Fractal Coagulation Kinetics

Bruce E. Logan
Department of Civil & Environmental Engineering
The Pennsylvania State University

Email: blogan@psu.edu
http://www.engr.psu.edu/ce/enve/logan.html
Global Carbon Cycling

Atmosphere

Photic zone

Carbon cycled

Deep Ocean

Marine snow

Net carbon sink

Sediments
Marine snow can’t form… but it does!

- Too few particles (phytoplankton), and therefore too few collisions to make large aggregates.

- Marine snow aggregates exist, so our calculations must be wrong.

- How can basis of calculations be improved?
Coagulation

No collisions  Unsuccessful collisions  Successful collisions & coagulation
Coagulation Theory

• Coagulation theory is quite old, dating back to Schmoluchowski (1917)

• Coagulation rate proportional to particle concentration squared.

• All particles are spheres.
Coagulation mechanisms

- Brownian motion
- Fluid shear
- Differential sedimentation
What paradigm shift is needed to explain the formation of marine snow?
Birth of Fractal Geometry

• In 1982, Benoît Mandelbrot publishes “Fractal Geometry” and fractal mathematics is born.

• Fractal scaling relationships are observed to apply in a variety of fields including geography, hydrology, turbulence, and mathematical solution sets.

• Colorful fractal pictures are developed.
Bacterial aggregates produced in the laboratory have a variety of shapes.

*Shewanella putrefaciens* grown on two different growth substrates under otherwise identical mixing and media conditions.
Particles that coagulate in nature are not spheres...

Source: Cover photograph of Deep-Sea Res. II 42(1), 1995; photograph by A.L. Alldredge
Acridine orange staining shows large holes in non-spherical biological aggregates...
...and acridine orange staining reveals interesting shapes of fractal objects!
Objectives

• Mathematically define “fractal” and “fractal dimension”

• Demonstrate that biological aggregates formed by shear and differential coagulation in the laboratory are fractal…

• … and that marine snow aggregates formed in nature are fractal.

• Show coagulation rates of fractal aggregates can be 1 million times faster than those of spheres.
Fractal: An object that is similar to the whole (in some fashion).

Fractal generator

Fractal object

Objects that are not fractal
Stochastic fractal

Deterministic fractal

Stochastic fractal
**Fractal dimension: definition**

Definition: Power ($D_n$) that characterizes how an aggregate property changes with size.

Examples: $N \sim l^{D_3}$ and $m \sim l^{D_3}$

$A \sim l^{D_2}$

$P \sim l^{D_1}$

where:

- $n =$ value of $D$ for Euclidean object
- $N =$ number of particles in aggregate
- $m =$ mass of aggregate
- $A =$ cross sectional area of aggregate
- $P =$ perimeter of aggregate.
Aggregate Properties: Euclidean Geometry

Volume (encased)

\[ V_e = \frac{\Pi}{6} d^3 = \xi d^3 \]

Number of particles in an aggregate

\[ N^* = \zeta \frac{V_p}{V_e} = \zeta \frac{\frac{\pi}{6} d^3}{\frac{\pi}{6} d_0^3} = \left( \frac{\xi_0 \xi}{\xi_0} \right) d^3 \]

where:

- \( \xi \) = shape factor
- \( \zeta \) = packing factor
Aggregate Properties: Fractal Geometry

Number of particles

\[ N^* = b_D \left( \frac{l_{ag}}{l_p} \right)^{D_3} \]

Volume

\[ V_{ag} = \xi_p l_p^3 b_D \left( \frac{l_{ag}}{l_p} \right)^{D_3} \]

Mass

\[ m_{ag} = \rho_p \xi_p l_p^3 b_D \left( \frac{l_{ag}}{l_p} \right)^{D_3} \]

where:

\[ b_D = \left( \frac{\xi \xi}{\xi_0} \right)^{D_3} \]
<table>
<thead>
<tr>
<th>Fractal Property</th>
<th>Fractal Scaling Relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid volume</td>
<td>$V = a_v l^{D_3}$</td>
</tr>
<tr>
<td>Mass</td>
<td>$m = a_m l^{D_3}$</td>
</tr>
<tr>
<td>Area</td>
<td>$A = a_A l^{D_2}$</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho = a_\rho l^{D_3-3}$</td>
</tr>
<tr>
<td>Porosity</td>
<td>$\epsilon = a_\epsilon l^{D_3-3}$</td>
</tr>
</tbody>
</table>
Simulations demonstrate a variety of fractal dimensions possible for Colloidal-Sized Aggregates

