On the impacts of locally adaptive signal control on urban network stability and the Macroscopic Fundamental Diagram

Vikash V. Gayah\textsuperscript{a}, Xueyu (Shirley) Gao\textsuperscript{a}, Andrew S. Nagle\textsuperscript{a}

\textsuperscript{a}Department of Civil and Environmental Engineering, The Pennsylvania State University, University Park, PA, 16802

Abstract

Urban traffic networks are inherently unstable when congested. This instability causes a natural tendency towards spatially inhomogeneous vehicle distributions and less consistent and reproducible relationships between urban traffic variables. It is important to find ways to mitigate this unstable behavior since well-defined relationships between average network flow and density—the MFD—are useful to aid network design and control.

This paper examines the impacts of locally adaptive traffic signals—e.g., those that allocate green times proportionally to upstream approach densities—on network stability and the MFD. A family of adaptive signal control strategies is examined on two abstractions of an idealized grid network using an analytical model and an interactive simulation. The results suggest that locally adaptive traffic signals provide stability when the network is moderately congested, which increases average flows and decreases the likelihood of gridlock. These benefits increase with the overall adaptivity of the signals. However, adaptive signals appear to have little to no effect on network stability or the MFD in heavily congested networks as vehicle movement becomes more constrained by downstream congestion and queue spillbacks. Under these conditions, other strategies should be used to mitigate the instability, such as adaptively routing drivers to avoid locally congested regions. These behaviors are verified using more realistic micro-simulations and are consistent with other observations documented in the literature.

Keywords: Macroscopic Fundamental Diagram, Traffic network instabilities, Traffic congestion, Urban mobility, Adaptive traffic signal control

1. Introduction

Researchers have studied aggregate urban traffic models for nearly fifty years (e.g., Smeed, 1966; Godfrey, 1969; Zahavi, 1972). Godfrey (1969) appears to have been the first to propose a relationship between the average network flow and density that realistically describes the range of potential traffic conditions. This idea was later refined using theoretical, empirical and simulation-based studies of macroscopic network behavior (Herman and Prigogine, 1979; Ardekani and Herman, 1987; Mahmassani et al., 1984, 1987; Olszewski et al., 1995, among others).
The theoretical basis for the existence of well-defined relationships between urban traffic variables was provided in Daganzo (2007). This work postulated that average network flow and density are related by a well-defined unimodal curve, known more commonly now as the Macroscopic Fundamental Diagram (MFD), provided that vehicles are uniformly distributed across space. This conjecture was later verified analytically (Daganzo and Geroliminis, 2008) and empirically using data from Yokohama, Japan (Geroliminis and Daganzo, 2008). Since then, a number of studies have shown that the MFD has tremendous potential to inform the design and control of urban traffic networks (e.g., Geroliminis and Levinson, 2009; Daganzo et al., 2012; Gayah and Daganzo, 2012; Zheng et al., 2012; Keyvan-Ekbatani et al., 2012; Haddad et al., 2013; Keyvan-Ekbatani et al., 2013).

However, network-wide relationships between flow and density are not always well-defined—significant amounts of scatter may arise in which multiple flows are observed for a given value of density (Mazloumian et al., 2010). Hysteresis patterns may also arise for which flows during the onset of congestion are significantly different from those during the dissipation of congestion (Buisson and Ladier, 2009). Similar patterns also emerge in the MFD of freeway networks (Geroliminis and Sun, 2011a; Saberi and Mahmassani, 2013). Daganzo et al. (2011) and Gayah and Daganzo (2011a) provide an explanation for this behavior: congested multi-route networks are inherently unstable and this causes traffic to naturally tend towards spatially inhomogeneous vehicle distributions. This innate instability arises in the simplest networks with multiple routes and may persist and grow with time until the network completely gridlocks. Haddad and Geroliminis (2012) examined the ability of network-wide perimeter metering control to mitigate instabilities in two-region networks, but the study ignores congestion inhomogeneities that are likely to arise within a region. Such inhomogeneous congestion distributions are likely to arise in both under- and over-saturated networks (Doig et al., 2013) and significantly affect the shape and functional form of the MFD (Knoop et al., 2013). Although inhomogeneous vehicle distributions do not preclude the existence of a well-defined MFD, congestion patterns must at least be reproducible for an MFD to be useful (Geroliminis and Sun, 2011b).

Fortunately, well-defined MFDs still often arise in real networks. One potential explanation is that drivers tend to adaptively route themselves to avoid localized pockets of congestion. This helps to mitigate the innate instability and should result in more consistent congestion patterns and MFDs (Daganzo et al., 2011; Gayah and Daganzo, 2011b). The impacts of adaptive driving behavior have been verified in large-scale simulations of realistic traffic networks (Saberi et al., 2014; Mahmassani et al., 2013). However, drivers do not always have sufficient information or the ability to route themselves adaptively. Thus, it would be useful to know if other strategies can be used to provide stability within a network.

