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General density gradients in general domains: the "two-tank" method revisited

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Abstract An experimental technique for the creation of any statically stable density profile is presented. This technique is essentially a generalized, unsteady version of the "two-tank" method that is well known to the stratified flow community. It involves specifying the desired density profile and then solving an inverse problem in order to determine the necessary flow rates of light and dense fluid into the test-section tank. In addition to creating nonlinear density profiles, this technique is also useful for creating linear profiles in tanks whose planform areas vary with the vertical coordinate. The execution of this technique is carried out with computer-controlled peristaltic pumps. Several tests of the method are presented. The first consists of creating a hyperbolic tangent density profile in a rectangular tank. The second consists of creating, again in a rectangular tank, a density profile that is representative of those found in oceans and lakes. Finally, the third test involves creating a linear profile in a tank whose planform area is not constant. In all cases, the measured density profile shows excellent agreement with the requested profile.

1 Introduction

The great interest in stratified flows stems from the fact that most environmental and many industrial flows exhibit variations in density. This interest has resulted in a tremendous number and variety of laboratory investigations over the past decades. These controlled laboratory studies have led to the identification and quantification of numerous phenomena unique to stratified fluid mechanics. A thorough review is provided by Turner (1973).

Early laboratory studies of stratified flows were carried out for the case of discrete fluid layers. Two-layer systems of immiscible fluids such as water and carbon tetrachloride (Long 1954) or water and a freon–kerosene mixture (Lewis et al. 1974) enjoyed the benefit of a very sharp interface, ideal for interfacial wave studies, but have been largely abandoned due to safety and environmental considerations.

Two-layer systems of miscible fluids, such as fresh water and sugar water or fresh water and salt water (Thorpe 1968) have also proven to be very popular. These stratifications are not only inexpensive and safe, but they are, of course, more appropriate if mixing is being studied. With care, a relatively sharp interface, on the order of 1 cm, can be obtained and selective withdrawal can be used to "re-sharpen" an interface throughout the course of an experiment. While most investigators have sought to minimize the interface in this way, others (e.g., Davis and Acrivos 1967) took advantage of the finite region of continuous stratification, in this case to study internal wave interaction phenomena.

While two-layer systems are the most common, three or more layers are certainly possible as well. Schooley and Stewart (1963) and Thorpe (1968) carried this notion to the extreme limit by carefully layering a tank, from below, with progressively heavier fluid layers. After completing this task, a wait of a few hours yielded a fluid with a more or less linear density gradient, due to diffusion. In the same vein, an ambitious facility employed by Stillinger et al. (1983) created a remarkably linear density profile in a water tunnel by bringing together a total of 10 fluid layers via splitter plates.

The important and heavily cited work of Fortuin (1960) provided the fluid mechanics community with a much easier method of obtaining linearly stratified fluids. Known familiarly as the "two-tank" method, it essentially required establishing and maintaining constant flow rates between three containers of fluid – a storage tank, a mixing tank, and the test-section tank. Additional discussion of this method, and others, is provided by Oster (1965).

The vast majority of stratified flow experiments since these initial works have used two-layer or linear density profiles as well. This is because (i) these profiles are easy to construct, (ii) they allow for analytic progress in terms of solving the equations of motion and making scaling arguments, and (iii) they are often reasonable approximations to observed density profiles.

Clearly, more complex, piecewise-linear density profiles can easily be obtained (Linden 1975) by combining layers of constant and linearly varying density. Additionally, it is possible, to a degree, to "create" more complex profiles by disturbing two-layer profiles in a host of ways. For example, Linden (1980) found that, by dropping a grid vertically through a density interface, the interface would thicken considerably, yielding a diffuse pycnocline. Alternatively (Prych et al. 1964), towing an object, such as a flat plate, horizontally at a density interface can yield a similarly altered density profile.

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The current study reports on the development of a method whereby statically stable density profiles of *any* shape may be constructed by blending together supplies of light and dense fluid, such as fresh water and brine. Moreover, these profiles may be established in laboratory tanks whose planform areas vary with the vertical coordinate. This generalization of the classic "two-tank" method is first considered theoretically. Upon specification of the tank geometry, desired density profile and other experimental parameters, the flow rates of the light and dense fluid are determined.

Next, the implementation of this procedure is detailed and several test cases are discussed. Peristaltic pumps, which are easily controlled with an analog voltage input, are used to deliver the required flow rates. In each test case, the measured density profile is found to agree extremely well with the requested density profile.

