n" developed for uniform flow is therefore applicable for varied flow. This is not absolutely true but the errors associated with the assumption are generally considered quite small. In this discussion of the gradually varied flow equations some further restrictions are applied, namely:

1. The slope of the channel is small, i.e.:
   a. The depth of flow is the same whether the vertical or normal direction is used. The cosine of the slope angle is sufficiently small to allow $\cos\theta = 1$.

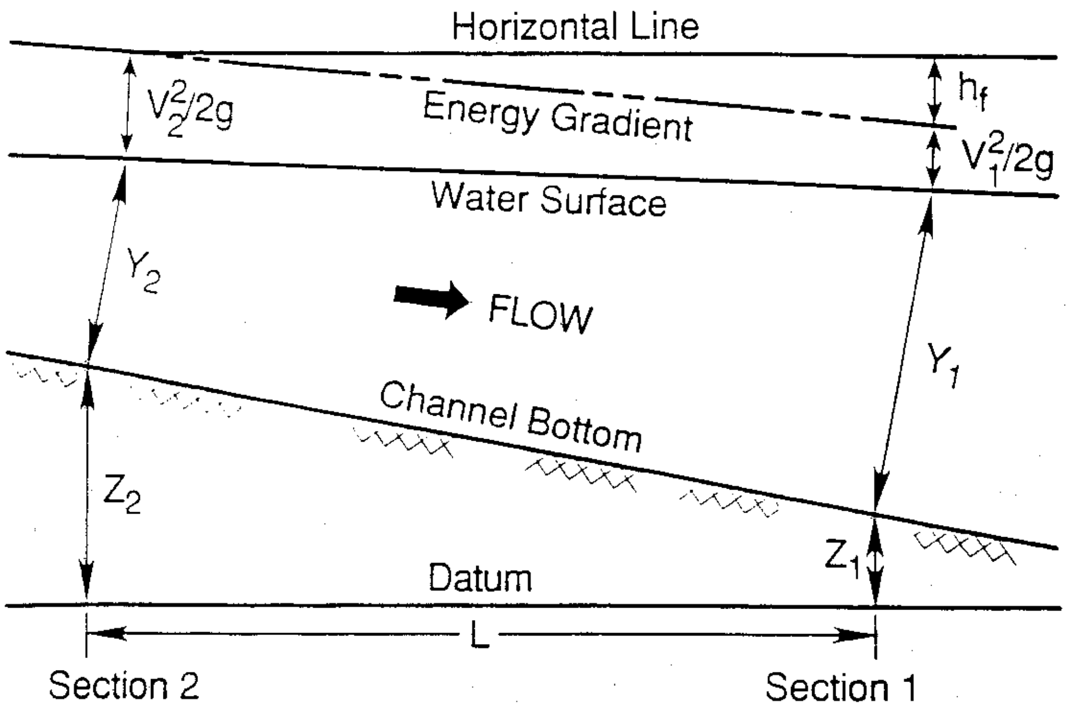


FIGURE 3 DEFINITION SKETCH - NONUNIFORM FLOW

b. No air entrainment occurs.

2. The channel is prismatic; the channel has constant alignment and shape.

3. The velocity distribution in the channel section is fixed (the velocity distribution coefficients are constant).

4. The roughness coefficient is independent of the depth of flow and constant throughout the channel reach under consideration.

III. Basic Theory

To develop the theory for the computations of flow profiles the energy principle (Bernoulli's theorem) is used. The energy associated with section 1 and section 2 in Figure 3 can be stated as follow: the elevation of the water surface plus the velocity head at section 2 is equal to the elevation of the water surface and the velocity head at section 1 plus the energy losses through the reach.

The total energy or head above an arbitrary datum can be written as

$$H = Z + Y + \frac{V^2}{2g} \qquad \text{Eq. 3}$$

where H is the total head in ft, Z is the vertical distance of the channel bottom above the datum in ft, Y is the depth of flow in ft, and $\frac{V^2}{2g}$ is the velocity head in ft. Taking the bottom of the channel as the longitudinal X axis and differentiating equation 3 with respect to the length X, the following equation is obtained

$$\frac{dH}{dX} = \frac{dZ}{dX} + \frac{dY}{dX} + \frac{d}{dX}\left[\frac{V^2}{2g}\right] \qquad \text{Eq. 4}$$

It should be noted that the slope is defined as the sine of the slope angle and is considered negative if it ascends and positive if it descends in the direction of the flow. Thus, the energy slope $S_f = -\frac{dH}{dX}$ and the channel slope $S_o = \sin\theta = -\frac{dZ}{dX}$. Substituting these slopes into equation 4 and noting that $\frac{d}{dX} = \frac{d}{dY}\frac{dY}{dX}$ (the velocity does not vary with X but does vary with respect to Y) and rearranging, equation 4 can be rewritten to solve for $\frac{dY}{dX}$ as

$$\frac{dY}{dX} = \frac{S_o - S_f}{H\,\frac{d}{dY}\left[\frac{V^2}{2g}\right]} \qquad \text{Eq. 5}$$

Substituting $\frac{Q}{A} = V$, where Q is constant discharge in cfs and A is the cross sectional area in ft^2, and noting that, the top width can be represented by $\frac{dA}{dY}$, the derivative of the velocity head can be represented by

$$\frac{d}{dY}\left[\frac{V^2}{2g}\right] = -\frac{Q^2T}{gA^3} \qquad \text{Eq. 6}$$

Equation 5 is the varied flow equation restricted to the assumptions stated earlier. Equation 5 can be further modified by substituting equation 6 into equation 5.

$$S_f = \frac{Q^2}{K^2}$$ Eq. 11

and

$$K_n = \frac{Q}{S_o^{1/2}}$$ Eq. 12

where K_n is the normal conveyance at uniform flow (i.e. uniform flow occurs at normal depth). For any discharge Q the critical section factor and the normal conveyance term can be calculated independent of the actual depth of flow. Thus, Z_c and K_n become useful tools in computational procedures.

Solving for the ratio of the friction slope to the bed slope yields

$$\frac{S_f}{S_o} = \frac{K_n^2}{K^2}$$ Eq. 13

By factoring S_o and substituting equations 10 and 13 into equation 5, yields

$$\frac{dY}{dX} = S_o \frac{1 - (K_n/K)^2}{1 - (Z_c/Z)^2}$$ Eq. 14

Equation 14 is useful when computing water surface profiles by hand and reference texts are available to determine the section factors and the conveyance terms[2].

For simplicity, in development of the various flow profiles, a wide rectangular channel is

[2]Brater's Handbook of Hydraulics, McGraw-Hill.

$$\frac{dY}{dX} = \frac{S_o - S_f}{1 - \frac{Q^2 T}{g A^3}}$$ Eq. 7

It is advantageous in developing the twelve possible flow profiles to change the form of equation 7 to something that is more easily manipulated. To accomplish this, a section factor defined as $Z = \sqrt{A^3/T}$ (Z in ft) and a conveyance term defined as $K = 1.49/n \, AR^{2/3}$ is introduced. Equation 6 would become

$$\frac{Q^2 T}{g A^3} = - \frac{Q^2}{g Z^2}$$ Eq. 8

Suppose that a discharge occurs at critical conditions, the section factor would be defined as

$$Z_c = Q/\sqrt{g}$$ Eq. 9

The value of Z_c is the critical section factor which is computed as if the flow occurs at minimum energy. Substituting equation 9 into equation 8 yields

$$\frac{Q^2 T}{g A^3} = \frac{Z_c^2}{Z^2}$$ Eq. 10

Critical conditions were used to represent Z_c for Q in equation 8. Conveyance will be used in place of the friction slope and in particular a "normal conveyance" for normal or uniform flow conditions. Thus, represents the slope of the water surface with respect to the bottom of the channel. If the water surface converges to the bottom of the channel (i.e. the depth of the flow decreases with distance downstream) the slope is negative. Alternatively, if the water surface diverges from the bottom of the channel (i.e. the depth of flow increases with distance downstream) the slope is positive. To simplify the development of water surface profile curves the following notation illustrated in Figure 4 is used to describe depths, regions of flow, and slopes.