One strategy with the potential to create more homogeneous vehicle distributions is the implementation of locally adaptive traffic signal control schemes that allocate green time based on existing traffic conditions. Dynamic algorithms have long been used to efficiently allocate green time at individual signalized intersections between competing directions based

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1The term adaptive here is used in the traffic engineering sense and is not related to adaptive control from a control theoretic perspective.
on real-time data (e.g., Miller, 1963; Robertson and Bretherton, 1974; Diakaki et al., 2002). It is possible that traffic signals that adapt to local intersection demands may help to uniformly distribute congestion across an entire region by prioritizing movements from more congested areas and restricting movement from less congested areas. This should result in more consistent MFDs and higher average network-wide flows when a network is congested.

A recent study provides some evidence to this effect (Zhang et al., 2013). This study found that congestion inhomogeneity did not always grow with time when three adaptive signal control schemes were implemented in a cellular automata simulation of an urban network. The presence of adaptive signal control was also found to significantly influence the functional form of the MFD, particularly in the congested regime. As illustrated in Figure 1, the more flexible scheme (SOTL) provides higher average flows in the congested branch of the MFD than the less flexible schemes (SCATS-L and SCATS-F). However, in all cases very low flows representing near gridlock conditions still occur when the network is heavily congested (e.g., for densities greater than 0.5). Thus, it appears that adaptive signals provide a stabilizing influence for some congested densities but not all.

The purpose of this paper is to examine the effects of locally adaptive traffic signals on network-wide traffic relationships in a more systematic and theoretical way to see if the results of Zhang et al. (2013) should generally be expected. The effects of adaptive signals and adaptive driver routing are also compared to see which mitigates this instability more robustly. To do this, traffic on an idealized grid network with periodic boundary conditions is considered. This particular network is chosen because it can be abstracted to simpler networks that are easier to study. Simulation and theoretical analyses of these network abstractions suggest that locally adaptive traffic signals can help create more uniform vehicles distributions at moderate congestion levels, resulting in more consistent and reproducible MFDs. These results are true even if the signals are only partially adaptive. However, unlike adaptive vehicle routing, adaptive signals are not beneficial when the network is extremely
congested.

The rest of this paper is organized as follows. Section 2 describes the idealized network and the two abstractions of the network that will be considered. Section 3 studies the impact of locally adaptive traffic signals analytically using one of these abstractions. Section 4 and 5 confirm these results with simulations of increasing complexity and realism. Finally, Section 6 provides some insights and concluding remarks.

2. Idealized network and abstractions

The network considered here is a homogeneous square grid comprised of alternating one-way streets; see Figure 2a. Traffic on each of the links is assumed to be described by the same fundamental diagram, \( Q(k) \). For simplicity, a triangular fundamental diagram is chosen with free flow speed, \( v \), backward wave speed, \( w \), and capacity \( q_m \); see Figure 3. The jam density can be determined from the other parameters, \( k_j = q_m (1/v + 1/w) \).

![Figure 2: (a) Homogeneous grid network; (b) two-ring abstraction; and, (c) two-bin abstraction.](image)

![Figure 3: Fundamental diagram describing traffic on each link.](image)

Vehicles are assumed to travel within the network without any destinations. Instead, routing is entirely determined by turning probabilities at each intersection. A fraction \( 1 - \alpha \) of the vehicles in the network turn randomly at each intersection with some known
probability that is independent of downstream congestion. These vehicles can be assumed to be following predetermined routes, since routing decisions are made independent of traffic. The remaining \( \alpha \) fraction of vehicles are also subject to the random turning probabilities, but they may alter these decisions in response to current traffic conditions. These vehicles represent drivers that dynamically adapt their routes to avoid congestion. For these adaptive drivers, if a random turning maneuver requires a turn onto a link that is less congested (i.e., has a lower traffic density) than the alternative through link, the turn is made. However, if the random turning maneuver requires a turn onto a more congested link, the vehicle foregoes that decision and instead takes the through link to avoid congestion.

Since there are no destinations in the network, vehicles will travel indefinitely until they reach the downstream-most end of a particular street. Once a vehicle exits the network at the downstream end, another is simultaneously inserted at the upstream-most end of that same street. In this way, the network maintains periodic boundary conditions and the number of vehicles remains fixed.