There are numerous benefits to this method. First, the high level of experimental control that it affords will allow investigators to study phenomena in fluids whose density profiles deviate significantly from the limiting cases of a two-layer or linear profile. As an example, the density profile in a lake is often characterized by a well-mixed layer overlying a fairly diffuse thermocline. Below this intermediate region of rapidly varying density gradient, the gradient slowly tapers off with depth.

Second, the current method offers substantial savings, in terms of time and cost, to the investigator. As discussed above, a density profile of any shape can be, in principle, created by manually layering a tank in thin increments. This is highly labor intensive, however, requiring potentially dozens of solutions to be prepared and pumped into the laboratory tank. As another example, some techniques for creating linear density profiles in tanks with sloping boundaries require the tanks to be double their desired size. Not only does this result in increased equipment costs, but the necessity of repeatedly stratifying twice as much fluid as is required results in increased supply costs.

2 Formulation and procedure

Consider the experimental schematic shown in Fig. 1. At the bare minimum, three tanks and a mixer are required. To provide the flow between the tanks, it is, in principle, possible to use gravity, but this is not terribly practical. As such, two pumps will be required to deliver the required flow rates.

Looking at the configuration in detail, there is a storage tank, which contains salty water of some known concentration C_S (kg/m^3). This water is pumped at a flow rate of $Q_1(t)$ into a second tank, called the mixing tank. As indicated, in this general case, Q_1 is unsteady and therefore shown as a function of time. In the mixing tank, an electric (or air) mixer is used to blend the incoming salty water with the (initially) fresh water. As a result, the concentration of salt in the mixing tank will be a monotonically increasing function of time and is given by $C_M(t)$. Additionally, the volume of the mixing tank will be time varying and is given by $V_M(t)$. The initial volume of the mixing tank is given by V_{Mi}.

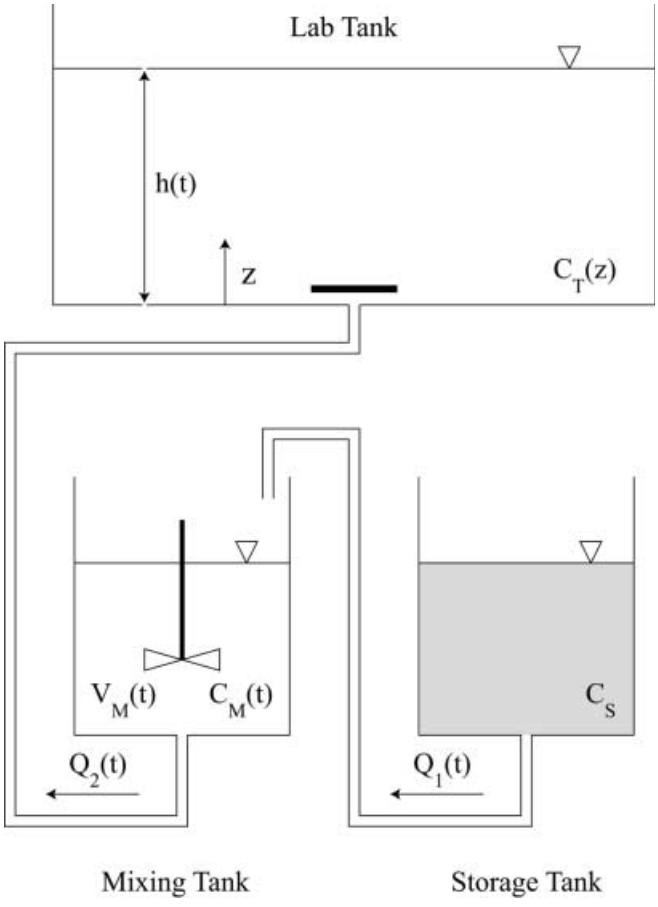

Fig. 1. Schematic of the experimental apparatus

The mixed fluid is then pumped at a flow rate of $Q_2(t)$ from the mixing tank into the laboratory, or test-section, tank. This third and final tank will, of course, be specific to the experiment and can be, generally speaking, of any shape and size. The volume of this tank is given by

$$V = \int_0^H A(z)\,\mathrm{d}z \ , \qquad (1)$$

where z is a coordinate with its origin fixed at the bottom of the tank (positive upward), $A(z)$ is the planform area of the tank and H is the total depth of the filled tank. The mass of salt in the tank, upon completion of filling, is given by

$$M_T = \int_0^H A(z)C_T(z)\,\mathrm{d}z \ , \qquad (2)$$

where $C_T(z)$ is the vertical distribution of concentration. The amount of time that it takes to fill this tank is denoted by T.