There are five types of channel slopes: (1) adverse, (2) horizontal, (3) mild, (4) critical, and (5) steep. The mechanics of determining if a channel has an adverse or horizontal slope is quite simple. If the bed of the river has a positive slope, the slope is adverse; if the channel bottom lies in a horizontal plane, the slope is horizontal. To distinguish between mild, critical, and steep slopes is not as straightforward, however, comparison of normal and critical depth is probably the easiest way to categorize these slopes.

Normal depth can be solved using Manning's formula, equation 2. Critical depth can be solved for by determining the minimum specific energy at a section and solving for the depth at that minimum energy. The specific energy can be written as

$$E = Y + \frac{V^2}{2g} \qquad \text{Eq. 20}$$

By taking the derivative of E (E in ft) with respect to Y and setting it equal to zero yields

$$\frac{dE}{dY} = 0 = \frac{dY}{dY} + \frac{d}{dY}\left[\frac{V^2}{2_g}\right] \qquad \text{Eq. 21}$$

Substituting equation 6 into equation 21 and solving equation 22 can be written as assumed and the Manning's formula is utilized to represent the resistance. By noting that Q/B = q (where B is the bottom width of the rectangular channel in ft and q is the discharge per unit width in cfs/ft) equation 6 can be expressed as

$$\frac{Q^2T}{gA^3} = \frac{q^2}{gY^3} \qquad \text{Eq. 15}$$

At critical conditions $q^2 = gY_c^3$, substituting this relationship into equation 15 yields

$$\frac{q^2}{gY^3} = \left[\frac{Y_c}{Y}\right]^3 \qquad \text{Eq. 16}$$

From the Manning's formula

$$\frac{q}{\sqrt{S_o}} = \frac{1.49}{n} Y_n^{5/3} = K_n \qquad \text{Eq. 17}$$

therefore

$$\left[\frac{K_n}{K}\right] = \left[\frac{Y_n}{Y}\right]^{5/3} \qquad \text{Eq. 18}$$

And, equation 5 for a wide rectangular channel using Manning's formula becomes

$$\frac{dY}{dX} = S_o \frac{1 - (Y_n/Y)^{10/3}}{1 - (Y_c/Y)^3} \qquad \text{Eq. 19}$$

Eq. 19 can be utilized to obtain the shape of water surface profiles. Recall that dY/dX

$$1 = \frac{Q^2 T}{g A^3} \qquad \text{Eq. 22}$$

Solving for the depth in equation 22 (this can be done because cross section shape is a function of the depth) yields the critical depth of flow. The slopes can now be defined as

	Mild Slope	$Y_n > Y_c$
	Critical Slope	$Y_n = Y_c$
and	Steep Slope	$Y_n < Y_c$

It is necessary to understand how the water surface profiles can occur over these various slopes. Mild, critical, and steep channels are the most important in a practical sense and attention will be focused here first. In the development equation 19 will be used

$$\frac{dY}{dX} = S_o \frac{1 - (Y_n/Y)^{10/3}}{1 - (Y_c/Y)^3} \qquad \text{Eq. 19}$$

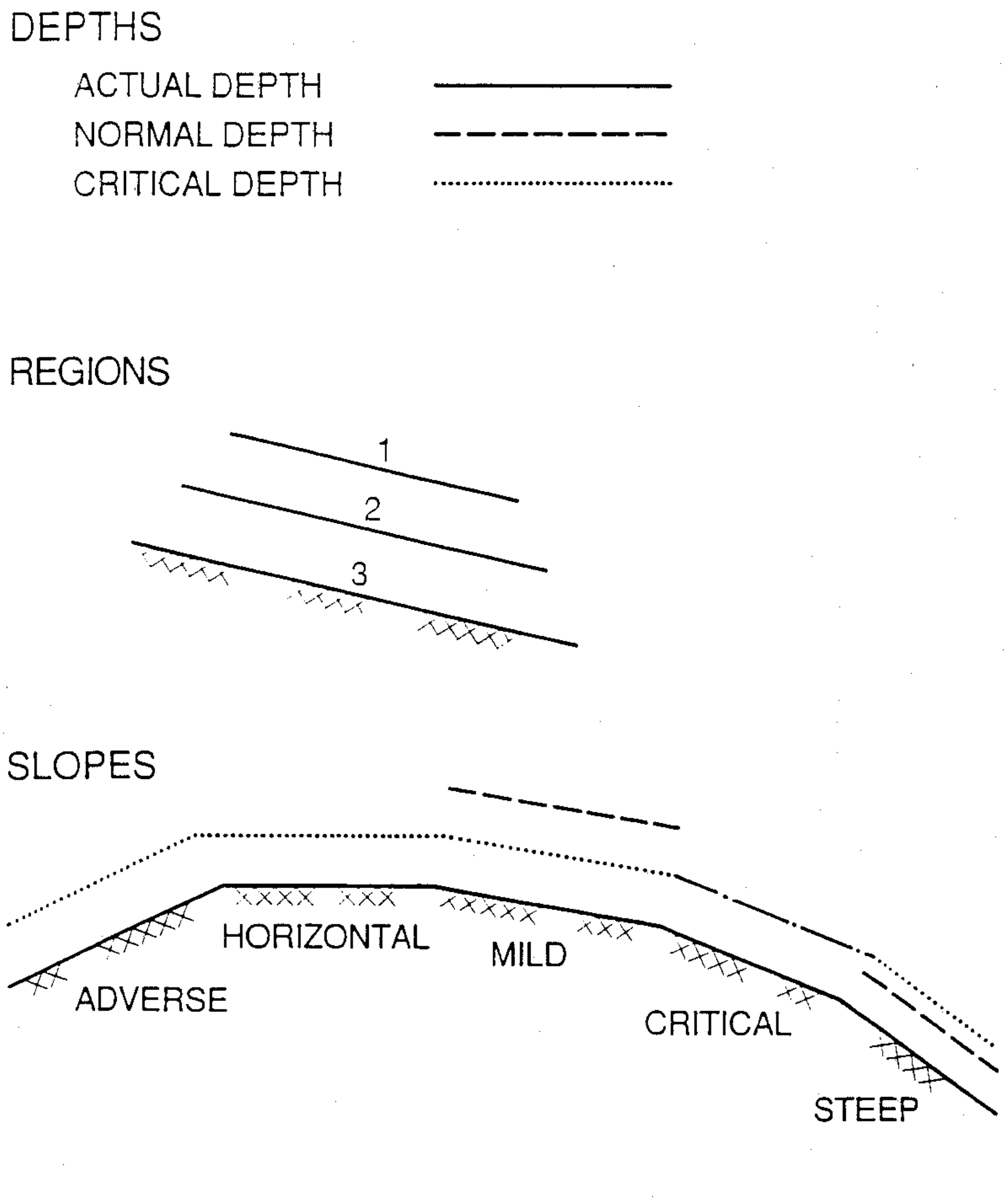


FIGURE 4 NOTATION

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<u>1st Assume</u> $Y \rightarrow$ (Approaches) Y_n i.e. $Y_n/Y \rightarrow 1$

$$\frac{dY}{dX} = S_o \frac{1 - 1}{1 - (Y_c/Y_n)} = S_o \frac{(0)}{1 - (Y_c/Y_n)^3} = 0$$

If $\frac{dY}{dX} = 0$

(the water surface profile (w.s.p.) becomes asymptotic to the normal depth line)

<u>2nd Assume</u> $Y \rightarrow Y_c$

$$\frac{dy}{dx} = S_o \frac{1 - (Y_n/Y_c)^{10/3}}{1 - (Y_c/Y_c)^3} = S_o \frac{1 - (Y_n/Y_c)^{10/3}}{0} = S_o (\infty)$$

$$\frac{dY}{dX} \rightarrow \infty$$

(the w.s.p. is perpendicular to the critical depth line)