<table>
<thead>
<tr>
<th></th>
<th>Reaction-limited</th>
<th>Ballistic</th>
<th>Diffusion-limited</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monomer-cluster</td>
<td>Eden</td>
<td>Vold</td>
<td>Witten-Sander</td>
</tr>
<tr>
<td></td>
<td>$D = 3.00$</td>
<td>$D = 3.00$</td>
<td>$D = 2.50$</td>
</tr>
<tr>
<td>Cluster-cluster</td>
<td>RLCA</td>
<td>Sutherland</td>
<td>DLCA</td>
</tr>
<tr>
<td></td>
<td>$D = 2.09$</td>
<td>$D = 1.95$</td>
<td>$D = 1.80$</td>
</tr>
</tbody>
</table>
“Universality” of fractal scaling relationships for Colloidal-Sized Aggregates

- For colloidal aggregates formed by Brownian motion, fractal dimensions independent of type of colloid (gold, silica, latex spheres)

- Diffusion-limited aggregation (DLA)
  \[ D_3 = 1.8 \]

- Reaction-limited aggregation (RLA)
  \[ D_3 = 2.1 \]

- When \( D_3 < 2 \), then: \( D_3 < D_2 \)
Is there a “universality” of fractal dimensions for aggregates formed by mechanisms other than Brownian motion?

- **Type of particle**
  - Bacteria
  - Yeast
  - Inorganic microspheres

- **Coagulation mechanism**
  - Laminar shear
  - Turbulent shear
  - Differential sedimentation
Laboratory studies:

Biological aggregates
Methods to calculate fractal dimensions

Power law relationships:
- \( A \sim l^{D_2} \)
- \( N \sim l^{D_3} \)

Size Distributions:
- Steady State (SS)
- Two slope method (TSM)
- Particle concentration technique (PCT)
Fractal dimensions of biological aggregates

- Aggregates using pure cultures
  (Zoogloea ramigera, Saccharomyces cerevisiae)

- Aggregates sized, dispersed, and cells counted using acridine orange staining

- Fractal dimensions determined from log-log plots of:
  - size (l) and
  - number of particles (N) or Area (A)
Bacterial aggregates (Zoogloea ramigera)
### Fractal Dimensions of Different Biological Aggregates

<table>
<thead>
<tr>
<th>Microbe</th>
<th>Reactor</th>
<th>$D_2$ (± S.D.)</th>
<th>$D_3$ (± S.D.)</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>S. cerevisae</em></td>
<td>Test tubes (rotating)</td>
<td>1.92 (±0.08)</td>
<td>2.66 (±0.34)</td>
</tr>
<tr>
<td><em>Z. ramigera</em></td>
<td>Aerated bioreactor</td>
<td>1.78 (±0.11)</td>
<td>2.99 (±0.36)</td>
</tr>
<tr>
<td><em>Z. ramigera</em></td>
<td>Test tubes (rotating)</td>
<td>1.69 (±0.08)</td>
<td>1.79 (±0.28)</td>
</tr>
</tbody>
</table>

**Observations:**
- $D$ varies for different microorganisms
- $D$ is a function of growth conditions
Hypothesis:
Values of D are a function of the fluid mechanical environment

- Laminar shear (couette device)
- Turbulent shear (paddle mixer)
- Sedimentation (rolling cylinder)
Laboratory studies:
Inorganic aggregates
Couette device: laminar shear

From: Jiang and Logan (1996) J. AWWA
Paddle mixer: turbulent shear
Rolling cylinder: gravitational sedimentation
Image Analysis System - Particle length
(Galai - Microscope system)
Resistance-type Particle Counter-solid volume (Coulter Counter)
Fluorescent Microsphere Coagulation Experiments
Fluorescent Microsphere Coagulation Experiments
Fractal dimensions of microsphere aggregates from power laws

From: Logan and Kilps (1995) 
Water Research

Paddle Mixer
$D_2 = 1.89$

Roller
$D_2 = 1.68$
Methods to calculate fractal dimensions

Power law relationships:
- \( A \sim l^{D_2} \)
- \( N \sim l^{D_3} \)