The mixed fluid is introduced into the laboratory tank by way of a diffuser plate, affixed to the bottom of the tank, which spreads out the fluid gently and minimizes mixing during the filling process. It should be noted that some researchers use the approach of introducing the fluid by way of a floating diffuser plate, which remains on the free surface of the laboratory tank, rather than the bottom. In this approach, the fresh water tank is the storage tank and

the (initially) salty water tank is the mixing tank. While the subsequent analysis is for the case of the bottom diffuser, the two problems are essentially mirror images of each other and solution of the surface diffuser case is a simple extension.

The main equations in the analysis are the conservation of salt and water in the mixing tank. These are given by

$$C_M(t)\frac{dV_M(t)}{dt} + V_M(t)\frac{dC_M(t)}{dt} = C_S Q_1(t) - C_M(t)Q_2(t) \tag{3}$$

$$\frac{dV_M(t)}{dt} = Q_1(t) - Q_2(t) \ . \tag{4}$$

Additionally, use will be made of the equations of volume and salt conservation in the laboratory tank.

$$\frac{d}{dt}\int_0^{h(t)} A(z)\, dz = Q_2(t) \tag{5}$$

$$\frac{d}{dt}\int_0^{h(t)} A(z)C_T(z)\, dz = C_M(t)Q_2(t) \ . \tag{6}$$

2.1
Constant *N*, constant *A*

If the laboratory tank is of constant planform area and a constant buoyancy frequency is desired, the problem reduces to the "steady" form of the two-tank method. Note, first of all, that the buoyancy frequency is defined as

$$N^2 = -\frac{g}{\rho_0}\frac{\partial \rho}{\partial z} \ ,$$

where ρ_0 is a reference density. A constant N amounts to a linear density (or salt concentration) profile.

It is straightforward to show, by combining (3) and (4), that a linear density profile is obtained if $Q_2 = 2Q_1 =$ constant. Facilitating this in the laboratory, to a satisfactory degree, is a simple matter not requiring any sort of active flow control. Most commonly, two small centrifugal pumps are used to move fluid between the three tanks. Since the flow through such a pump varies with the head across the pump, which in this case is changing as the water surfaces rise and fall, periodic monitoring of Q_1 and Q_2 and adjustment of valves between the three tanks is required.

From a design point of view, it is a simple matter to show that the minimum initial volumes in both the mixing and storage tanks are equal to $V/2$. Additionally, if the maximum desired concentration of salt in the laboratory tank is specified ($C_T(0)$), the required storage tank concentration to achieve this is given by

$$C_S = 2\frac{V_{Mi}}{V}C_T(0) \ . \tag{7}$$

2.2
Variable *N*, variable *A*

If the restrictions of constant planform area and constant buoyancy frequency are now removed, the problem becomes rather more interesting. The forward problem, in which the time-varying flow rates, tank hypsographic data, storage tank concentration, and initial mixing tank volume are specified and the resulting laboratory tank density profile is computed is relatively easy. First, the laboratory tank volume and filling time are computed; the latter from

$$\int_0^T Q_2(t)\, dt = \int_0^H A(z)\, dz \ . \tag{8}$$

Next, $V_M(t)$ and $C_M(t)$ are obtained by numerically integrating (3) and (4).

Finally, the temporal profile of concentration in the mixing tank must be converted to the spatial (vertical) profile of concentration in the laboratory tank. In other words, it is possible to make a change of variables by noting that fluid leaving the mixing tank at some time t' ends up at some particular elevation z' in the laboratory tank. The easiest approach is to work backwards in time from $t = T$ to $t = 0$. Specifically, during an interval of time Δt, a layer of fluid of concentration $C_M(t')$ and initial thickness

$$\Delta z_i = \int_{t'}^{t'+\Delta t} \frac{Q_2(t)\, dt}{A(0)}$$

is introduced at the bottom of the laboratory tank. As this layer moves upward, its thickness deforms if the planform area changes. Thus, the thickness of the layer at its final location, as illustrated in Fig. 2, is given by

$$\Delta z_f = \Delta z_i \frac{A(0)}{A(z')} \ .$$

While of predictive use, this forward problem is not helpful from a design point of view. After all, the stated goal is to be able to specify the final density profile and then figure out what it takes to accomplish this. This *inverse* problem is rather more complicated and there is no unique combination of suitable initial conditions.