<u>3rd Assume</u> $Y \rightarrow 0$

$$\frac{dY}{dX} = S_o \frac{1 - (Y_n/o)^{10/3}}{1 - (Y_c/o)^3}$$

Rewriting $$\frac{dY}{dX} = \frac{Y^3 - Y_n^3 (Y_n/Y)^{1/3}}{Y^3 - Y_c^3}$$

substituting $$\frac{dY}{dX} = S_o \frac{0 - \infty (Y_n)^3}{0 - Y_c^3} = \infty$$

(the w.s.p. is perpendicular to the bottom of the channel)

<u>4th Assume</u> $Y \rightarrow \infty$

$$\frac{dY}{dX} = S_o \frac{1 - (Y_n/\infty)^{10/3}}{1 - (Y_c/\infty)^3} = S_o$$

(the w.s.p. parallels the channel bottom)

The first four assumptions yield details of what the slope of the water surface looks like at four important depths. To summarize these

Depth	0	Y_c	Y_n	∞
$\frac{dY}{dX}$	∞	∞	0	S_o

If uniform flow were present in a channel, there would be no problem in determining the stage of a river. The depth would be calculated using the Manning's formula and it would occur at normal depth. However, because uniform flow usually does not exist the depth, Y, must be calculated in all the reaches. There are three conditions that can occur over a mild slope, two over a critical slope, and three over a steep slope.

A. Mild Slope ($Y_n > Y_c$)

Condition 1 ($Y > Y_n > Y_c$ Water surface profile (w.s.p.) is in region 1

$$\frac{dY}{dX} = S_o \frac{1 - (Y_n/Y)^{10/3}}{1 - (Y_c/Y)^3} = S_o \frac{1 - (\text{No. less than } 1)}{1 - (\text{No. less than } 1)}$$

$$\frac{dY}{dX} = S_o\,(+) = +$$

This implies that the depth increases in the direction of flow

Zone 1, this is called an M_1 curve

Condition 2 $Y_n > Y > Y_c$ w.s.p. curve is in Region 2

$$\frac{dY}{dX} = S_o \frac{1 - (no.\ greater\ than\ 1)}{1 - (no.\ less\ than\ 1)} = S_o \frac{(-)}{(+)}$$

$$\frac{dY}{dX} = -$$

This implies that the depth decreases in the direction of the flow

Zone 2, this is called an M_2 curve

Condition 3 $Y_n > Y_c > Y$ w.s.p. is in Region 3

$$\frac{dY}{dX} = S_o \frac{1 - (no.\ greater\ than\ 1)}{1 - (no.\ greater\ than\ 1)} = S_o \frac{(-)}{(-)}$$

$$\frac{dY}{dX} = +$$

This implies that the depth increases in the direction of flow

Zone 3, this is called an M_3 curve

B. Steep Slope $Y_c > Y_n$

Condition 1 $Y > Y_c > Y_n$ w.s.p. is in region 1

$$\frac{dY}{dX} = S_o\ (+) = +$$

This implies that the depth increases in the direction of flow

Zone 1, this is called an S_1 curve

Condition 2 $Y_c > Y > Y_n$ w.s.p. is in region 2

$$\frac{dY}{dX} = S_o \; (-) = -$$

This implies that the depth decreases in the direction of flow

Zone 2, this is called an S_2 curve

Condition 3 $Y_c > Y_n > Y$ w.s.p. in region 3

$$\frac{dY}{dX} = S_o \; (+) = +$$

This implies that the depth increases in the direction of flow

Zone 3, this is called an S_3 curve

C. Critical Slope $Y_n = Y_c$

Condition 1 $Y > Y_n = Y_c$ w.s.p. in region 1

$$\frac{dY}{dX} = S_o \; (+) = +$$

This implies that the depth increases in the direction of flow

Zone 1, this is called a C_1 curve

Condition 2   Does not exist

Condition 3 $Y_n = Y_c > Y$ w.s.p. is in region 3

$$\frac{dY}{dX} = S_o\,(+) = +$$

This implies that the depth increases in the direction of flow

Zone 3, this is called a C_3 curve

D. Adverse Slope

Normal depth does not exist. Therefore we need to modify equation 20; namely

$$\frac{dY}{dX} = \frac{S_o - S_f}{1 - (Y_c/Y)^3}$$

Using our sign convention established earlier, S_o, for an adverse slope is negative.

Condition 1 Does not exist

Condition 2 $Y > Y_c$ w.s.p. is in region 2

$$\frac{dY}{dX} = \frac{-S_o - S_f}{1 - (Y_c/Y)^3} = \frac{(-)}{+} = -$$

$$\frac{dY}{dX} = -$$

This implies that the depth decrease in the direction flow

Zone 2, this is called an A_2 curve

Condition 3 $Y_c > Y$ w.s.p. is in region 3

$$\frac{dY}{dX} = \frac{-S_o - S_f}{1 - (Y_c/Y)^3} = \frac{-}{-} = +$$

$$\frac{dY}{dX} = +$$

This implies that the depth increases in the direction of flow

Zone 3, this is called an A_3 curve

E. Horizontal Slope

Again normal depth does not exist and we will revert to equation 24 for our analysis.

Condition 1 Does not exist

Condition 2 $Y > Y_c$ w.s.p. is in region 2

$$\frac{dY}{dX} = \frac{0 - S_f}{1 - (Y_c/Y)^3} = \frac{-S_f}{1} = -$$

$$\frac{dY}{dX} = -$$

This implies that the depth decreases in the direction of flow

Zone 2, this is called an H_2 curve

Condition 3 $Y_c > Y$ w.s.p. is in region 3

$$\frac{dY}{dX} = \frac{0 - S_f}{1 - (Y_c/Y)^3} = \frac{-S_f}{-} = +$$

$$\frac{dY}{dX} = +$$

This implies that the depth increases in the direction of flow

Zone 3, this is called an H_3 curve

If we combine all of the above conditions associated with the various slopes and the four assumptions made earlier, we can develop Figure 5. By combining all of the above conditions associated with the various slopes and the four initial assumptions. these curves are represented in Figure 5.

