A parametric study of charging a full-scale radial inlet diffuser in a cylindrical stratified chilled water storage tank was performed by applying factorial experimental theory to the results of simulations performed with a validated computational fluid dynamics (CFD) model. Part 1 describes the development and validation of the CFD model. Part 2 summarizes the results of the parametric study. Dimensional storage tank and inlet diffuser parameters having the potential to influence inlet thermal performance were identified, then formed into dimensionless groups using Buckingham Pi analysis. These included the inlet Richardson number \( Ri \), inlet Reynolds number \( Re_i \), ratio of diffuser radius to tank radius \( RD/R_w \) and ratio of diffuser radius to diffuser inlet height \( RD/hi \). Thermal performance was measured in terms of thermocline thickness and equivalent lost tank height. Sixteen simulations comprising a full \( 2^k \) factorial experiment were completed and analyzed. Parameter ranges considered were as follows: \( Ri \) from 1.0 to 11.1, \( Re_i \) from 1,000 to 12,000, \( RD/R_w \) from 0.2 to 0.4 and \( RD/hi \) from 5 to 10. Within these ranges, \( Ri \), \( RD/R_w \) and \( RD/hi \) were found to be of first-order significance while \( Re_i \) was not. Regression models of thermal performance metrics as functions of \( Ri \), \( RD/R_w \) and \( RD/hi \) that are sufficiently simple to be useful for design were developed. These models agreed well with CFD simulations from which they were derived and with field data.

INTRODUCTION

Naturally stratified chilled water storage tanks are widely used in large chilled water systems for load shifting. In a naturally stratified tank, cooler, denser water is stored beneath warmer, less dense water. The two bodies of water are prevented from mixing by buoyancy forces. A relatively thin thermal transition layer called a thermocline forms at the interface between warm and cool water.

In order to create and maintain good stratification, cool water must always enter and leave a stratified tank at the bottom and warm water must enter and leave at the top. Diffusers at the top and bottom of the tank reduce inlet velocity and promote stratification. The characteristics of the diffuser have a significant effect on thermal performance because of their influence on thermocline formation.
The radial parallel plate design is one of two types commonly used in cylindrical tanks. A radial diffuser consists of a plate located near a spreading surface, which is the floor of the tank in the case of a lower diffuser and the water free surface in the case of an upper diffuser (Figure 1). Inlet flow from a radial parallel plate diffuser enters a cylindrical tank as a horizontal current that flows outward radially from the perimeter of the gap formed by the diffuser disk and spreading surface. During charging, cool water enters the tank through the lower diffuser as warm water is withdrawn through the upper diffuser. During discharging, flow reverses and cool water is withdrawn through the lower diffuser while warm water enters through the upper diffuser.

Current diffuser design guidance (Dorgan and Elleson 1993) is based on research performed with scale model and prototype tanks (Wildin and Truman 1989, Wildin 1990). There is evidence to suggest that these studies may not scale up properly to full-size systems, which may be more than two orders of magnitude larger (Andrepont 1992, Musser and Bahnfleth 1999). Further, existing methods do not take into account tank geometry and are qualitative. When certain criteria are met, “good” performance is predicted, but the effects of design parameter changes are not quantified, which precludes design optimization.

The research described in this paper was undertaken to address the deficiencies of existing design methodology. The objective of the project was to develop first order correlations between the design parameters of full-scale cylindrical stratified chilled water storage tanks with radial parallel plate diffusers and expected inlet thermal performance using the results of computational fluid dynamic (CFD) analysis. The development and validation of the CFD model are described in Part 1 (Musser and Bahnfleth 2001). Part 2 summarizes the parametric analysis of inlet performance conducted with the aid of the CFD model.

BACKGROUND

Stratified Tank Thermal Performance

Thermal losses in a stratified chilled water storage tank occur by three mechanisms: conduction through the tank walls, conduction across the thermocline, and mixing and conduction near the inlet diffuser when the thermocline is forming. Of these mechanisms, conduction in tank
walls and through the thermocline are well understood and can modeled without difficulty. One-dimensional conduction through the thermocline is assumed in a number of computational models (Truman and Wildin 1985, Hussain 1989, Gretarsson 1990, Homan et al. 1996). The cited models also account for ambient heat gains and mixing near the inlet; however, inlet mixing is treated in a highly simplified manner and the user must provide mixing performance parameters based on empirical knowledge of diffuser performance.