To carry out the inverse problem, it is necessary to specify the desired laboratory tank density profile and, as with the forward problem, the hypsographic data of the laboratory tank. Furthermore, $Q_2(t)$ is taken to be an input to the problem. Depending upon the particular geometry (i.e., the diffuser size and gap height) of the individual facility, there will be a range of fairly optimal values of Q_2. For example, if the flow rate through the diffuser is too high, there will be significant and undesirable local mixing at the diffuser exit; too low and it may take prohibitively long to fill the laboratory tank. For the experiments detailed in Sect. 3.2, $Q_2 \sim 0.2$–0.8 gpm proved to be fairly ideal. It must be stressed that Q_2 is, in general, a function of time, i.e., not necessarily constant as is the case in the "classic" two-tank method. Assuming $Q_2 =$ constant certainly simplifies the present analysis and is therefore desired, but, as is discussed in Sect. 3.2, there are cases where it may be necessary to select Q_2 to be a more complex function.

In principle, the storage tank concentration C_S can be treated, as was done in Sect. 2.1, as an output of the problem, with the goal perhaps of minimizing the amount of salt that must be used per experiment. However, it turns out, from a practical point of view, to be more convenient

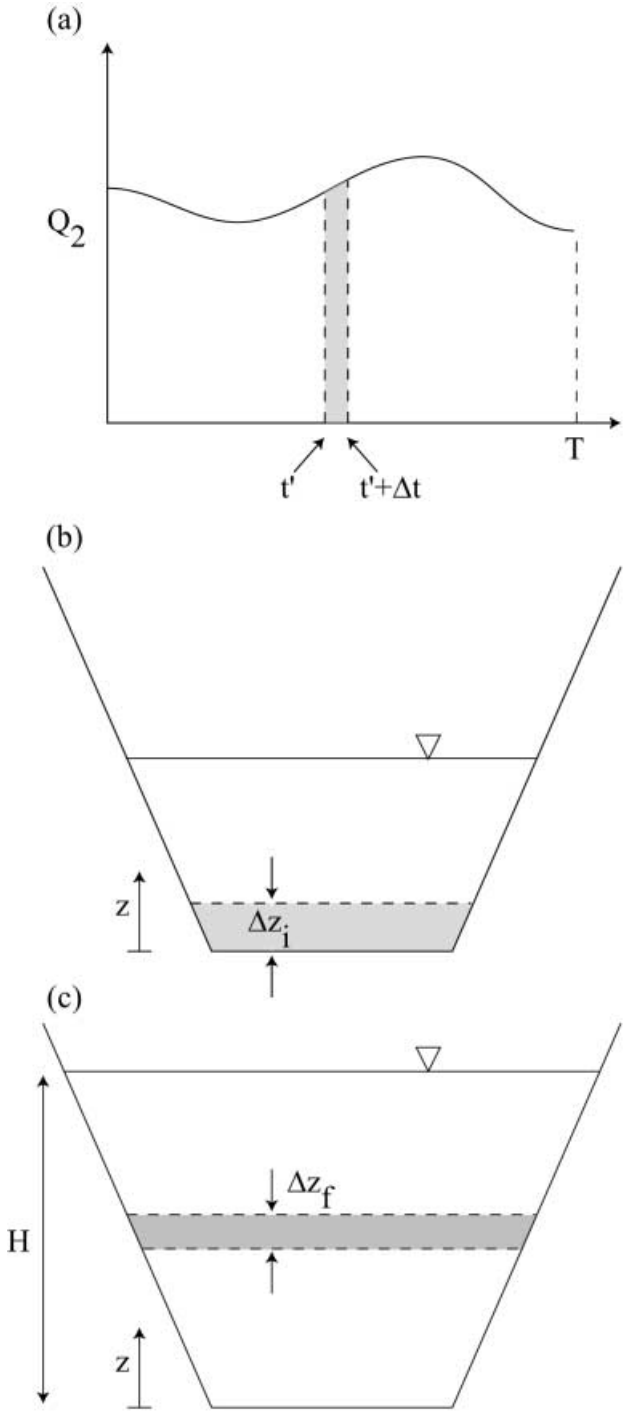

Fig. 2. **a** Volume of fluid leaving the mixing tank and entering the test-section tank at time t'; **b** initial layer thickness of this volume; **c** final layer thickness of this volume

to treat it as an input to the problem. In the former case, a researcher would need to adjust the density of a large volume of fluid to a fairly precise value. This is an iterative and cumbersome task that would involve multiple additions of salt and water. In the latter case, the researcher needs only to prepare the storage tank solution to some reasonable value of concentration. Upon measuring this concentration a single time, the subsequent analysis yields the $Q_1(t)$ required to obtain the desired $C_T(z)$.