GRADUALLY VARIED FLOW

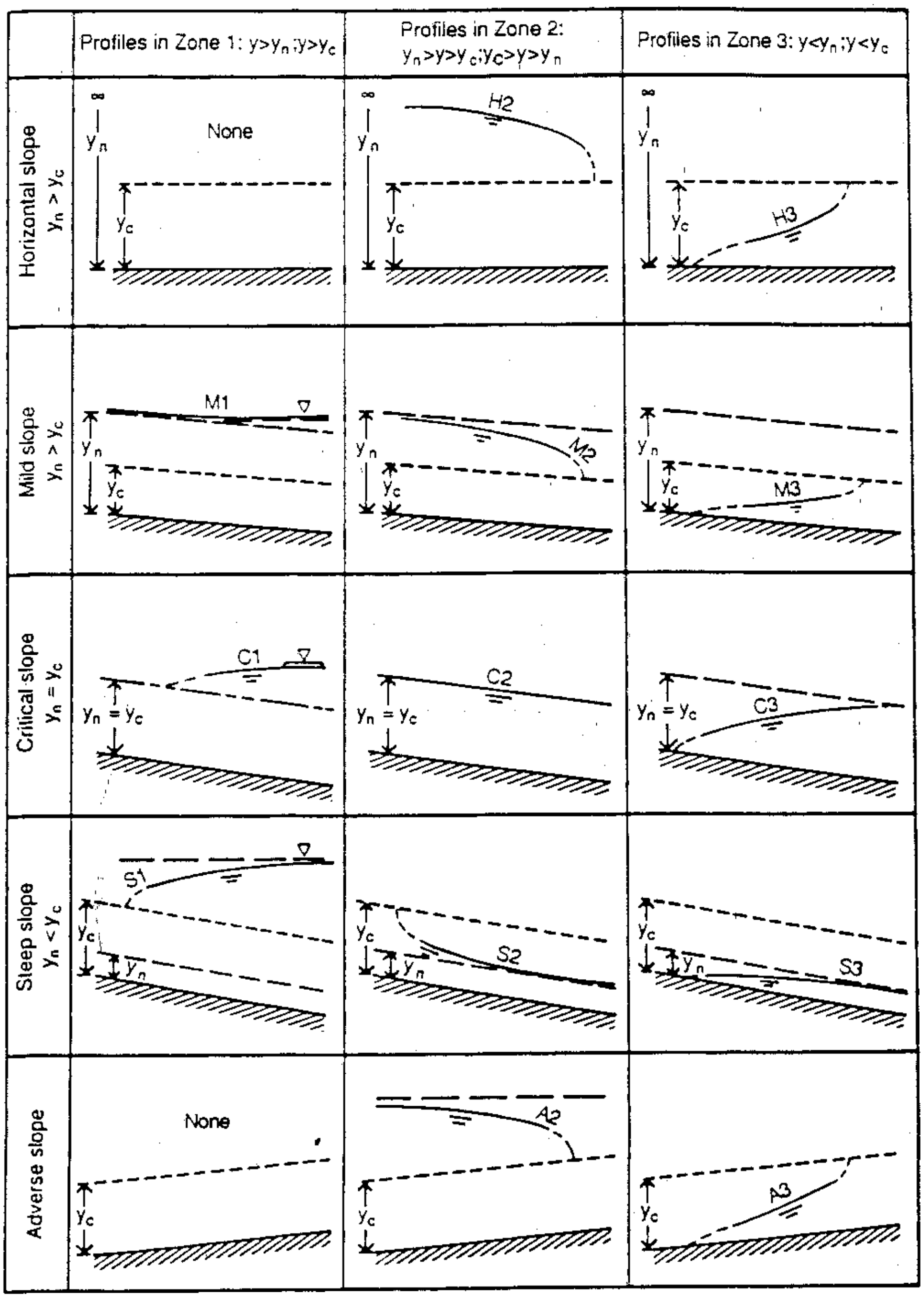

FIGURE 5

Analysis of Flow Profile[3]

Flow profile analysis is a procedure used to predict the general shape of the flow profile. It enables the engineer to learn beforehand the possible flow profiles that may occur in a given channel layout. This procedure constitutes a very significant part of all problems in channel design for gradually varied flow.

A. Prismatic Channel with Constant Slope

Typical examples of these flow profiles are shown in Figure 5.

B. Prismatic Channel with a Change in Slope.

This channel is equivalent to a pair of connected prismatic channels of the same cross section but with different slopes. Twenty typical flow profiles in a long prismatic channel with a break in slope are shown in Figure 6. These profiles are self explanatory. However, some special features should be mentioned:

1. The profile near or at the critical depth cannot be predicted precisely by the theory of gradually varied flow, since the flow is generally rapidly varied.

2. In passing a critical line, the flow profile should, theoretically, have a vertical slope. Since the flow is usually rapid when passing the critical line, the actual slope of the profile cannot be predicted precisely by the theory. For the same reason, the critical depth may not occur exactly above the break of the

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[3]Extracted from Chow, V. T., Open-Channel Hydraulics, McGraw-Hill Book Company, Inc., 1959.

channel bottom and may be different from the depth shown.

3. In some cases a hydraulic jump may occur either in the upstream channel or in the downstream channel, depending upon the relative steepness of the two slopes. In case g, for instance, the jump will occur in the downstream channel if the normal depth in this channel is comparatively small. When the slope of the downstream channel decreases and, accordingly, the normal depth increases, the jump will move upstream, eventually into the upstream channel.

4. If the upstream channel has an adverse slope, the discharge is fixed not by upstream channel conditions but by the elevation of the upstream pool level, which is the horizontal asymptote of the A2 profile.

C. Prismatic Channel with Several Changes in Slope.

For such channels the general procedure of analysis is as follows:

1. Plot the channel profile with an exaggerated vertical scale.

2. Compute Y_n for each reach, and plot the normal depth line, shown by dashed lines, throughout the channel.

3. Compute Y_c for each reach, and plot the critical depth line, shown by dotted lines, throughout the channel.

4. Locate all possible control sections. At the control section, flow must pass

through a control depth which may be the critical depth, the normal depth or any other known depth. There are three types of control sections:

a. Upstream control sections. This occurs in any steep reach at the upstream end, since the flow in a steep channel has to pass through the critical section at the upstream end and then follow either the S1 or S2 profiles. The critical depth is, therefore, the control depth. If the downstream water surface is very high, it may raise the flow surface at the upstream control. When several steep reaches occur in succession, the control section is at the upstream end of the upper-most reach. Upstream control also occurs in long mild reaches, because the M1 or M2 curve will approach the normal depth at the upstream end.

b. Downstream control section. This occurs at the downstream end in any long steep reach, because the flow will approach the normal depth at the downstream end. If the downstream end of a mild channel terminates at a free overfall, the control section may be assumed at the brink where the depth is critical.

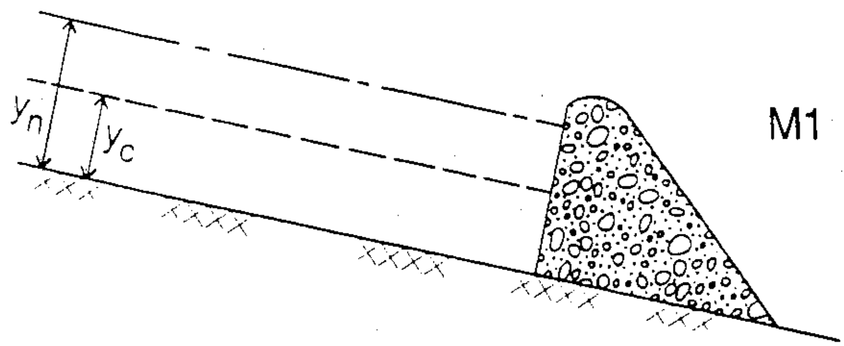
c. Artificial control section. This occurs at a control structure, such as a weir, dam, or sluice gate, at which the control depth either is know or can be determined.

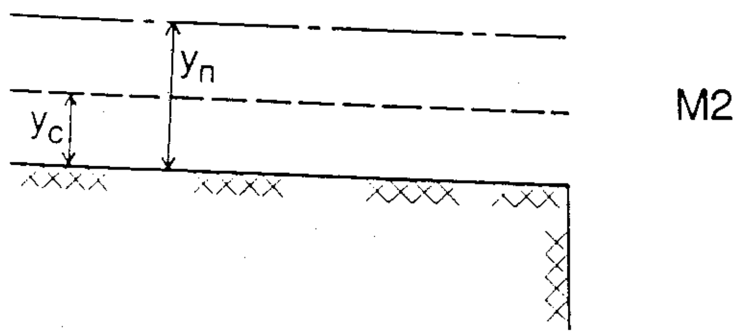
5. Starting at the control depth at each control section, trace in each reach a continuous profile. This position of the profile in each reach can be correctly located with respect to the normal and critical depth lines.

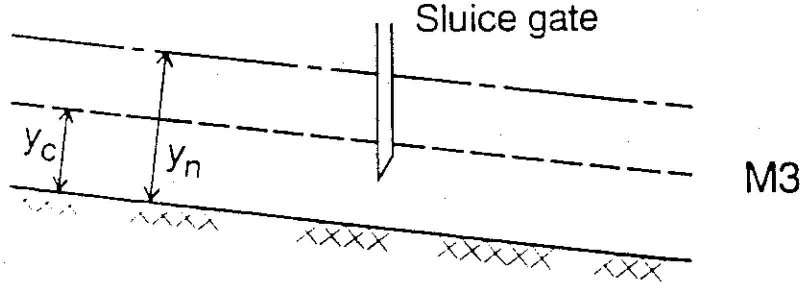
6. When flow is supercritical in the upstream portion of a reach but subcritical in the downstream portion, the flow profile has to pass the critical depth somewhere in the reach. In crossing the critical depth line, a hydraulic jump is usually created in raising the water surface from a low depth to its sequent depth.

