There are 9 questions on 8 pages. Please read each problem carefully before starting to solve it. For each multiple choice problem 4 answers are given, only one of which is correct. Mark only one choice. For partial credit questions, all work must be shown - credit will not be given for an answer unsupported by work.

NO CALCULATORS ARE ALLOWED.

PLEASE DO NOT WRITE IN THE BOX BELOW.

A TABLE OF LAPLACE TRANSFORMS IS ATTACHED.
1. (6 points) Which of the following systems corresponds to the 2nd order differential equation:

\[ 2y'' + 4y' + 8y = 0 \]

(a) \[
\begin{align*}
x_1' &= x_2 \\
x_2' &= -4x_1 - 2x_2
\end{align*}
\]

(b) \[
\begin{align*}
x_1' &= 4x_1 + 2x_2 \\
x_2' &= 2x_1 + 4x_2
\end{align*}
\]

(c) \[
\begin{align*}
x_1' &= x_2 \\
x_2' &= 4x_1 + 2x_2
\end{align*}
\]

(d) \[
\begin{align*}
x_1' &= -x_2 \\
x_2' &= -4x_1 - 8x_2
\end{align*}
\]

2. (6 points) Find the inverse Laplace transform for \( s > a \) of:

\[
\frac{-2s + 1}{s^2 - 2s + 5}
\]

(a) \(-2e^{-t} \cos(2t)\)
(b) \(-2e^{-t} \cos(2t) + 3e^{-t} \sin(2t)\)
(c) \(-2e^{t} \cos(2t) + e^{t} \sin(2t)\)
(d) \(-2e^{t} \cos(2t) - \frac{1}{2} e^{t} \sin(2t)\)
3. (6 points) Find the Laplace transform of $u_{\pi/2}(t) \cos(2t)$. The following identity may be useful:

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

(a) $-\frac{s}{s^2+4}e^{-s\pi/2}$
(b) $-\frac{2}{s^2+4}e^{-s\pi/2}$
(c) $\frac{1}{(s^2+4)}e^{-s\pi/2}$
(d) $\frac{s}{s^2+4s+4}e^{-s\pi/2}$

4. (6 points) Suppose $f(t) = 2 - u_2(t) + t(u_3(t) - u_6(t))$, what is $f(5)$?

(a) 2
(b) 4
(c) 6
(d) 7
5. (12 points) When using the method of undetermined coefficients to solve the following equation, what is the form of the particular solution? **Do not solve for the constants.**

\[ y'' - y' - 2y = t^2 e^{3t} + 3e^{-t} + te^{2t} \cos(2t) \]
6. (18 points) A mass of 1 kg stretches a spring \( \frac{5}{8} \) m. The system has a damping constant of \( 4 \text{ kg s}^{-1} \). The mass is pulled down 1 m from its equilibrium position and released. You may take \( g = 10 \text{ m s}^{-2} \).

(a) Set up an initial value problem modeling this system.
(b) Solve this initial value problem.
(c) At what quasi-frequency does the system oscillate?
(d) In the absence of damping (i.e. \( \gamma = 0 \)), what is the system's natural frequency?
7. (12 points) Rewrite the following function in terms of unit step functions and find it's Laplace transform.

\[ f(t) = \begin{cases} 
  t + 1 & 0 \leq t < 3 \\
  t^2 + e^t & 3 \leq t 
\end{cases} \]
8. (20 points) Solve the initial value problem:

\[ y'' - 4y' + 4y = 2\delta(t - 4) \quad y(0) = 1 \quad y'(0) = 0 \]
9. (14 points) Solve the initial value problem:

\[
\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} X' = \begin{bmatrix} 0 \\ 3 \end{bmatrix}
\]