The procedure is as follows. First, as with the forward problem, the tank volume and filling time are computed. Next, the vertical profile of concentration in the laboratory tank is converted to $C_M(t)$. Here, the filled tank is discretized into thin slices, each of known concentration and volume. Starting from the bottom of the tank and working upwards, the correlation between z and t (i.e., $C_T(z)$ and $C_M(t)$) is determined by calculating how much time it took for each of these layers to flow into the laboratory tank. Finally, once $C_M(t)$ is known, (3) and (4) are integrated to find, in this case, $V_M(t)$ and $Q_1(t)$.

3 Experiments

3.1 Apparatus

Having determined the flow rates necessary to create a desired density profile, the second step is the delivery of these flow rates, for which there are several options. Benielli and Sommeria (1998) used stepper motors to advance what amounted to the plungers of large syringes in their study of the Faraday resonance of internal waves in linearly stratified fluids. This is a fine and precise method, but one that does not lend itself well to large volumes of water.

A second alternative is to utilize controlled proportional solenoid valves. A number of vendors offer integrated flow control devices, which consist of a valve and an electronic flow sensor to provide feedback in the system. In these cases, a 4–20 mA input control signal will open or close the valve, thereby controlling the flow. As these controllers do not include any means for driving the flow, an external pump will have to be included in the system. Special care must be exercised, therefore, in selecting an appropriate pump so as not to overload the controller.

A third option, and the one adopted in this study, is to control the flow by controlling the pumps directly. Controlling standard centrifugal pumps is difficult for a variety of reasons, so peristaltic pumps were chosen instead. Specifically, two Masterflex pumps (Cole–Parmer, model 07549–52), which accept a 4–20 mA input and each deliver a flow rate of 0.2–2.0 gpm were used. While peristaltic pumps have the distinct advantage of being quite linear in operation, initial tests indicated that they were not nearly as linear as purported to be, particularly at the extremes of their range. Therefore, a simple calibration and 4th-order polynomial fit between current input and flow rate output was performed. Finally, note that the heads on peristaltic pumps are "stackable." In other words, for applications to very large laboratory tanks, two heads can be placed on the pump delivering Q_2, thereby minimizing the fill time.

The rest of the experimental apparatus consists of components familiar to those who work with stratified flows: a MicroScale conductivity and temperature instrument (MSCTI) (Head 1983), a computer-controlled translation stage (Velmex unislide assembly) for traversing the probe, and associated data acquisition equipment (National Instruments A/D PCI-6024E, Labview). Highly resolved density profiles were obtained by vertically traversing the MSCTI through the water column at a speed of

12.5 mm/s while sampling at a rate of 50 Hz. Temperature and conductivity data were converted to density data using the procedure outlined by Rehmann (1995).

3.2 Results

To test the ability of the designed system to faithfully reproduce requested density profiles, three trials were carried out. All were facilitated by stratifying a laboratory tank having a width of 45.7 cm and a total depth of 30.5 cm. The first two trials involved creating nonlinear density profiles in a tank of constant planform area. For these cases, the length of the domain was 61.0 cm. The third trial involved creating a linear density profile in a domain whose length and, therefore, planform area varied with elevation.

The goal of the first trial was to create a density profile having a hyperbolic tangent shape. In contrast to a two-layer model, which has an infinitely thin pycnocline, the smooth transition of the hyperbolic tangent profile yields a more diffuse pycnocline. This is often more representative of density profiles actually observed in the natural environment. While a more or less tanh profile can be obtained by agitating an initially two-layer system (Prych et al. 1964; Linden 1980), the current method offers a greater degree of control and eliminates the trial-and-error nature of those approaches.

As demonstrated by Fig. 3, the agreement between the requested and the measured density profiles is exceptional. Also shown are the flow rates of the two pumps that were used to achieve this profile. In this case, the chosen profile for $Q_2(t)$ was driven by the fact that the peristaltic pumps do not operate down to a flowrate of zero. Rather, there is a cutoff, found to be at a value of ~ 0.1 gpm, below which the rotors cease to rotate. Thus, $Q_2(t)$ was chosen to help minimize the amount of time during which the computed