All intersections are signalized and share the same fixed cycle length, \( C \), with zero offset between adjacent signals. We consider a family of adaptive traffic signal control strategies that allocates green time as follows. A fraction \( 1 - \gamma \) of the cycle length is divided evenly between the two competing directions, providing a minimum green time of \( (1 - \gamma)C/2 \) to each. The remaining \( \gamma \) fraction of the cycle is dynamically allocated every cycle to the two competing directions proportional to vehicle densities on the upstream links.\(^2\)

The parameter \( \gamma \in [0, 1] \) represents the level of signal adaptivity. The extreme value \( \gamma = 0 \) represents traffic signals that have fixed timings; in this case, a constant \( C/2 \) green time is allocated to each direction. At the other extreme, \( \gamma = 1 \) represents fully adaptive signals in which all of the green time is allocated proportional to upstream densities. Of course, other adaptive control schemes exist that consider additional factors and offer more flexibility, such as dynamically changing the cycle length or signal offsets. For analytical completeness, we consider this simpler strategy that can be studied exhaustively through simulations and analytical models. Furthermore, this simple strategy replicates the primary goal of many adaptive signal control schemes: to provide more green time to the approach(es) with more traffic.

While the network described here is relatively simple, analytical formulations are not feasible due to the large number of links and intersections. Simulations also suffer from stochastic behavior, and the noise associated with stochastic simulations often precludes the ability to obtain fundamental insights (Daganzo et al., 2012). To alleviate these issues, two physically equivalent abstractions of this network are proposed that help unveil insights on the impact of adaptive signal control on congested networks.

2.1. Two-ring abstraction

Assume now that vehicles are initially distributed such that the density profile on all east-west and north-south links are identical. Further assume that the random turning

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\(^2\)For simplicity, here we ignore the fraction of the cycle that is lost for vehicle movement during signal phase changes. This extension can be easily introduced into this adaptive signal control algorithm, but it will not affect the qualitative results presented.
probabilities and proportion of adaptive drivers are equal at all intersections and constant over time. Denote these values $P_T$ and $\alpha$, respectively. In this case, the density profile on all east-west links (and similarly north-south links) will always be identical to each other as traffic evolves on the network. Under these conditions, the entire network can be modeled by a physically equivalent system of two interconnected rings and the single intersection that connects them. In this abstraction, one ring represents all east-west links and the other all north-south links; see Figure 2b.

While simpler than the idealized grid network, analytical descriptions of this two-ring system are still difficult. Jin et al. (2013) studies the equilibrium behavior and dynamics of an unsignalized two-ring network analytically using a kinematic wave model. However, this method is not feasible when signals exist and signal timings are updated every cycle. To overcome this challenge and gain analytical insights, this two-ring system is abstracted further in Section 2.2.

2.2. Two-Bin abstraction

Assume now that vehicles within each ring are always uniformly distributed across its length and that turning vehicles force their way into the receiving ring if this ring is not completely jammed. Under these conditions, the state of each ring is completely described by the number (or density) of vehicles on it at any moment in time. Thus, the two-ring network can be modeled as a simpler system of just two bins of vehicles that interact through the flow of vehicles turning between them, as shown in Figure 2c.

This two-bin network abstraction was previously used in Daganzo et al. (2011) to unveil the instabilities that exist in congested urban networks. The impacts of adaptive signal control on this two-bin system, specifically with respect to these instabilities and the MFD, are discussed next.

3. Evidence of limited stability in the two-bin model

Consider the abstracted two-bin network described in Section 2.2. Let $i = \{1, 2\}$ be used as an index for each bin, and denote the density and flow of vehicles within each bin as $k_i$ and $q_i$, respectively. Since the network is homogeneous, the average density, $k_T$, and average flow, $q_T$, in the homogeneous network are unweighted averages of the individual densities and flows within each bin:

$$k_T = (k_1 + k_2)/2, \quad (1)$$

and

$$q_T = (q_1 + q_2)/2. \quad (2)$$

A function $Q_i(k_i)$ is used to relate the density and flow within each bin. If intersections were not controlled by traffic signals, $Q_i(k_i)$ would be equal to the fundamental diagram for each link, i.e., the relationship shown in Figure 3. However, the presence of traffic signals disrupts flows at intersections and changes this relationship. Recent empirical work has shown that the relationship between average flow and density on signalized arterials generally follows a trapezoidal shape (Wu et al., 2011). The height of this trapezoid should be
proportional to the amount of green time provided to the direction of interest. If the impact of traffic signals on the increasing and decreasing branch of the flow-density relationship is ignored, the flow of vehicles within each bin can be described by:

$$q_i = Q_i(k_i) = \min \{Q(k_i), g_i q_m\} = \min \{v k_i, w(k_j - k_i), g_i q_m\},$$

(3)

where $Q(k_i)$ is the fundamental diagram of each link, and $g_i$ is the fraction of green time allocated to the streets represented by bin $i$. An illustration of a single bin MFD is shown in Figure 4 for the special case where $g_i = 0.5$. This special case is also used to define two key densities, $k_a$ and $k_b$, that are also depicted in Figure 4. As will be discussed later in this section, $k_a$ and $k_b$ bound the range of densities for which our simple adaptive control scheme impacts network behavior.