The flow near radial inlet diffusers is less well understood, but specific cases have been modeled using computational fluid dynamics (Cai et al. 1993, Homan and Soo 1997). These simulations require a great deal of time and computing power to complete, and generally require more resources than are available to the typical designer. Therefore, simplified relationships derived from a validated parametric study using computational fluid dynamics would allow designers to predict tank performance based on tank and diffuser characteristics.

**Published Design Guidance**

Designers of naturally stratified chilled water thermal storage tanks typically reference either the EPRI *Stratified Chilled-Water Storage Design Guide* (Mackie and Reeves 1988) or the ASHRAE *Design Guide for Cool Thermal Storage* (Dorgan and Elleson 1993) for design guidance. These sources present design procedures based primarily on dimensionless parameters identified in the literature as being of significance.

The two governing parameters identified by prior investigations are the densimetric inlet Froude number \( Fr_i \) and the inlet Reynolds number \( Re_i \). Inlet Froude number, which represents the ratio of characteristic inertial and body forces, is defined as follows:

\[
Fr_i = \frac{q}{\eta \left( \frac{h_i g}{\rho} \right)^{\frac{3}{2}}} \frac{\Delta \rho}{\rho} \tag{1}
\]

where

- \( q \) = inlet flow rate per unit length of diffuser
- \( h_i \) = diffuser inlet height
- \( g \) = gravitational acceleration
- \( \rho \) = ambient fluid density
- \( \Delta \rho \) = difference between ambient and inlet fluid densities.

For a radial diffuser, the inlet height is the width of the gap between the diffuser plate and spreading surface. The diffuser length used to define \( q \) is the perimeter of the diffuser plate.

The inlet Froude number has been observed to influence the behavior of a cold gravity current entering along the floor of a tank filled with warm water (Yoo 1986, Nakos 1987). While stratification was observed to take place for inlet Froude numbers less than 2, the best performance was obtained for values of 1 or less.

Inlet Reynolds number represents the ratio of characteristic inlet inertial and viscous forces and is customarily defined as follows:

\[
Re_i = \frac{q}{\nu} \tag{2}
\]

where \( \nu \) is kinematic viscosity and \( q \) is volumetric flow per unit diffuser length, as defined for Equation (1). Experiments performed at constant inlet Froude number with two radial diffusers of different size and one octagonal diffuser system suggest that inlet Reynolds number may also influence inlet performance (Wildin 1989).
The EPRI guide recommends that diffusers be designed for an inlet Froude number (Fr_i) less than or equal to one. This guide does not discuss or set limits on Re_i; however, it recommends the use of inlet flow rates of 2 to 9 gpm/ft of diffuser length (0.41 to 1.86 L/s·m). This corresponds to a range of Re_i of roughly 300 to 1400. The inlet flow rate criterion determines the range of diffuser length. For a given inlet flow rate, the limit set on Fr_i determines diffuser inlet height.

The ASHRAE guide recommends design for an inlet Froude number of one or less and specifies criteria for the inlet flow rate in terms of Re_i. Based upon the experiments of Wildin and Truman (1989) conducted with a 14 ft (4.3 m) water depth prototype tank, a maximum Re_i of 200 is proposed for tanks with less than 15 feet (4.57 m) water depth, and an upper limit of 400 to 850 for tanks 15 ft (4.57 m) or taller. A maximum Re_i of 2,000 is suggested for tanks 40 ft or taller, although it is noted that insufficient data exist to provide rigid guidelines. According to this procedure, inlet Reynolds number determines the required diffuser length and, as in the case of the EPRI guide method, Fr_i determines the inlet height.

The similarity between the Re_i range suggested in the ASHRAE guide and the range implied by the inlet velocity criteria of the EPRI guide is noteworthy. Equally of note, however, is the accumulating evidence that stratified tanks that violate these limits by a wide margin provide acceptable performance. It has been reported that tall tanks may stratify well with inlet Reynolds numbers as high as 10,000 (Andrepon 1992) and the authors have published data for four full-scale tanks operating with Re_i as great as 12,000 that exhibit good stratification (Musser and Bahnfleth 1999).

Both the ASHRAE and EPRI design guides recommended that radial diffusers be designed so that the diffuser area does not exceed 50% of the tank plan area. It is not possible, in most cases, for a radial diffuser to satisfy both this criterion and the recommended Re_i limits. Indeed, the Re_i limit would require diffuser radius to exceed tank radius in many cases. Investigation of the dimensions of radial diffusers in full-scale installations reveals that they typically occupy only 5 to 10% of the plan area. Evidently, designers of radial diffusers apply different, and as yet unpublished, criteria than are recommended in open sources.