Size Distributions:
- Steady State (SS)
- Two slope method (TSM)
- Particle concentration technique (PCT)
If the same population of particles are measured, the two size distributions must be equal, or

$$N(l) = N(V) \quad \text{or} \quad B_l l^{S_l} = B_V V^{S_V}$$

From fractal geometry,

$$B_l l^{S_l} = B_V \left( \xi_p l^3 b_D l^{-D} l^D \right)^{S_V}$$

The exponents on $l$ must be the same, so that

$$S_l = D S_V \quad \text{or} \quad D = \frac{S_l}{S_V} \quad (TSM)$$
TSM: a direct method for calculating $D_3$ if the slope is constant

From: Logan and Kilps (1995)
Water Research
PCT (Particle Concentration Technique): used for non-linear size distributions
PCT: linked data from two size spectra

\[ D_3 \] calculated as the slope of a line generated from a plot of solid volume (\( ds \)) versus length (\( l \))
Fractal Dimensions of microsphere aggregates vary for different fluid environments

<table>
<thead>
<tr>
<th>Aggregate Type</th>
<th>Method</th>
<th>Fractal dimension</th>
<th>Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laminar shear</td>
<td>TS (high salt)</td>
<td>1.43 - 1.74&lt;sup&gt;a&lt;/sup&gt;</td>
<td>b</td>
</tr>
<tr>
<td></td>
<td>TS (low salt)</td>
<td>1.92 (±0.04)</td>
<td>b</td>
</tr>
<tr>
<td>Turbulent Shear</td>
<td>TS</td>
<td>1.92 (±0.04)</td>
<td>c</td>
</tr>
<tr>
<td></td>
<td>Power (Size-area)</td>
<td>1.89 (±0.02)</td>
<td>c</td>
</tr>
<tr>
<td>Sedimentation</td>
<td>TS</td>
<td>1.59 (±0.16)</td>
<td>c</td>
</tr>
<tr>
<td></td>
<td>Power (Size-area)</td>
<td>1.68 (±0.02)</td>
<td>c</td>
</tr>
</tbody>
</table>

<sup>a</sup> D varies as a function of shear rate  
<sup>b</sup> Jiang and Logan (1996) *JAWWA*  
<sup>c</sup> Logan and Kilps (1995) *Water Research*
CONCLUSIONS

- There is no “universality” of fractal dimensions for biological aggregates
- $D_3$ varies from 1.4 - 3.0
Natural Systems:
Tank Experiment
Fractal dimensions during a coagulation event: a simulated phytoplankton bloom

Hypothesis:
Marine snow aggregates formed by physical coagulation.

Fractal dimensions should decrease during coagulation

Start: $D_3=3$
Finish: $D_3<2$

Tank experiment conducted by SIGMA group to monitor a coagulation event.
A phytoplankton bloom was simulated in a mesocosm (tank) in the laboratory to study coagulation.
Tank Inoculated on “Day 0” with:
-Seawater from near UC Santa Barbara
-Nutrients

Tank was mixed and lighted to match existing photocycle

Fig. 4. Phytoplankton biomass and particle number. Bars represent 95% confidence intervals. (A) Chlorophyll $a$; (B) phytoplankton abundance expressed as discrete particles, i.e. numbers of chains and single cells.

Nutrients were depleted by day 12

Fig. 5. Nutrient concentrations over the study. (A) Nitrate and silicate. (B) Phosphate and ammonium. Error bars were too small to be resolved graphically and are not shown.

Diatom bloom dominated by Chaetoceros spp.
The particle spectra indicated large increases in particles after day 6.

Particles measured here were 2-300 µm in length (by image analysis).

The measurements of solid particle volume (Coulter Counter, 2-300 um size fraction) parallel the measurements of dry weight.

Fig. 3. Total solid volume of particles in the size range of $2 \leq d_s \leq 50 \mu m$ (Coulter counter data from this study) and total particulate dry mass (data from Alldredge et al., 1994) in the mesocosm.

Note the largest size particles (aggregates) developed after day 6.