The green time provided to each intersection approach, $g_i$, is a function of the adaptivity of the traffic signals, $\gamma$, and the distribution of vehicles across the two bins:

$$g_i = (1 - \gamma)1/2 + \gamma k_i/(2k_T).$$

(4)

The flow of vehicles out of bin $i$ (and thus into the other) is denoted $f_i$. Based on the routing rules previously described, the fraction of vehicles passing the intersection that turn out of bin $i$ at any moment, denoted $p_i$, is:

$$p_i = \begin{cases} P_T & \text{if } k_i \geq k_T, \\ (1 - \alpha)P_T & \text{if } k_i < k_T. \end{cases}$$

(5)

The exiting flow from bin $i$ is the product of the average flow within the bin and the proportion of vehicles turning:

$$f_i = F_i(k_i) = p_i q_i.$$

(6)

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3Note that when links are short, the maximum network flow is not always bounded by $g_i q_m$; see Daganzo and Geroliminis (2008) for more details.
If units are selected such that the total length of streets in each bin is 1, the evolution of the two-bin system is described by:

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\begin{align*}
\frac{dk_1}{dt} &= F_1(k_2) - F_1(k_1) \quad \text{and} \\
\frac{dk_2}{dt} &= F_1(k_1) - F_2(k_2), \quad \text{if } k_1, k_2 < k_j; \quad \text{and,} \\
\frac{dk_1}{dt} &= \frac{dk_2}{dt} = 0, \quad \text{if } k_1 \text{ or } k_2 = k_j. 
\end{align*}
\] (7a) (7b)

As described in Daganzo et al. (2011), the two-bin system following (7) will be in equilibrium when the exiting flows from each bin are the same:

\[ F_1(k_1) = F_2(k_2) \] (8)

Two types of equilibrium states were shown to exist: stable and unstable. Unstable equilibria are short-lived and will not be reproducible over the long run. However, stable equilibria are those that are robust to disturbances and are used to describe the reproducible aggregate behavior of vehicles on the network (i.e., its MFD). Thus, it is important to identify these stable equilibria. The character of equilibrium states can be determined by examining how the system evolves after a minor perturbation. These minor perturbations will shrink with time at stable equilibria and grow with time at unstable equilibria. Mathematically, an equilibrium is stable if and only if:

\[ \frac{dF_1(k_1)}{dk_1} + \frac{dF_2(k_2)}{dk_2} > 0 \] (9)

The set of conditions described by (8) and (9) are now used to determine the set and character of all equilibrium states that arise in this network when adaptive signal control is applied. We first consider the case where signal timings are fixed to provide a baseline for comparison. Then, we examine the behavior when adaptive signals and the combination of adaptive signals and drivers are introduced.

3.1. Behavior without adaptive signals

We first determine the set of equilibrium states when the network operates with fixed traffic signals. In this case, the green ratio is constant and the flow-density relationship for each of the bins (referred to as the “bin MFD”) is also constant. The complete set of equilibrium states when signal timings are fixed is presented on a phase diagram in Figure 5a as was done in Daganzo et al. (2011). Each phase diagram graphically presents the set of possible states that the system can take. Diagonal thin dotted lines on each phase diagram are iso-density contours that represent a fixed number of vehicles that exist in the system. The red solid lines depict stable equilibrium states, while the red dotted lines depict unstable equilibrium states. The hatched area represents a third type of equilibrium state that exists for signalized street networks, which we call quasi-stable states. At quasi-stable states,
minor perturbations will not grow or shrink with time. Instead, after a perturbation the
system will remain at the perturbed state until another change occurs. These diagrams
differ from the ones provided in Daganzo et al. (2011) because the previous study did not
consider traffic signals and their impacts on the MFD of the two-bin system. As shown here,
the presence of traffic signals significantly changes the set of stable and unstable equilibrium
states and also introduces new quasi-stable equilibrium states when signal timings are fixed.

The phase diagrams show that a unique equilibrium state only arises when there are very
few vehicles in the network \((k_T < k_a)\). At this unique equilibrium, \(k_1 = k_2\) and vehicles are
evenly distributed across the two bins. As the density increases past \(k_a\), multiple equilibrium
states emerge and even distributions are no longer sustainable. Instead, stable equilibrium
states at these high densities only exist for \(k_1 \neq k_2\). The regions shaded on the phase
diagrams represent initial states for which the density difference between the two bins will
grow with time as the network moves towards further imbalance. The area of this shaded
region provides an indication of the general tendency of the network toward inhomogeneous
vehicle distributions—larger shaded regions indicate a stronger push towards unevenness and
smaller shaded regions indicate a weaker push. Another metric that describes the stability
of the network is the bifurcation density, \(k_S\)—the density at which stable equilibrium states
emerge with inhomogeneous vehicle distributions. For fixed signals, \(k_S = k_c/2 + k_j/4\).

