There are 9 questions on 8 pages.
Please read each problem carefully before starting to solve it. For each multiple choice problem 4 answers are given, only one of which is correct. Mark only one choice.
For partial credit questions, all work must be shown - credit will not be given for an answer unsupported by work.

NO CALCULATORS ARE ALLOWED.
PLEASE DO NOT WRITE IN THE BOX BELOW.
A TABLE OF LAPLACE TRANSFORMS IS ATTACHED.
1. (5 points) If a mass-spring system described by the equation $2u'' + ku = 10 \cos 4t$ exhibits resonance, then what is the value of $k$?
   (a) 4
   (b) 8
   (c) 16
   (d) 32

2. (5 points) The inverse Laplace transform of $\frac{8}{s^3 - 4s}$ is
   (a) $-2 + e^{2t} + e^{-2t}$
   (b) $-2t + e^{2t} + e^{-2t}$
   (c) $2t + \cos 2t$
   (d) $-2 + \frac{1}{2} \sin 2t$

3. (6 points) True of false:
   i. $\mathcal{L}\{f(t)g(t)\} = \mathcal{L}\{f(t)\}\mathcal{L}\{g(t)\}$, T F
   ii. $\mathcal{L}\{u_\pi(t) \sin t\} = e^{-\pi s} \frac{1}{s^2 + 1}$, T F
   iii. If $f(t) = 2 - u_2(t) + 3u_4(t)$, then $f(3) = 1$, T F
4. (16 points)

(a) When using the method of undetermined coefficients to solve the following non-homogeneous equation, what is the form of the particular solution? (Do not solve for the coefficients!)

\[ y'' + 4y' + 4y = 3t^2 - te^{-2t} + 2t \cos 3t. \]

(b) Find the general solution of

\[ y'' - y = 3e^{-t}. \]
5. (14 points) A mass of 1 kg stretches a spring 50 cm. The mass-spring system has damping of $\gamma = 4 \text{ N} \cdot \text{sec/m}$. At $t = 0$ the mass is set in motion from its equilibrium position with downward velocity of $2 \text{ m/sec}$. Assume that the gravitational constant, $g = 10 \text{ m/sec}^2$.

(a) Set up an initial value problem that describes this situation.

(b) Solve the initial value problem.

(c) What is the quasi-frequency of the system?
6. (14 points) Rewrite the following function in terms of step functions and find its Laplace transform:

\[ f(t) = \begin{cases} 
  t^2, & 0 \leq t < 2 \\
  e^{3t} - t, & t \geq 2
\end{cases} \]
7. (12 points) Solve the initial value problem:

\[ y'' + 2y' + 5y = 3 \delta(t - 2\pi), \quad y(0) = 1 \quad y'(0) = 0. \]
8. (14 points)
    (a) Find the general solution of
    \[ X' = \begin{bmatrix} 2 & 5 \\ 2 & -1 \end{bmatrix} X. \]
    (b) Suppose \( X(0) = \begin{bmatrix} -5 \\ \alpha \end{bmatrix} \) and \( \lim_{t \to \infty} X(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \), what is the value of \( \alpha \)?
9. (14 points) Solve the initial value problem:

\[
X' = \begin{bmatrix} 1 & 3 \\ -3 & 1 \end{bmatrix} X, \quad X(0) = \begin{bmatrix} -2 \\ 5 \end{bmatrix}.
\]