Half lives of TEP in a mesocosm during a simulated phytoplankton bloom

Fig. 2. (A) Size distributions, (B) concentrations, and (C) half-lives of TEP and phytoplankton particles in the mesocosm.
Fractal dimensions of particles (2-200 μm) decreased during the phytoplankton bloom in the tank as predicted.
Many of the particles measured in the 2-300 um size fraction were not phytoplankton.

Fig. 5. Abundance of phytoplankton (data from Alldredge et al., 1995) and non-phytoplankton particles (image analyzer data from this study) in different size classes on Days 7 and 12.

Natural Systems:

Field Measurements
Fractal dimensions of particles formed in natural systems

- Are large aggregates formed in lakes and marine systems fractal?
  - Ocean Cruises (10-day) off S. California (UCSB)
  - Field studies (daily excursions):
    - Monterey Bay, CA
    - Friday Harbor, (near Seattle, WA)
  - Lake Constance, Germany

- Formation of aggregates was found to be linked to the abundance of TEP
Friday Harbor, WA

Monterey Bay, CA
Oceanographic Cruises - Two Week Studies off the Coast of Southern California
Multi-investigator study included launching camera systems...

Alice Alldredge and Chris Gottchalk launch Sno-cam
...and on-board laboratory studies (Point Sur)
Smaller ships were used in the Monterey Bay Study
A smaller ship meant individual samples
A number of camera systems were used in the Monterey Bay Study: The UCSB Sno Cam
Camera system used by group from University of Florida
The Monterey Bay Aquarium Bay Research Institute (MBARI) Camera System
The depth and location of the camera system was remotely controlled from onboard the ship.
Field Measurements of Fractal dimensions ($D_3$) using PCT

- Monterey Bay
- Friday Harbor

Image analysis of Marine Snow using Photographs taken by Divers: Determining $D_2$

Diatom floc

Larvacean house
Image analysis done on particles in seawater by staining all particles with Acridine Orange
Fractal dimensions from Monterey Bay and UCSB studies: Marine Snow and Smaller Particles

<table>
<thead>
<tr>
<th>Aggregate Type</th>
<th>Method</th>
<th>Fractal dimension</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>MARINE SNOW (Divers photographs, large aggregates)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Miscellaneous</td>
<td>Power (area)</td>
<td>1.28 (±0.11)</td>
<td>a</td>
</tr>
<tr>
<td>Fecal pellets</td>
<td>Power (area)</td>
<td>1.34 (±0.16)</td>
<td>a</td>
</tr>
<tr>
<td>Amorphous</td>
<td>Power (area)</td>
<td>1.63 (±0.72)</td>
<td>a</td>
</tr>
<tr>
<td>Diatoms</td>
<td>Power (area)</td>
<td>1.86 (±0.13)</td>
<td>a</td>
</tr>
<tr>
<td>General</td>
<td>Power (mass)</td>
<td>1.52 (±0.19)</td>
<td>b</td>
</tr>
<tr>
<td>OCEAN PARTICLES (Particle size spectra; &lt;300 um particles)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&gt;40 µm</td>
<td>SSM (size distrib.)</td>
<td>1.48 - 1.92</td>
<td>c</td>
</tr>
</tbody>
</table>

Particle Formation & Lake Snow

- Lake Constance (Konstanz, or the Bodensee) is located at the intersection of Germany, Switzerland, and Austria.
- It is a deep, oligotrophic Lake formed by runoff from the Swiss Alps.
- Large aggregates, or Lake snow, have been observed to form there.
- In 1993, we studied particle formation during the spring to determine if TEP contributed to snow formation in the manner observed in California.
Particles in Lake Constance: Sample was double-stained with DAPI (fluorescence) for total bacteria and alcian blue
Particles in Lake Constance: Alcian blue (AB) was used to count TEP particles (AB stains negatively-charged polysaccharides)
Notice how the concentration and size of TEP increased during the spring season.
Half lives of TEP in Lake Constance

In the last week of April, there was a storm that increased shear in the water.

This produced a massive coagulation event and loss of TEP.

TEP loss was reflected in sediment trap data taken (data not shown).