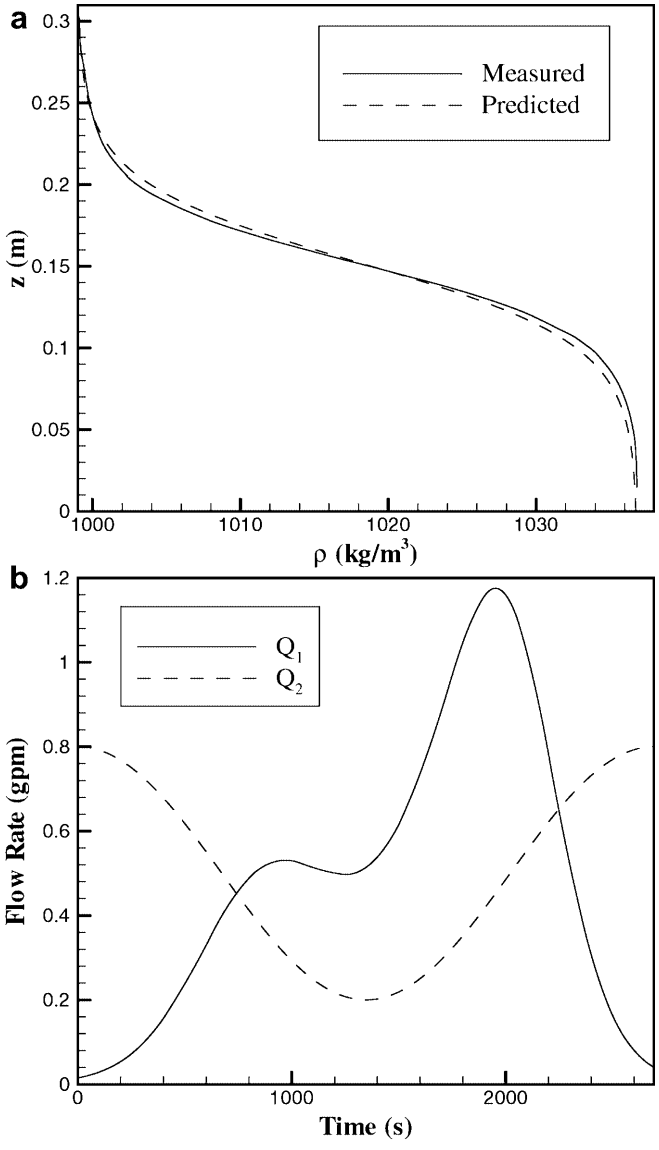

Fig. 3. **a** Comparison between requested and measured density profiles for a hyperbolic tangent shape; **b** required flow rates. $C_S = 61$ kg/m^3, $V_{Mi} = 25$ gal

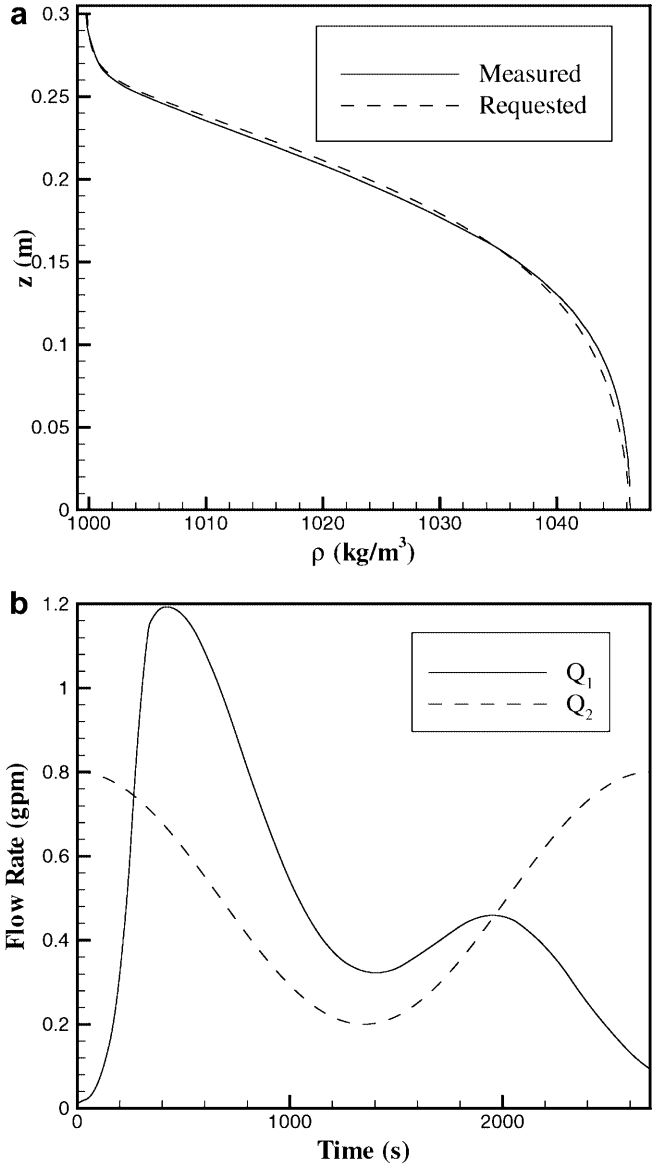

Fig. 4. **a** Comparison between requested and measured density profiles for a double-tanh shape; **b** required flow rates. $C_S = 81$ kg/m^3, $V_{Mi} = 25$ gal