Twelve Profiles:

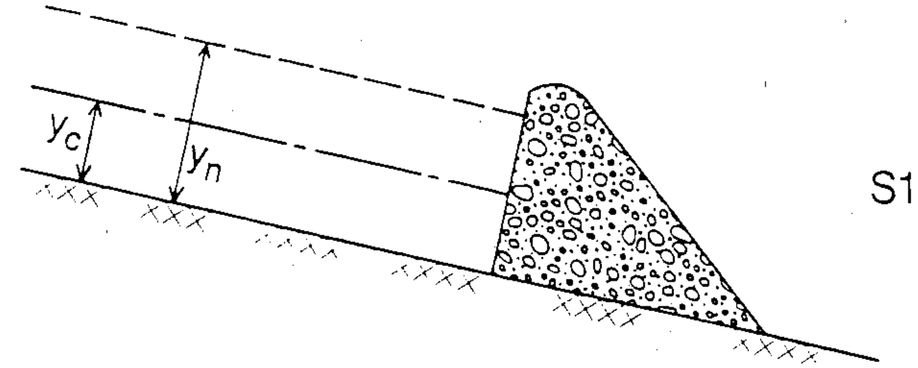
Mild slope $y_n > y_c$

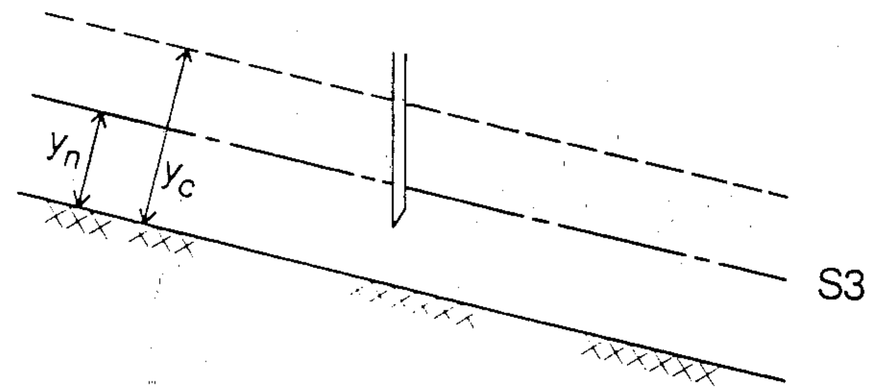


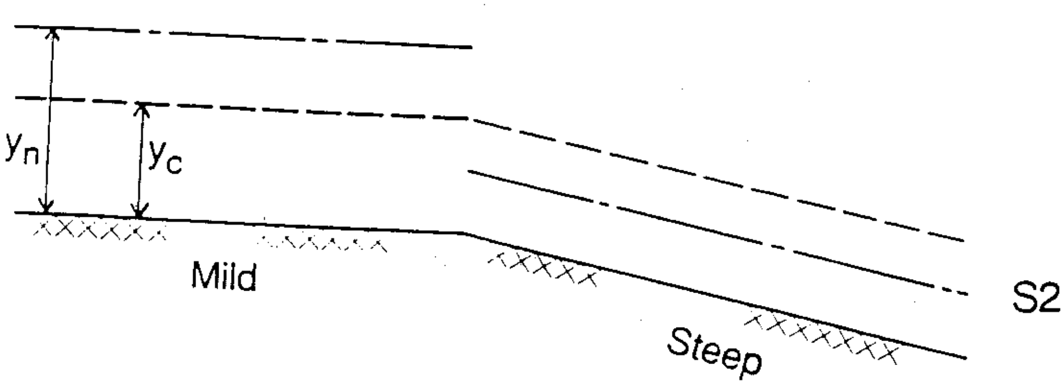




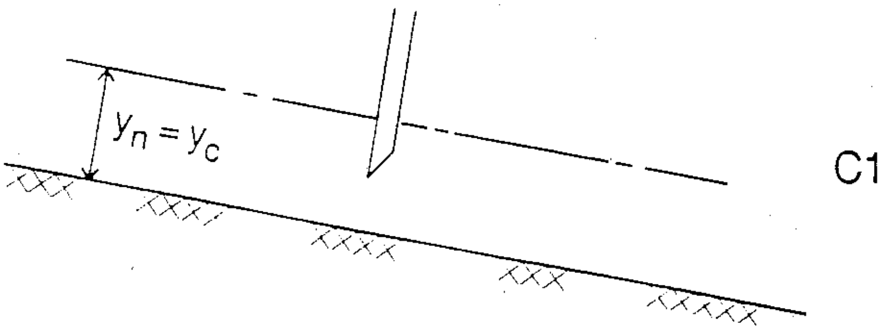
Steep slope $y_c > y_n$

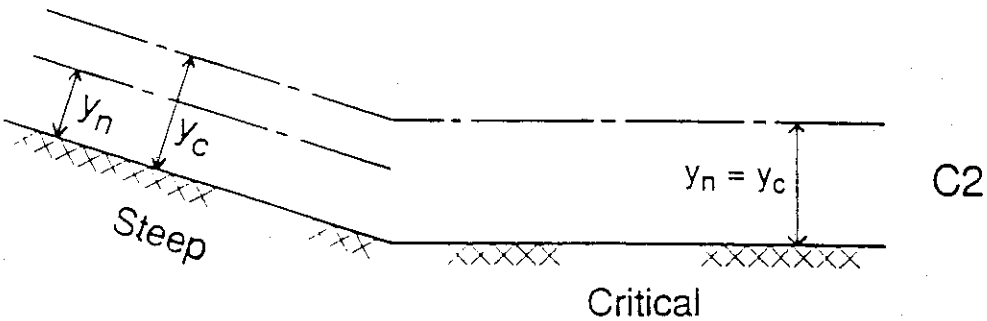




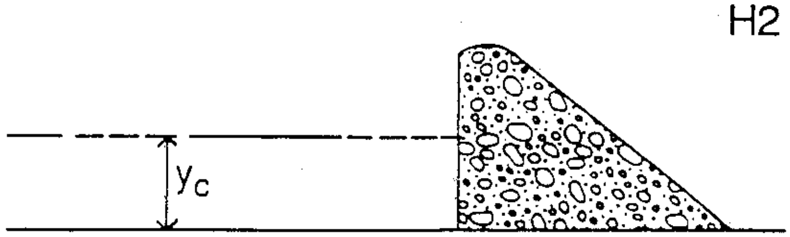


Critical slope $y_n > y_c$

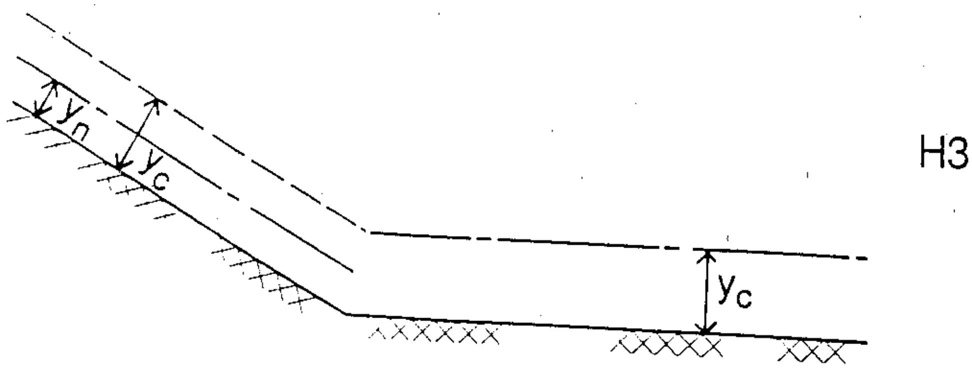




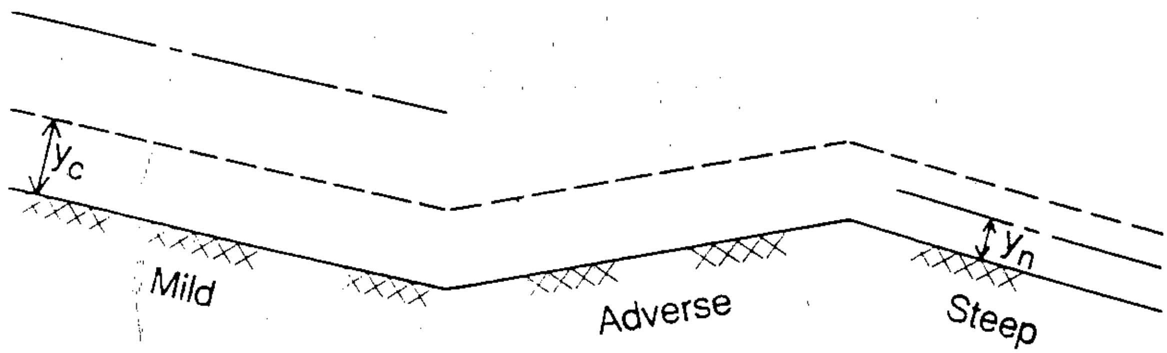
Horizontal slope $S_o = 0$, $y_n \rightarrow \infty$