Fig. 1. (A) The concentration of TEP; (B) sedimentation rates of particulate mass; and (C) the half lives of TEP particles in surface waters of Lake Constance during the spring of 1993. The peak in sedimentation follows the peak and disappearance of TEP from surface waters. Short half-lives (<1 day) support high TEP coagulation rates. (Calculations assume $G = 1 \text{ s}^{-1}$ and $\alpha = 1$.)
Conclusions From Lab & Field Studies

- \( D_3 \) is lower for aggregates formed in natural systems than those made in the laboratory
  - Natural systems: \( 1.28 < D_3 < 1.92 \)
  - Paddle mixers: \( 1.89 < D_3 < 1.92 \)
- \( D_3 \) are lower for aggregates formed by sedimentation than by shear
  - Sedimentation: \( 1.6 < D_3 < 1.7 \)
  - Shear: \( 1.8 < D_3 < 2.5 \)
- Low fractal dimensions of particles typical of systems where coagulation is important
  - Monterey Bay: \( D_3 = 1.6 \)
  - Friday Harbor: \( D_3 = 2.5 \)
Implications of the Fractal Structure of Aggregates

Settling Velocities

Coagulation Rates
Settling velocities of fractal aggregates versus spheres

- Settling velocities power laws differ:
  - Sphere: $U_s \sim d^2$
  - Fractal: $U_s \sim d^x$ where $x < 2$

- Fractal aggregates settle faster than spheres
Settling velocity: Spheres

From Stokes’ Law, the settling velocity of a sphere is:

\[
\dot{U}_s = \frac{g \Delta \rho (1 - \theta_{ag})}{18 \nu_w \rho_w} d_{ag}^2
\]

or in terms of number of particles in an aggregate,

\[
\dot{U}_s = \frac{g \Delta \rho v_{ag} N^*}{3 \pi \nu_w \rho_w d_{ag}}
\]

where \( N \sim d_{ag}^3 \)

Therefore, \( \dot{U}_s \sim d_{ag}^2 \)
Settling velocity: Fractals

From fractal scaling relationships,

\[
U_s = \left[ \frac{\Pi^2 g \xi \rho I_{\rho}^3}{18 b_d \rho_w \xi_2 V^{b_d}_{\rho}} \left( \rho_{\rho} - \rho_w \right) b_D I_g^{D_2-D-2} I_{ag}^{D-D_2+b_d} \right]^{\frac{1}{2-b_d}}
\]

The scaling relationship between \( U_s \) and \( I_{ag} \) is therefore

\[
U_{set} \sim I^{\frac{D_3-D_2+b_d}{2-b_d}}
\]

where \( b_d \) is a drag coefficient (unknown for fractals, but known for spheres).
Experimental measurement of settling velocities of fractal aggregates

Aggregates introduced one at a time. Settling velocity measured using a camera system.

Aggregate recovered for additional analysis

Recovery well.
Parameters measured for each aggregate

\[ l = \text{size of aggregate} \]
\[ A = \text{cross-sectional area} \]
\[ U_s = \text{settling velocity} \]
\[ N_p = \text{number of particles in aggregate, and therefore,} \]
\[ m = \text{mass of aggregate} \]
\[ v = \text{solid volume of aggregate} \]
Settling velocities of microsphere aggregates

Fractals settle 4-10 times faster than predicted by Stokes’ law

From: Johnson et al. (1997) ES&T
Implications of the Fractal Structure of Aggregates

Settling Velocities

Coagulation Rates
Aggregate coagulation rates

Particle coagulation rate can be described by

\[
\frac{dn_h}{dt} = \frac{1}{2} \sum_{i=j=h} \alpha \beta (v_i, v_j)n_i n_j - \sum_{i=1}^{\infty} \alpha \beta (v_i, v_h)n_i n_h
\]

where:

- \( n = \) particle concentration in a size interval
- \( \alpha = \) sticking efficiency
- \( \beta = \) collision efficiency

\[
\beta = \beta_{Br} + \beta_{sh} + \beta_{ds}
\]
Collision function: spheres

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Collision Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brownian motion</td>
<td>$\beta_{Br} = \frac{2 k_B T}{3 \mu_w} \left( \frac{d_i + d_j}{d_i d_j} \right)^2$</td>
</tr>
<tr>
<td>Turbulent shear</td>
<td>$\beta_{sh} = \frac{G}{6 \cdot 18} \left( d_i + d_j \right)^3$</td>
</tr>
<tr>
<td>Differential sedimentation</td>
<td>$\beta_{ds} = \frac{g \pi \Delta \rho}{72 \nu \rho} \left</td>
</tr>
</tbody>
</table>
Collision function: fractals