$Q_1(t)$ fell below this cutoff value. Specifically, for this and the subsequent test, $Q_2(t)$, in gpm, was chosen to be

$$Q_2(t) = 0.5 + 0.3 \cos\left(\frac{2\pi t}{T}\right) ,$$

where the fill time T was 2,695 s.

With this as a successful first attempt, a second and more challenging profile was attempted. Lamb (2000) recently demonstrated that numerical simulations of internal solitary wave breaking are extremely sensitive to the shape of the density profile. Specifically, he indicated that the presence of a mixed layer at the free surface had a profound effect upon the dynamics. As indicated by Fig. 4, an attempt was made at creating a density profile characterized by a thin mixed layer, fairly sharp pycnocline, and slow decay of N below that. Essentially, this profile is two hyperbolic tangent profiles matched at the depth of the pycnocline. Again, the agreement between the requested and measured density profiles is excellent.

The third test was to create a linear density profile in a domain with variable A. The ability to do this is of use, for example, in studies of breaking internal waves on sloping boundaries. A traditional wave flume has a vertical wavemaking paddle at one end and a sloping beach at the other end. If the "steady" two-tank method were employed in this case, the resulting density profile would be nonlinear due to the fact that the planform area varies with the vertical coordinate. There are, it should be pointed out, several ways to circumvent this in addition to the current method. For example, Ivey and Nokes (1989) used a wave tank that was shaped like a parallelogram and had a hinged wavemaker located in the center of the tank. In this case, half of the tank functioned as the test section, and the other half was not utilized. The main disadvantage of this approach is that the experimental tank needs to be twice the size of the desired domain.

Another possibility is to linearly stratify a rectangular flume and then carefully slide a sloping boundary into place (Cacchione 1970). Given sufficiently tight tolerances or some type of gasket material, the slope will effectively isolate the trapezoidal working portion of the tank. This method was also used by Phillips et al. (1986), although in a slightly different context. While simple to carry out, this method is appropriate only in the case of a planar slope. While planar slopes are often sufficient approximations and, indeed, they have been the focus of intensive study in the past (Cacchione and Wunsch 1974; Ivey and Nokes 1989; Taylor 1993), recent theoretical (Gilbert and Garrett 1989; Muller and Liu 2000) and numerical (Legg and Wunsch 1999) studies have demonstrated significant effects of higher-order derivatives of topography.

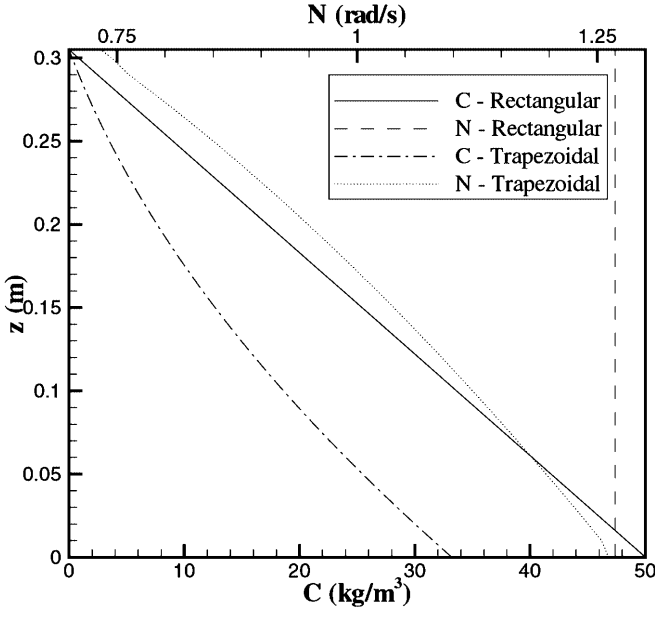

Fig. 5. Comparison of computed density profiles and buoyancy frequency profiles in rectangular and trapezoidal domains for the conditions $Q_1 = 0.25$ gpm, $Q_2 = 0.5$ gpm, $C_S = 80$ kg/m^3, $V_{Mi} = 18$ gal