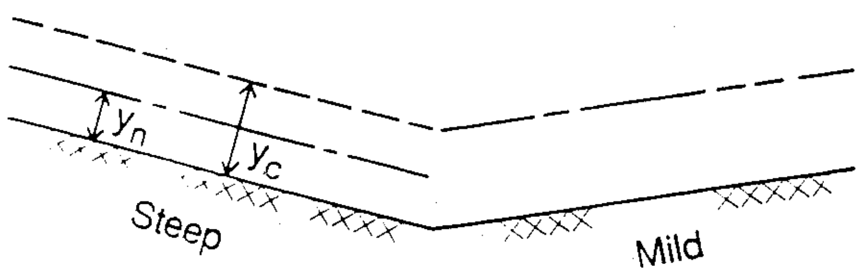


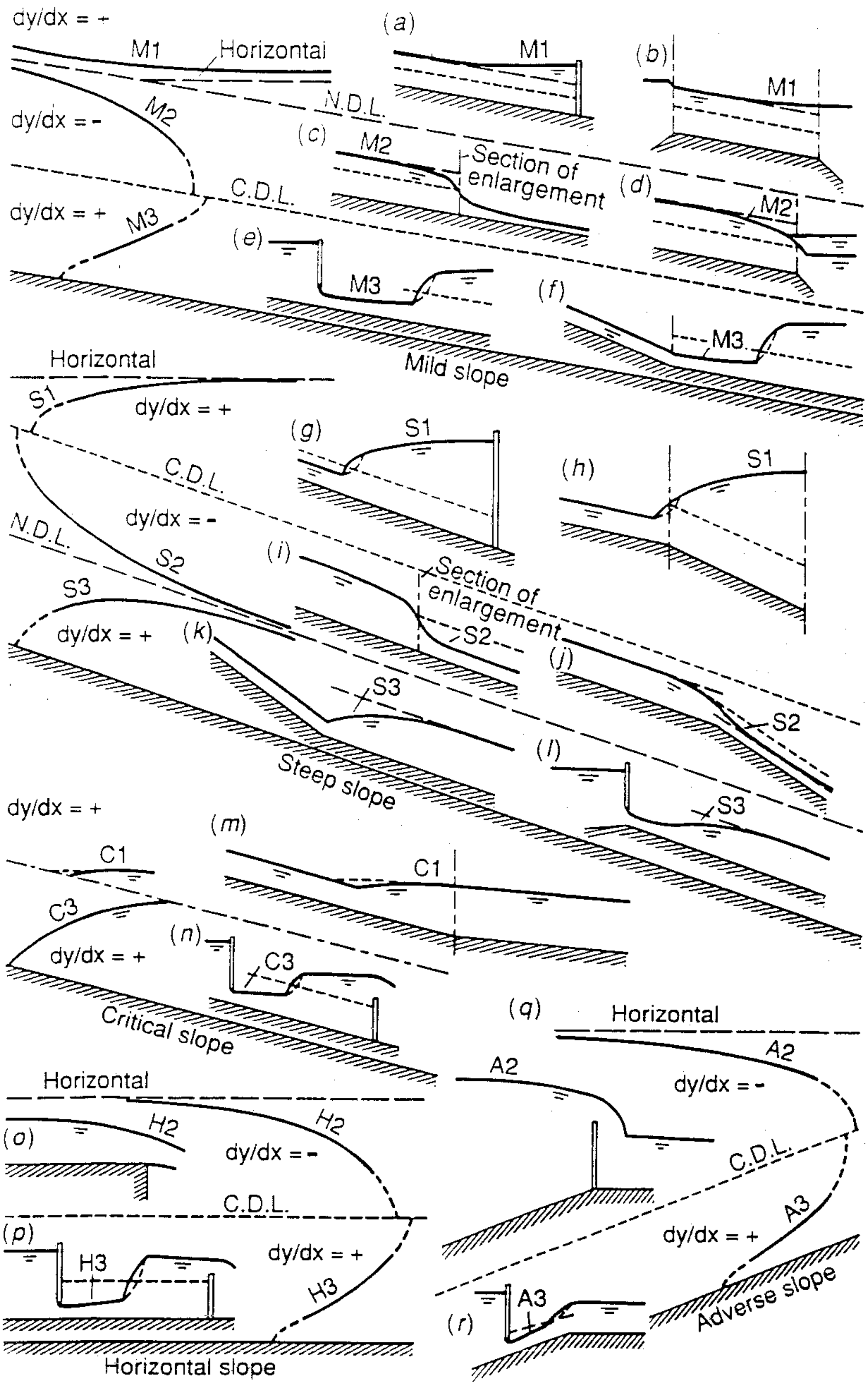
Horizontal slope $y_n \rightarrow \infty$