The flow-density relationship associated with the equilibrium states is also shown in
Figure 5a. The MFD of the system is described by flow-density pairs that represent stable
equilibrium states. Notice that the uneven vehicle distributions at higher densities result in
MFDs with lower network flows, and equilibrium states with zero flows (i.e., gridlock states)
emerge at densities lower than jam density.

3.2. Behavior with adaptive signal control

We now examine how the set of equilibrium states changes when adaptive signal control
is implemented and the implications for the MFD. Figures 5b and 5c present the phase
diagrams and aggregate flow-density relationships for partially adaptive signals \((0 < \gamma < 1)\)
and fully adaptive signals \((\gamma = 1)\), respectively, when drivers are not adaptive \((\alpha = 0)\).

Comparison of these phase diagrams with the one in Figure 5a reveals that quasi-stable
equilibrium states are unique to the case of fixed traffic signal control without adaptive
drivers. More importantly, two other fundamental and systematic changes occur when adap-
tive signals are implemented. The first change is that the bifurcation density \(k_S\) increases.
Therefore, a unique stable equilibrium state in which vehicles are evenly distributed across
both bins persists for a wider range of densities when signals are adaptive. The value of \(k_S\)
increases monotonically with \(\gamma\); see the light line in Figure 6a. The second change is that
the range of states for which the network has a tendency towards further imbalance (i.e.,
the shaded regions on the phase diagrams) shrinks when adaptive signals are implemented.
As illustrated by the light line in Figure 6b, the shaded regions shrink with \(\gamma\). This stronger
push towards more balanced states results in higher average network flows for a given den-

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4These states satisfy condition (9) after replacing the inequality with an equal sign.
Figure 5: Phase diagram and aggregate flow-density relationship when driver routing is not adaptive ($\alpha = 0$) for: (a) fixed traffic signals ($\gamma = 0$); (b) partially adaptive signals ($0 < \gamma < 1$); and, (c) fully adaptive signals ($\gamma = 1$).
Figure 6: Measures of stability in the two-bin network: (a) bifurcation density; and, (b) fraction of states leading to network imbalance.

Interestingly, the set of equilibrium states with average densities greater than $k_b$ is invariant to $\gamma$. This suggests that adaptive traffic signals are not able to mitigate the network’s tendency towards unevenness when the network is extremely congested. One reason for this is that the adaptive traffic signal control becomes less flexible as the network becomes more congested: as the overall density increases, the bin with fewer vehicles demands a larger fraction of available green time and this limits the flexibility of adaptive signal control. Additionally, the flow within an extremely congested bin is constrained by the decreasing branch of the bin MFD. From (3), we can see that increasing the amount of green time provided to an extremely congested bin will have no influence on the flow within (and out of) that bin. Thus, the presence of adaptive traffic signals has no impact on network stability or the MFD in heavy congestion; this can be verified by examining the flow-density relationships illustrated in Figure 5.

Overall, these phase diagrams suggest that adaptive signal control provides a beneficial stabilizing influence when the network is moderately congested and both bins operate at or near a capacity state. However, adaptive signal control has no significant effect when the network is heavily congested and the bins operate in the congested branch of the MFD.

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5We have assumed thus far that the adaptive signal control only impacts the height of the bin MFD and not the increasing or decreasing branches. However, Daganzo and Geroliminis (2008) shows that signal timings also influence these portions of the MFD. Including this complexity into the two-bin model is not analytically tractable; however, numerical simulations of the two-bin model including this complication verify that the qualitatively behavior is still the same.
3.3. Behavior with adaptive signal control and adaptive driver routing

Figure 7 presents the phase diagrams and flow-density relationships for the case in which drivers are adaptive (α > 0) both when signal timings are fixed (γ = 0) and signals are adaptive (γ > 0). Figure 7a shows that the presence of adaptive drivers alone influences the entire range of network densities, not only the range $k_a < k_T < k_b$ that are impacted by adaptive signals. In this case, an even distribution of vehicles becomes sustainable for all network densities. Furthermore, the bifurcation density and fraction of states that lead to imbalance also improve significantly with α; see the plotted values in Figure 6 when $γ = 0$. Thus, it appears that adaptive driving more robustly mitigates the inherent instability than adaptive signal control when each are applied alone.

When adaptive signal control and adaptive drivers are combined (Figure 7b), the benefits of each strategy are amplified. However, the additional benefits obtained by combining the two strategies are only experienced within the range $k_a < k_T < k_b$. The magnitude of the benefits to the bifurcation density and fraction of states leading to imbalance are also presented in Figure 6. Notice that the metrics improve significantly more as α increases than as γ increases. This is further evidence that suggests adaptive driving is more beneficial than
adaptive signal control at providing stability, although both strategies are useful.