As this brief review suggests, there remain many questions about the currently recommended method for diffuser design. The importance of the inlet Froude number is not in dispute, however, its effect on stratification is known only in a qualitative way. The significance of inlet Reynolds number, on the other hand, is unclear, with some attaching great importance to it and others claiming that it is irrelevant for practical purposes. The effects of tank dimensions are noteworthy by their absence from any design parameters. These are the issues addressed by the present work with respect to radial diffusers.

**STUDY METHODOLOGY**

The project was subdivided into the following tasks:

- Formulation of the computational problem.
- Development and validation of the CFD model.
- Identification of parameters affecting inlet performance.
- Design of test plan for parametric simulations.
- Performance of parametric CFD analyses.
- Analysis of results.

Formulation of the charge inlet problem and development and validation of the CFD model are described in detail in Part 1 (Musser and Bahnfleth 2001), but will be summarized briefly for completeness. The problem considered is the charging at constant temperature and flow rate of a tank initially containing water at a uniform warm temperature. Charging continues until a fully
developed thermocline is established. The geometry of the two-dimensional cylindrical domain and the boundary and initial conditions applied to the Navier-Stokes and energy equations are shown in Figure 2. This problem was implemented in a commercial finite element code (FDI 1993) and validated with field data from two full-scale tanks.

Identification of Parameters Influencing Inlet Performance

The performance of a radial inlet diffuser can be measured in terms of thermocline thickness, $h_t$, or equivalent lost tank height, ELH. Thermocline thickness is essentially a thermal boundary layer thickness. Equivalent lost tank height is the thickness of a layer of water that would represent the unusable capacity lost to mixing and other mechanisms if it experienced a temperature rise equal to the difference between the nominal discharge and charge inlet temperatures. Detailed discussion of these and other performance metrics have been provided elsewhere by the authors (Bahnfleth and Musser 1998, Musser and Bahnfleth 1999).

Buckingham Pi analysis (White 1986) reduces these seven dimensional variables (either performance metric and the six system variables) to five nondimensional parameters: the non-

![Figure 2. Domain and boundary conditions](image-url)
dimensional performance metric, $R_i$, $Re_i$ and two geometric parameters (Bahnfleth and Musser 1999). Arbitrarily, it was decided to non-dimensionalize $h_i$ and ELH with $R_w$. It follows that inlet performance can be represented by functions of the form:

$$
\left( \frac{h_i}{R_w} \right) = fnc \left( Re_i, Ri, \frac{R_D}{h_i}, \frac{R_D}{R_w} \right)
$$

(3)

or

$$
\left( \frac{ELH}{R_w} \right) = fnc \left( Re_i, Ri, \frac{R_D}{h_i}, \frac{R_D}{R_w} \right)
$$

(4)

where

$$
Re_i = \frac{Q}{\nu 2\pi R_D}
$$

(5)

$$
Ri = \frac{(2\pi R_D)^2 g' h_i^3}{Q^2} = \frac{1}{Fr_i^2}
$$

(6)

Note that the Richardson number ($R_i$) in Equations (3) and (4) is the inlet Richardson number, which is the inverse square of the inlet Froude number.

Not all parameters generated by a Pi analysis are necessarily significant. The objective of this investigation was to determine which of the parameters or combinations of parameters in Equations (3) and (4) are significant and to estimate the magnitude of these effects to first order. This was accomplished through the application of factorial experimental design theory.

**Factorial Experimental Design**

Factorial design theory enables the significance of the effects of parameters and their interactions on the value of a result of interest to be assessed through a minimum number of experiments (Montgomery 1997). Analysis of a factorial experiment identifies which parameters or multiple parameter interactions have significant effects on the dependent parameter (also called the response variable) over the range of values considered.

Factorial design is frequently applied to physical experiments, but it is equally applicable to the analysis of simulations. Factorial theory was of considerable importance to the present research because of the time and computational resources required for each CFD simulation. Running on a workstation, typical simulations took roughly four days of real time to execute. The range of execution times varied from two to eight days.