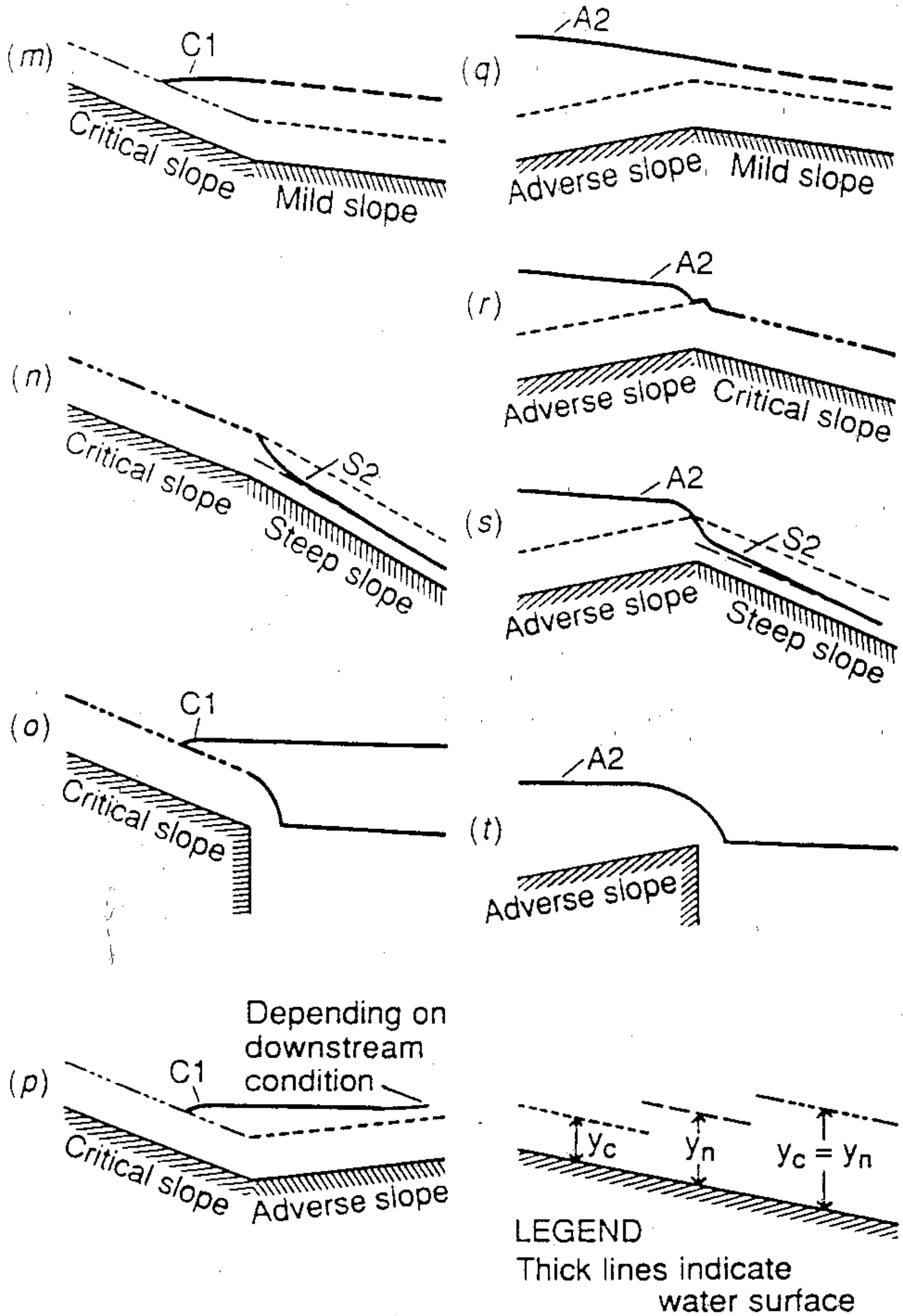


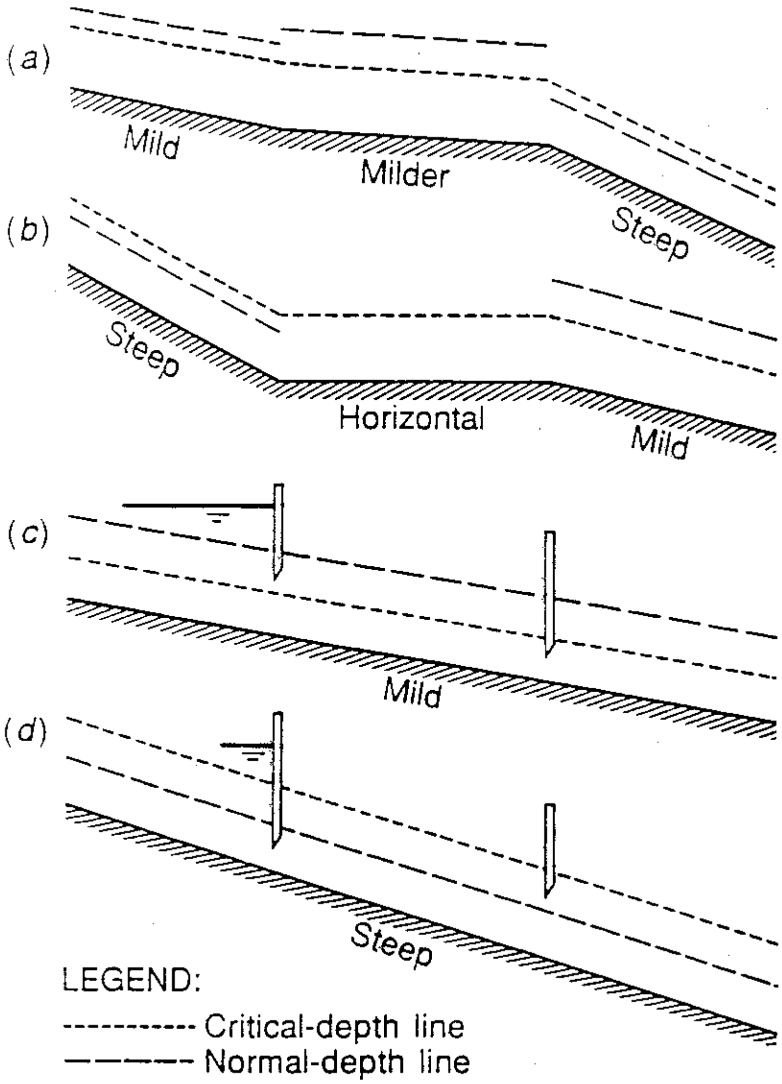
Adverse slope y_n undefined

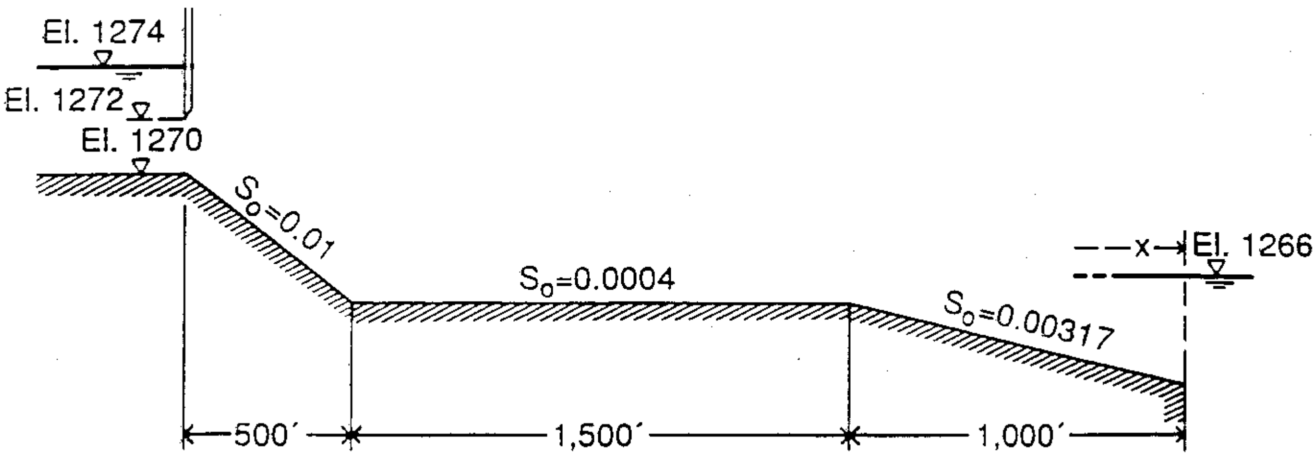





dy/dx = +
(a)
M1
Horizontal
(b)
M1
N.D.L.
dy/dx = -
M2
(c)
M2
Section of enlargement
(d)
M2
C.D.L.
dy/dx = +
M3
(e)
M3
(f)
M3
Mild slope
Horizontal
S1
dy/dx = +
(g)
S1
(h)
S1
C.D.L.
dy/dx = -
N.D.L.
S2
(i)
Section of enlargement
S2
S3
dy/dx = +
(k)
S3
(j)
S2
Steep slope
(l)
S3
dy/dx = +
(m)
C1
C1
C3
dy/dx = +
(n)
C3
Critical slope
(q)
Horizontal
A2
A2
dy/dx = -
Horizontal
H2
(o)
H2
dy/dx = -
C.D.L.
C.D.L.
(p)
H3
dy/dx = +
H3
dy/dx = +
A3
A3
(r)
Adverse slope
Horizontal slope


(m)
C1
Critical slope
Mild slope
(n)
S2
Critical slope
Steep slope
(o)
C1
Critical slope
(p)
Depending on downstream condition
C1
Critical slope
Adverse slope
(q)
A2
Adverse slope
Mild slope
(r)
A2
Adverse slope
Critical slope
(s)
A2
S2
Adverse slope
Steep slope
(t)
A2
Adverse slope
y_c
y_n
$y_c = y_n$
LEGEND
Thick lines indicate water surface