4. Confirmation of stability on two-ring network

The two-bin model results suggest that adaptive signal control provides a stabilizing influence on network traffic, but only for a narrow range of densities representing moderate congestion levels. It also suggests that adaptive driver routing is more robust than adaptive signal control since it provides a larger benefit and does so for a wider range of densities. However, this analytical model does not consider the impacts of vehicular dynamics along the ring and potential queue spillbacks at intersections. To verify that the same insights and behaviors hold when these impacts are included, simulations of the two-ring network abstraction are now used.

A two-ring network similar to the one proposed here was previously examined using an interactive simulation in Gayah and Daganzo (2011b). In this simulation, traffic on all links is modeled using a cellular automata model consistent with kinematic wave theory assuming a triangular fundamental diagram (Nagel and Schreckenberg, 1992; Daganzo, 2006). The simulation uses the following parameters: \( v = 60 \text{ mi/hr} \), \( w = 15 \text{ mi/hr} \), \( q_m = 1800 \text{ veh/hr} \) and \( k_j = 150 \text{ veh/mi} \) (equivalent to 60 vehicles per 0.4 mi ring). As the simulation runs, 1-minute averages of network flow and density are calculated using the generalized definitions of Edie (1965).

This previous simulation considers only traffic signals with fixed timings. We have expanded this simulation to include the adaptive signal control strategy described in Section 2. The parameter \( \gamma \) is included as an interactive variable, labeled Proportion of Cycle Adaptively Allocated. The updated version of the two-ring simulation is available online at the following address: http://www.engr.psu.edu/gayah/two_ring_adapt/two_ring_adapt.html. The reader is encouraged to interact with the simulation to verify the results that are presented in the remainder of this section. First, the behavior without adaptive signals is discussed as a base case, then the behavior with adaptive signals and the combination of adaptive signals and drivers is examined.

4.1. Behavior without adaptive signals

Simulations of the original two-ring network with fixed signal timing confirm that an instability exists in congested networks that causes vehicles to distribute themselves unevenly across space. If enough vehicles are present to completely fill one of the rings, the network will eventually become so unbalanced that it will gridlock (defined as when the average flow in the network reaches 0 veh/hr). To verify this, try the following steps with the online simulation: reset the simulation; set the probability of turning to 0.2, number of vehicles to 60 (enough to completely fill one ring so that \( k_T = 0.5k_j \) and proportion of cycle adaptive allocated to 0.0; and, start the simulation. The network quickly moves towards an unbalanced state where one ring has more vehicles than the other. Eventually, the rings become so unbalanced that one ring completely fills with vehicles while the other remains empty. At this point, the average flow becomes zero representing that the system is in complete gridlock.

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4.2. Behavior with adaptive signal control only

When the network is moderately congested (e.g., there are only enough vehicles to fill just one ring), the two-bin model suggests that adaptive signal control will help mitigate this innate instability. This should result in a reduced tendency towards gridlock, more homogeneous vehicle distributions and higher sustained flows in the network. To verify that this occurs, try the following: reset the simulation; set the probability of turning to 0.2, the number of vehicles to 60 and proportion of cycle adaptively allocated to 1.0; and, start the simulation. In this case, vehicles tend to remain fairly evenly distributed across the two rings. When stochastic fluctuations cause the system to become unbalanced, providing more green time to the more congested ring allows vehicles to exit this ring at a higher rate than at which they enter. This helps to move the system back towards a balanced state and to avoid gridlock.

The ability of the signals to mitigate the instability is related to the adaptivity parameter, $\gamma$. If signals are not very adaptive, the instability can dominate and gridlock may still occur. This can be observed by repeating the previous experiment but setting the proportion of cycle adaptively allocated to 0.2. However, even when lower signal adaptivity is unable to prevent gridlock in the moderately congested network, adaptive signal control still provides a benefit by delaying its occurrence. This can be seen in Figure 8a, which plots the average time required to reach gridlock over many simulation runs for different values of $\gamma$ along with the 95% confidence interval of the mean. When 60 vehicles are in the network, the time to gridlock increases with signal adaptivity. Gridlock is mostly avoided after the level of signal adaptivity increases beyond a critical value (around $\gamma = 0.65$ for this battery of tests). For $\gamma > 0.65$, increasing $\gamma$ provides an additional benefit by maintaining higher average flows; see Figure 8b, which presents the average flow measured over a period of 300 minutes across many simulation runs for the range of $\gamma$ at which the network does not gridlock. Adaptive signal control can sustain a flow as high as 850 veh/hr when 60 vehicles are in the network, compared to a long run flow of zero that would occur when adaptive signals are not implemented.