\[ \beta_{Br} = \frac{2k_B T}{3 \mu_w} (v_i^{-1/D} + v_j^{-1/D})(v_i^{1/D} + v_j^{1/D}) \]

\[ \beta_{sh} = \frac{G}{6 \xi pb} v^{1-(3/D)} (v_i^{1/D} + v_j^{1/D})^3 \]

\[ \beta_{ds} = \frac{\pi}{4} \left[ \frac{2g(\rho - \rho_w)}{b_d \rho_w \xi z^2 v^{b_d}} \right] \frac{1}{2-b_d} - \frac{1}{3} b_D \frac{1}{3} \frac{-1}{D} \frac{1}{2} \left( \frac{b_d - D_2}{2 - b_d} \right) \]

\[ \left| v_i^{D/(2-b_d)} - v_j^{D/(2-b_d)} \right| \left( \frac{1}{v_i^{D/2}} + \frac{1}{v_j^{D/2}} \right)^2 \]
Experimental measurement of fractal collision function: **Sedimentation**

1-um diameter yellow-green fluorescent microspheres

Aggregate recovered for additional analysis

Recovery well filled with water (no microspheres)
Example of fractal aggregate
Results: Differential sedimentation
Bead capture is a function of $D_3$.

From: Li & Logan (1997a):
ES&T 31(4):1229-36, Figure 7a

$D_3 = 1.81$

$D_3 = 2.33$
Explanation of curvilinear versus rectilinear collision model

Rectilinear model

Curvilinear model
Results: Differential sedimentation

From: Li & Logan (1997a):
ES&T 31(4):1229-36, Figure 7b

A: \( D_3 = 1.81 \)
B: \( x \quad D_3 = 2.33 \)
Experimental measurement of fractal collision function: Shear

Beaker filled with 1-um diameter yellow-green fluorescent microspheres

Paddle mixer apparatus used to measure collision efficiencies

Aggregate recovered for additional analysis after exposure time $t$ in beaker
Results: Shear

From: Li & Logan (1997b): ES&T 31(4):1237-42, Figure 7b
Experimental fractal collision functions: Shear, smaller particles

Aggregates and YG microspheres combined and coagulated.

Samples withdrawn, filtered, and aggregates examined on microscope.

Beaker filled with 1-um diameter yellow-green fluorescent microspheres.
Examples of small RB aggregates coagulated with monodisperse YG beads
Results: Fluid shear with small and large microsphere aggregates

Figure 5
Shear coagulation experiments repeated with bacterial aggregates and a monodisperse suspension of beads.

Bacterial aggregate stained with a fluorescent dye (acridine orange)
Results: Fluid shear with microsphere and bacterial aggregates

From: Serra & Logan (1999), Environ. Sci. Technol. Figure 6
CONCLUSIONS

1. Microbial and inorganic aggregates have fractal geometries with fractal dimensions ranging from ~1.5-2.5.

2. The fractal dimension, D, is a function of reactor type (paddle mixer, laminar shear, roller), particle type, and particle stickiness.
CONCLUSIONS ... cont'd

3. Settling fractal aggregates have lower drag coefficients than impermeable spheres (larger permeabilities) resulting in settling velocities that are an order-of-magnitude larger than spherical aggregates (of identical size and mass).

4. Collision frequencies of large fractal particles with much smaller particles are many times larger than those between spheres:
   \[ \sim 10 \times \text{larger for differential sedimentation} \]
   \[ \sim 10^6 \times \text{larger for turbulent shear} \]
Acknowledgements

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Office of Naval Research (ONR) (to B.E. Logan) Comissió Interdepartamental de Recerca i Innovaciò Technologica (CIRT)(to T. Serra)

<table>
<thead>
<tr>
<th>Students and Collaborators:</th>
<th>SIGMA team, especially:</th>
</tr>
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<tbody>
<tr>
<td>D. Wilkinson</td>
<td>A.L. Alldredge</td>
</tr>
<tr>
<td>J. Kilps</td>
<td>G. Jackson</td>
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<tr>
<td>Q. Jiang</td>
<td>U. Passow</td>
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<tr>
<td>C. Johnson</td>
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<tr>
<td>X. Li</td>
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<td>T. Serra</td>
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