Variables in a factorial experiment may be continuous or discrete. When the independent variables are continuous, each can be tested at any number of values. The number of values of each independent parameter tested determines the highest order of the correlation that can be obtained. For a study in which the goal is to identify important parameters and interactions, it is typical to select only two values of each variable. This procedure is called a $2^k$ factorial experiment, since the total number of combinations that can be generated by two values each of $k$ parameters, and hence, the number of experiments required, is $2^k$. In a $2^k$ experiment, first order correlations can be developed between the response variable and independent parameters. When higher order correlations are required, three or more values of each significant parameter must be tested.
The two values of each independent variable in a $2^k$ experiment should be the most representative high and low values. It is important to choose these values appropriately, because the effect of each parameter is linearized. Choosing a very wide interval may have the result that a parameter that affects the response variable strongly only in an extreme range erroneously appears to be significant over the entire range investigated. On the other hand, the range between high and low values should be sufficiently large that the analysis will identify all parameters that are significant for most realistic cases.

**Test Plan**

The test plan for the present study was a $2^k$ factorial experiment. High and low values of each of the four parameters defining inlet diffuser performance ($Re_i$, $Ri$, $R_{Iy}/h_i$, and $R_{Iy}/R_w$) were selected after reviewing the dimensions, temperature, and flow rate characteristics of a number of full scale chilled water storage systems. These limiting values are summarized in Table 1. The parameters of the two full-scale tanks used for validation of the CFD model (Musser and Bahnfleth 2001) both fall within these ranges at design conditions. The sixteen simulations required for the factorial experiment are listed in Table 2.

**Identification and Modeling of Significant Effects**

When a full factorial experiment has been performed, each main effect and interaction can be calculated. Effects are defined as the average change in response that occurs as a result of changing each variable from its low value to its high value. Formulas for computing effects can be found in statistics texts (Montgomery 1997).

In a physical experiment, repeated trials of each treatment combination should be performed so that differences due to measurement uncertainty can be distinguished from significant effects. An indication of significance can be obtained by the $t$-test, which can be performed graphically by constructing a normal probability plot of all calculated effects. In these coordinates, effects that are most likely due to measurement error will lie along a straight line while significant effects will deviate from that line. When factorial design theory is used to analyze results that do not require repetition significant effects can also be identified visually from a normal probability plot.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Low Value (−)</th>
<th>High Value (+)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Re_i$</td>
<td>1,000</td>
<td>12,000</td>
</tr>
<tr>
<td>$Ri$</td>
<td>1.0</td>
<td>11.1</td>
</tr>
<tr>
<td>$R_{Iy}/h_i$</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>$R_{Iy}/R_w$</td>
<td>0.2</td>
<td>0.4</td>
</tr>
</tbody>
</table>

**Table 1. Typical High and Low Values of Nondimensional Parameters**

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Parameter Value*</th>
<th>Parameter Value*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Re_i$</td>
<td>$Ri$</td>
</tr>
<tr>
<td>1</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>3</td>
<td>–</td>
<td>+</td>
</tr>
<tr>
<td>4</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>5</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>6</td>
<td>+</td>
<td>–</td>
</tr>
<tr>
<td>7</td>
<td>–</td>
<td>+</td>
</tr>
<tr>
<td>8</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>

*High (+) or low (−) value from Table 1
If no effects are discarded, a model can be obtained that reproduces each response exactly. If, on the basis of a t-test, some effects are neglected, a relationship is obtained that estimates the response as a function of the most significant variables. Linear regression can be used to estimate the value of the response variable as a function of the primary effects and interactions. For a $2^4$ factorial experiment involving parameters $A$, $B$, $C$, and $D$, there are fifteen possible effects: four main effects ($A$, $B$, $C$, and $D$), six two-factor effects ($AB$, $AC$, $AD$, $BC$, $BD$, and $CD$), four three-factor effects ($ABC$, $ABD$, $BCD$, and $ACD$), and one four-factor effect ($ABCD$). If all of these effects are included in the regression model, it has form:

$$\hat{y} = \beta_0 + \beta_A x_A + \beta_B x_B + \beta_C x_C + \beta_D x_D + \beta_{AB} x_A x_B + \beta_{AC} x_A x_C + \beta_{AD} x_A x_D + \beta_{BC} x_B x_C + \beta_{BD} x_B x_D + \beta_{CD} x_C x_D + \beta_{ABC} x_A x_B x_C$$

where

- $\hat{y}$ = predicted response
- $\beta_0$ = intercept
- $\beta_i$ = regression coefficient for effect $i$
- $x_j$ = values of independent parameters ($j = A, B, C, or D$)

Typically, many effects can be neglected. The value of $\beta_i$ for each neglected effect is zero.