Recall also that the two-bin model also suggests that adaptive signal control will not provide any significant stabilizing influence when the network is heavily congested. To verify this, try the following: reset the simulation; set the probability of turning to 0.2, the number of vehicles to 90, and the proportion of cycle adaptively allocated to 1.0; and, start the simulation. In this scenario, gridlock still occurs even though the entire cycle is dynamically allocated to the two competing approaches. Further examination of the green time provided per cycle (displayed in the bottom right-hand side of the simulation interface) shows that signals timings are not as flexible when the network is very congested. The presence of many vehicles in the less congested ring requires that some minimum amount of green time always be allocated to this ring, reducing the green time benefits experienced by the more congested ring. For example, in this case the less congested ring would require at least 20 seconds of green time even when the other was completely filled. The simulation also shows that flows become constrained by queue spillbacks at the intersection when the network is very congested. The reader could verify that gridlock still occurs with fully adaptive signals even when the number of vehicles is as low as 70, although gridlock might
take a long time to occur. At these high levels of congestion, the time to gridlock is also insensitive to signal adaptivity; see Figure 8a. This verifies that adaptive signal control provides no stabilizing influence when the network is very congested.

The network-wide flow-density relationships of the two-ring system confirms this behavior. As illustrated in Figure 9a, larger values of $\gamma$ are associated with MFDs that exhibit higher average flows for densities between 50 and 90 veh/mi. This demonstrates the additional stability provided by the adaptive signals in this region. However, for densities greater than 90 veh/mi, the adaptive signals have no impact on the MFD, as expected from the previous analytical results.

4.3 Behavior with adaptive signal control and adaptive driver routing

As suggested by the analytical model, adaptive routing is inherently different than adaptive signal control because adaptive routing provides a stabilizing influence for the entire range of congested states. The reader can verify that a very high proportion of adaptive drivers (e.g., $\alpha = 0.8$) can prevent gridlock even when the network has 90 vehicles and is very congested. For a lower proportion of adaptive drivers (e.g., $\alpha = 0.2$), gridlock cannot be prevented. However, for cases with low levels of adaptive driving, additional stability is still provided as the network is able to delay gridlock for a longer period of time and sustain higher average flows as illustrated in Figure 8. Thus, the simulation confirms that adaptive driver routing can provide stability even in cases where adaptive signal control cannot. The combination of adaptive signals and adaptive drivers provides additional stability when the network is moderately congested. Once again, this behavior manifests itself in the MFD. Figure 9b presents the results of the two-ring simulation when both signals and drivers are adaptive. Notice that the positive flows are maintained for a much wider range of densities than the case of adaptive signals alone.
Figure 9: Average network-wide flow-density relationship obtained from many iterations of the two-ring simulation when: (a) signals are adaptive; and, (b) drivers are adaptive.

5. Confirmation of stability on more realistic networks

Analysis of the two-bin and two-ring network abstractions reveal insights as to how adaptive signal control might influence the MFDs of urban traffic networks. To confirm that these same patterns and behaviors emerge in more realistic network conditions, micro-simulations of the grid network described in Section 2 with adaptive signal control were performed in the AIMSUN simulation environment.\(^6\)

For the first set of tests, the idealized grid network with periodic boundary conditions described in Section 2 was created in the micro-simulation. Two versions were considered—one with long links (800 ft) and one with short links (400 ft). A fixed cycle length of 60 seconds was selected with zero offset between adjacent signals. The adaptive signal control strategy proposed here was not native to the AIMSUN environment; instead, it was manually coded into the software using the Application Programming Interface (API). The resulting flow-density relationships for a long period of time and over many simulation runs are presented in Figure 10 for multiple values of \(\gamma\).

The MFDs shown in Figure 10 confirm the same general patterns as predicted from the simplified models: namely, that adaptive signal control improves network performance for a small range of densities representing moderate congestion. The range of densities for which large positive flows can be sustained increases with the level of adaptivity, as expected. However, the adaptive signals have no impact when the network is heavily congested (i.e., for densities greater than 150 veh/mi). Notice that the adaptive signal control strategy has the same effect on both networks when congested: the bifurcation density increases with the level of signal adaptivity and higher positive flows are able to be sustained for larger densities.

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\(^6\)More realistic micro-simulations of adaptive driver routing have already been performed in other studies; see Mahmassani et al. (2013) and Saberi et al. (2014) for more details. Therefore, the micro-simulation tests here focus only on the impacts of adaptive signal control.
When the network is extremely congested (i.e., for densities greater than about 150 veh/mi), the network gridlocks with extremely low flows independent of the level of adaptivity of the traffic signals. However, in these more realistic networks the free flow branch of the MFD can also be impacted by the traffic signals when links are short. Recall from Section 3 that in the two-bin model the impact of adaptive traffic signals on the increasing branch of the bin MFD is ignored. This is generally a valid assumption if links are long, as Figure 10a confirms. However, when links are short, adaptive traffic signals might also influence the shape of the MFD for lower densities, as well.