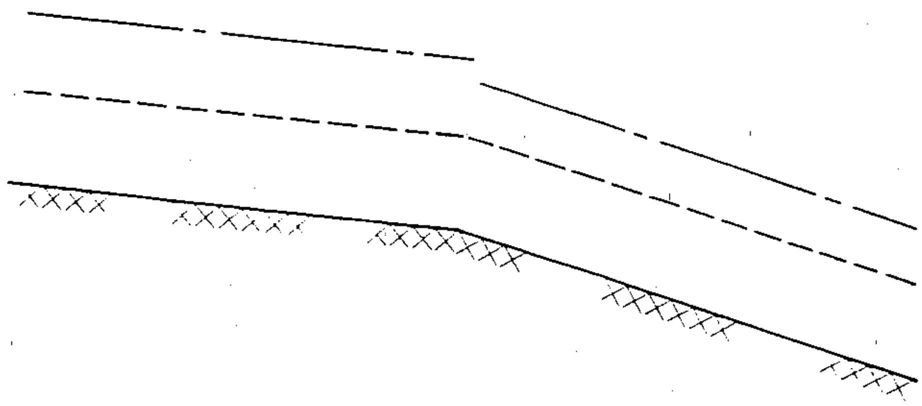


A rectangular channel, 20 ft wide, consists of three reaches of different slopes. The channel has a roughness coefficient n = 0.015 and carries a discharge of 500 cfs. Determine:

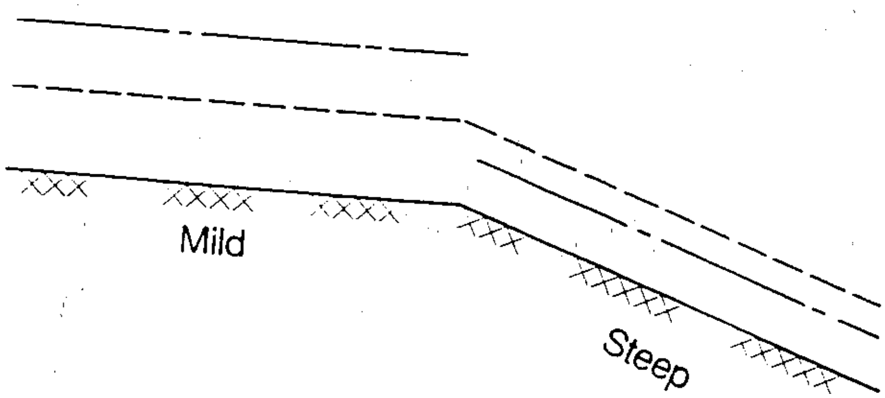
a. the normal and critical depths in each reach

Examples:

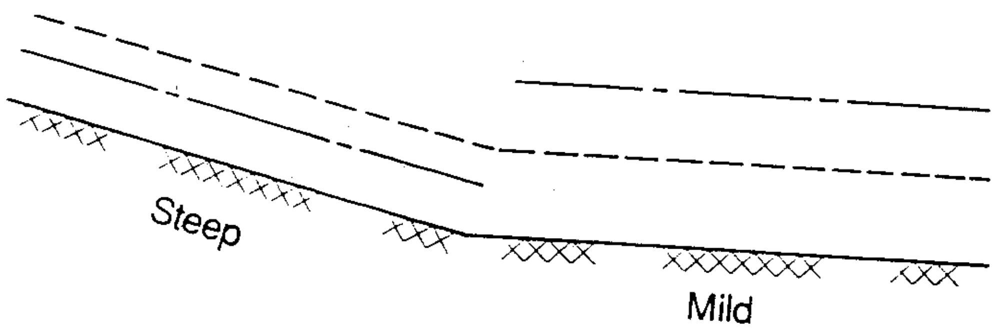
Case 1



Case 2



Case 3



TYPES OF FLOW PROFILES IN PRISMATIC CHANNELS

Channel Slope	Designation Zone 1	Designation Zone 2	Designation Zone 3	Relation of y to y_n and y_c (Zone 1 / Zone 2 / Zone 3)	General type of Curve	Type of Flow
Horizontal $S_o = 0$	None			$y > y_n > y_c$	None	None
		H_2		$y_n > y > y_c$	Drawdown	Subcritical
			H_3	$y_n > y_c > y$	Backwater	Supercritical
Mild $0<S_o<S_c$	M_1			$y > y_n > y_c$	Backwater	Subcritical
		M_2		$y_n > y > y_c$	Drawdown	Subcritical
			M_3	$y_n > y_c > y$	Backwater	Supercritical
Critical $S_o = S_c > 0$	C_1			$y > y_o = y_n$	Backwater	Subcritical
		C_2		$y_c = y = y_n$	Parallel to channel bottom	Uniform-critical
			C_3	$y_c = y_n > y$	Backwater	Supercritical
Steep $S_o\ S_c > 0$	S_1			$y > y_c > y_n$	Backwater	Subcritical
		S_2		$y_c > y > y_n$	Drawdown	Supercritical
			S_3	$y_c > y_n > y$	Backwater	Supercritical
Adverse $S_o < 0$	None			$y > (y_n)* > y_c$	None	None
		A_2		$(y_n)* > y > y_c$	Drawdown	Subcritical
			A_3	$(y_n)* > y_c > y$	Backwater	Supercritical

References

1. Backwater Curves in River Channels, Engineering Manual 1110-2-1409, U.S. Army Corps of Engineers, 1959.

2. Bradley, G. H., Hydraulics of Bridge Waterways, Hydraulic Design Series No. 1, Bureau of Public Roads, U.S. Government Printing Office, Washington, D.C., 1972, 111 p.

3. Bradley, J. N., The Use of Backwater in the Design of Bridge Waterways, Public Roads, Vol. 30, No. 10, October 1959, p. 221.

4. Bradley, J. N., Use of Backwater in Designing Bridge Waterways, Highway Research Board Bulletin 242, Washington, D.C., 1960, p. 57.

5. Chow, V. T., Open Channel Hydraulics, McGraw-Hill Book Co., Inc., New York, N.Y., 1959.

6. Computation of Backwater at Open Channel Constrictions, Open File Report, U.S. Geological Survey, 1955.

7. Eichert, B. S., Survey of Programs for Water-Surface Profiles, Journal of the Hydraulics Division, ASCE, Vol. 96, No. HY2, Proc. Paper 7098, February 1970, pp. 547-563.

8. Eichert, B. S., Critical Water Surface by Minimum Specific Energy Using the Parabolic Method, Hydrologic Engineering Center, U.S. Army Corps of Engineers, Davis, California.

9. Eichert, B. S. and Peters, J., Computer Determination of Flow Through Bridges, Hydrologic Engineering Center, U.S. Army Corps of Engineers, Davis, California.

10. Ezra, A. A., A Direct Step Method for Computing Water Surface Profiles, Trans. Am. Soc. Civil Engineers, Vol. 199, 1954, pp. 453-462.

11. Henderson, F. M., Open Channel Flow, The MacMillan Company, New York, N. Y., 1966.

12. Kindsvater, C. E. and Carter, R. W., Tranquil Flow Through Open Channel Constrictions, Trans. Am. Soc. Civil Engineers, Vol. 120, 1955, p. 955.

13. Liu, H. K., Gradley, J. N. and Plate, E. J., Backwater Effects of Piers and Abutments, Colorado State University, Report CER57HKL10, October 1957, 364 pp.

14. Tracey, H. J. and Carter, R. W., Backwater Effects of Open-Channel Constrictions, Trans. Am. Soc. of Civil Engineers, Vol. 120, 1955, p. 993.

15. Welty, K. H., Corry, M. L. and Morris, J. L., Hydraulics of Bridge Waterways, Bureau of Public Roads, Electronic Computer Program HY-4-69, Washington, D.C., 1969.

16. Yarnell, D. L., Bridge Piers as Channel Obstructions, U.S. Department of Agriculture, Tech. Bulletin No. 442, Washington, D.C., U.S. Government Printing Office, November 1934, 42 pp.

17. Yarnell, D. L. and Nagler, F. A., Flow of Water Over Railway and Highway Embankments, Public Roads, Vol. 11, No. 2, April 1930, p. 30.