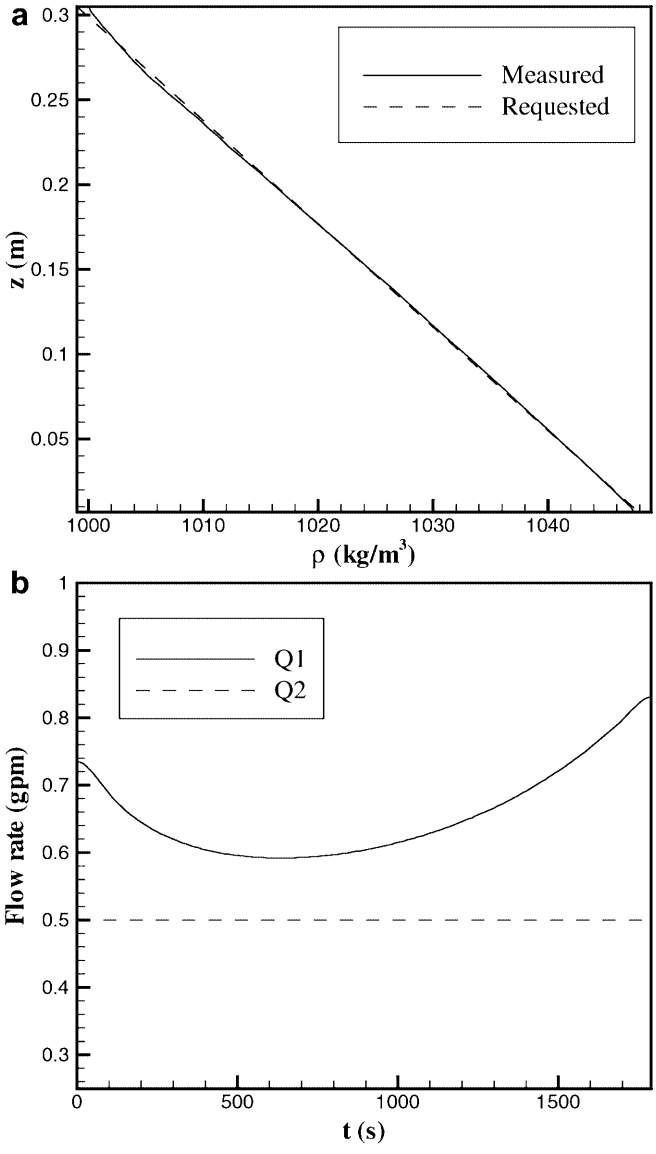

Fig. 6. **a** Comparison between requested and measured linear density profiles in a trapezoidal domain; **b** required flow rates. $C_S = 80$ kg/m^3, $V_{Mi} = 18$ gal

To demonstrate the ability of the generalized two-tank method to handle variable A, a trapezoidal domain was constructed where the length of the tank varied from 61.0 cm at the bottom to 20.0 cm at the free surface. The width and total depth were as in the previous examples. Consider, first, the effect of the shape of the domain. For example, if the conditions $Q_1 = 0.25$ gpm, $Q_2 = 0.5$ gpm, $C_S = 80$ kg/m^3 and $V_{Mi} = 18$ gal are applied to the rectangular domain used in the first two examples, a perfectly linear profile with $C_T(0) = 50$ kg/m^3 is obtained. If these same conditions are used with the trapezoidal domain, the resulting density profile demonstrates significant curvature, as indicated by Fig. 5. Note as well the differences in the vertical distribution of N.

In order to reproduce the linear profile, with the same $C_T(0)$, Q_2, C_S, and V_{Mi}, in the trapezoidal domain, the flowrate between the storage and mixing tanks (Q_1) must be as illustrated in Fig. 6b. The comparison between the requested linear profile and the measured profile is shown in Fig. 6a and indicates nearly perfect agreement.

4
Concluding remarks

An experimental technique, based upon the familiar two-tank method, for the creation of general density gradients has been presented. Upon specification of tank geometry, the desired density profile and other suitable initial conditions, the unsteady conservation equations were solved in order to determine the flow rates needed to achieve the requested profile. Implementation of the analysis was facilitated with externally controlled peristaltic pumps.

Three trials, designed to test the ability of the technique to (i) create nonlinear density profiles and (ii) create linear density profiles in tanks with variable planform area, were conducted. In all cases, the agreement between the requested and measured density profiles was excellent.

This technique may find application to the study of internal wave dynamics and breaking. Most previous studies have been confined to linear density gradients and linear sloping beaches, but recent theoretical and numerical work have suggested the significance of higher derivatives of both quantities.

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