The second set of tests are performed on a more realistic network that relaxes the assumption of periodic boundary conditions. Two other realistic extensions are also considered: 1) vehicle entries and exits; and, 2) a more realistic adaptive traffic signal control scheme. A 10x10 grid network was created with 800 ft links and origins and destinations at all intersections and the upstream/downstream end of all entry/exit links. A uniform traffic demand pattern was assumed in which demands at all origins were the same and all destinations were assumed to be equally likely. Furthermore, signals were assumed to operate using a modified version of the Sydney Coordinated Adaptive Traffic System (SCATS) algorithm to the traffic signals within the grid network.

This algorithm works by adjusting the green time allocated to each competing intersection approach and the total cycle length based on data obtained from detectors on the upstream approaches. The maximum cycle length is used as a measure of the signal adaptivity: higher maximum cycle lengths provide signal timings that are more flexible. The MFDs for different maximum cycle lengths in the SCATS algorithm and for a network simulated with fixed signal timings are displayed in Figure 11. Notice that the same pattern observed in Figure 10 is repeated in Figure 11: the range of densities for which positive flows are observed increases as the maximum cycle length increases.

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7This algorithm is the SCATS-F algorithm as described in Zhang et al. (2013).
6. Concluding remarks and consistency with other studies

In this paper, we use multiple abstractions of an idealized traffic network to examine the impact of locally adaptive traffic signals on a network’s ability to avoid spatially inhomogeneous vehicle distributions. A simple adaptive signal control strategy is exhaustively studied using an analytical model and an interactive simulation. This analysis shows that adaptive signal timings can help mitigate the network’s natural tendency towards unevenness when it is moderately congested (i.e., at densities just greater than densities associated with capacity) by prioritizing movements from more congested approaches over movements from less congested approaches. This results in delayed occurrence of gridlock and larger average network flows at densities that would typically become gridlocked without adaptive signals. The ability of the network to avoid gridlock and the magnitude of the flows sustained increases with the adaptivity of the signals. However, when the network is very congested, flows tend to be more limited by downstream congestion and queue spillbacks than by local signal timings, and adaptive signals have no effect on aggregate behavior of the network. Thus, it appears that adaptive signals might be more beneficial when applied simultaneously with another strategy that prevents heavy congestion, like perimeter metering or pricing (e.g., Geroliminis and Levinson, 2009; Haddad et al., 2013; Keyvan-Ekbatani et al., 2012; Aboudolas and Geroliminis, 2013; Keyvan-Ekbatani et al., 2013). These benefits
should occur even in the case where vehicles do not have sufficient information to re-route to avoid local congestion. However, adaptive routing should be more effective in general because it is able to influence a wider range of traffic states.

The results obtained using simple network abstractions were also verified to occur using micro-simulations under more realistic conditions. This includes the presence of unique vehicle entries and exits and under a more realistic adaptive signal control scheme (a modified version of the Sydney Coordinated Adaptive Traffic System). Of course, these micro-simulations still rely on some idealized assumptions, such as the use of a homogeneous and redundant grid network. Further work should be performed to verify that these results hold on more general network structures. In addition, we considered a network of one-way streets with only two competing approaches at each intersection. However, two-way street networks with more conflicts might create additional complexities that may change behavior.

Nevertheless, the trends observed here are consistent with those that were previously observed in Zhang et al. (2013), which examined the effect of different types of adaptive traffic control schemes on two-way arterial street networks with more complicated signalized intersections. Our analysis reveals that the higher congested flows observed in the MFDs of networks with more adaptive signal control schemes (Figure 1) should be generally expected. In addition, our results help to explain why all three MFDs in Figure 1 experience a sharp drop in flow in the congested branch—these abrupt drops appear to be associated with the density for which adaptive signals are no longer beneficial. The two-bin model suggests that the density at which this transition occurs increases with the adaptivity level of the traffic signals, which is exactly what occurs in Zhang et al. (2013). Finally, all three MFDs converge to near gridlock flows at densities of 0.5 or greater. Even in these more realistic simulations, locally adaptive traffic signals are still ineffective at preventing premature gridlock from occurring when the network is very congested, independent of the adaptiveness of the control scheme. Thus, it appears that the general phenomena observed in the simple networks studied here are also observed on more complicated and realistic networks.

These results might also shed additional insight as to why empirically derived MFDs (e.g., Yokohama and Toulouse as described in Geroliminis and Daganzo, 2008; Buisson and Ladier, 2009) generally do not exhibit the chaotic behavior associated with network instability. For one, heavy congestion is not observed in these networks and the instability only arises for very congested states. Additionally, the empirical data comes from real networks in which drivers are adaptive and traffic signals are responsive to traffic (e.g., actuated). The combination of adaptive signals and adaptive driver routing might provide enough of a stabilizing influence to yield consistent and reproducible MFDs for the entire range of densities that are observed. Overall, these results are very promising for the use of aggregate traffic relationships in the modeling of large-scale urban traffic networks.

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References


