There are 12 questions on 10 pages. Please read each problem carefully before starting to solve it. For each multiple choice problem 4 answers are given, only one of which is correct. Mark only one choice. For partial credit questions, all work must be shown - credit will not be given for an answer unsupported by work.

NO CALCULATORS ARE ALLOWED.
PLEASE DO NOT WRITE IN THE BOX BELOW.

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Total:_______
1. (4 points) Identify each of the following equations as linear or non-linear and also determine their order.
   
   (a) \( y \frac{dy}{dt} = t \)
   
   (b) \( \left( \frac{d^2y}{dt^2} \right)^3 + \left( \frac{dy}{dt} \right)^3 + y^3 = t^3 \)
   
   (c) \( \sin(t) \frac{dy}{dt} + t^5 y = (1 - t^2) \frac{d^2y}{dt^2} \)
   
   (d) \( (1 + y)\sin^2 t + (\frac{d^3y}{dt^3} + y)\cos^2 t = 1 \)

2. (6 points) The initial value problem
\[
(4 - t^2)y' + \ln(t)y = \sin(t) \quad y(1.33) = 3.14159
\]

is certain to have a unique continuous solution on the interval

(a) \((-4, 4)\)
(b) \((-2, 2)\)
(c) \((0, 2)\)
(d) \((0, 3.14159)\)
3. (6 points) The general solution to the equation

\[ y'' - 6y' + 9y = 0 \]

has the form

(a) \( ce^{-3t} \)
(b) \( c_1 e^{-3t} + c_2 e^{-3t} \)
(c) \( c_1 e^{-3t} + c_2 te^{-3t} \)
(d) \( c_1 e^{-3t} + c_2 e^{-3t^2} \)

4. (6 points) If \( y(t) \) is the solution to the initial value problem

\[ y' = y(y^2 - 4) \quad y(0) = 3. \]

Then \( \lim_{t \to \infty} y(t) = ? \)

(a) 3
(b) 2
(c) -2
(d) \( \infty \)
5. (6 points) Let $y_1$ and $y_2$ be two solutions to the linear equation

$$2t^2 y'' - ty' - y = 0.$$ 

Then the Wronskian of $y_1$ and $y_2$ must be a constant multiple of

(a) $\sqrt{t}$  
(b) $t$  
(c) 1  
(d) $t^2$

6. (6 points) Solutions to

$$(x \cos y) \frac{dy}{dx} = \sin y$$ 

are,

(a) $y(x) = \tan(x + C)$  
(b) $y(x) = \sin^{-1}(-\frac{1}{x} + C)$  
(c) $y(x) = \sin^{-1}(Cx)$  
(d) $y(x) = \sin^{-1}(\ln(x) - \frac{x}{2} + C)$
7. (10 points) Solve the initial value problem.

\[ t^3 y' + 4t^2 y = e^{-t} \quad y(1) = 0 \]
8. (10 points) A tank has 100gal of water and 100lb of salt mixed in it. Water enters the tank at the rate of 3gal/min with concentration of salt in it, at time $t$ given by $e^{t}$lb/gal. A well mixed solution leaves the tank at the same rate of 3gal/min. Find a formula for the amount of salt in the tank at any time $t$.

Find also the eventual concentration of salt in the tank.
9. (12 points) Show that the equation

\[(4xy - 3)y' + 2(y^2 + x) = 0\]

is exact.

Find the general solution.
10. (10 points) Solve the following initial value problems:

(a) \[ y'' + 2y' + 5y = 0, \quad y(0) = 0, \quad y'(0) = -1. \]

(b) \[ y'' + \frac{1}{t}y' = 1 \quad y(2) = -2ln(2), \quad y'(2) = 2. \]
11. (12 points) Find $\alpha$ so that the solution to the initial value problem

$$y'' + 3y' - 4y = 0, \quad y(0) = \alpha, \quad y'(0) = 1,$$

converges to 0 as $t \to \infty$. 

12. (12 points) Given that $y_1 = t$ is a solution to the equation

$$t^2 y'' - 4ty' + 4y = 0,$$

use the method of reduction of order to find a second solution which is not a constant multiple of $y_1$.

Show that the two solutions above are linearly independent on the interval $(0, \infty)$. 