In order to confirm that significant effects have been taken into account, Equation (7), with neglected effects set to zero, is used to predict each response ($y$) measured in the experiment. The residuals ($y - \hat{y}$) are then calculated and plotted on normal probability coordinates. If all significant interactions have been included in the linear regression model, the plot should approximate a straight line.

If a few higher order effects seem to have borderline significance, or if the residual plots show either a non-normal distribution of residuals or correlation between residuals and response value, Equation (7) may provide a better model of a transformed response. A common and useful procedure is square root transformation, in which regression is performed on $\sqrt{y}$, rather than $y$.

RESULTS

Thermocline thicknesses and equivalent lost tank height were calculated for each of the sixteen simulations in the factorial experiment. Thermocline thicknesses were calculated for tails truncated at 10% and 15% of the overall temperature difference, using the method described in Part 1 (Musser and Bahnfleth 2001). Equivalent lost height was calculated for the entire temperature profile remaining in the tank and for distributions truncated at 10 and 15% of the overall temperature difference with a limiting outlet temperature equal to the average of $T_h$ and $T_c$. The reason for generating several variations on both $h_t$ and ELH was to facilitate subsequent comparison of response variable models with field data.

These results were transformed into the non-dimensional response variable values shown in Table 3. For all five measured responses, statistical analysis indicated the importance of the three main effects: $R_i$, $R_p/R_w$, and $R_f/h_t$. Although the values of the various response variables differ from one another, the results follow nearly identical patterns. Therefore, only representative analysis for equivalent lost tank height ($ELH/R_w$) is presented here.
Table 3. Response Variables Computed from CFD Simulation Results

<table>
<thead>
<tr>
<th>Simulation*</th>
<th>$h_{1,10%}$/$R_w$</th>
<th>$h_{1,15%}$/$R_w$</th>
<th>ELH$_{1,10%}$/$R_w$</th>
<th>ELH$_{1,15%}$/$R_w$</th>
<th>ELH/$R_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0925</td>
<td>0.0741</td>
<td>0.0125</td>
<td>0.0111</td>
<td>0.0148</td>
</tr>
<tr>
<td>2</td>
<td>0.1841</td>
<td>0.1437</td>
<td>0.0233</td>
<td>0.0210</td>
<td>0.0271</td>
</tr>
<tr>
<td>3</td>
<td>0.1167</td>
<td>0.0827</td>
<td>0.0139</td>
<td>0.0113</td>
<td>0.0168</td>
</tr>
<tr>
<td>4</td>
<td>0.0622</td>
<td>0.0454</td>
<td>0.0070</td>
<td>0.0059</td>
<td>0.0086</td>
</tr>
<tr>
<td>5</td>
<td>0.1165</td>
<td>0.0885</td>
<td>0.0161</td>
<td>0.0144</td>
<td>0.0195</td>
</tr>
<tr>
<td>6</td>
<td>0.0719</td>
<td>0.0520</td>
<td>0.0075</td>
<td>0.0061</td>
<td>0.0087</td>
</tr>
<tr>
<td>7</td>
<td>0.0306</td>
<td>0.0260</td>
<td>0.0034</td>
<td>0.0033</td>
<td>0.0036</td>
</tr>
<tr>
<td>8</td>
<td>0.0633</td>
<td>0.0514</td>
<td>0.0066</td>
<td>0.0065</td>
<td>0.0080</td>
</tr>
<tr>
<td>9</td>
<td>0.1779</td>
<td>0.1471</td>
<td>0.0207</td>
<td>0.0189</td>
<td>0.0273</td>
</tr>
<tr>
<td>10</td>
<td>0.0945</td>
<td>0.0699</td>
<td>0.0125</td>
<td>0.0103</td>
<td>0.0142</td>
</tr>
<tr>
<td>11</td>
<td>0.0637</td>
<td>0.0492</td>
<td>0.0081</td>
<td>0.0070</td>
<td>0.0093</td>
</tr>
<tr>
<td>12</td>
<td>0.1265</td>
<td>0.0976</td>
<td>0.0144</td>
<td>0.0121</td>
<td>0.0161</td>
</tr>
<tr>
<td>13</td>
<td>0.0635</td>
<td>0.0525</td>
<td>0.0079</td>
<td>0.0073</td>
<td>0.0086</td>
</tr>
<tr>
<td>14</td>
<td>0.1122</td>
<td>0.0882</td>
<td>0.0122</td>
<td>0.0111</td>
<td>0.0165</td>
</tr>
<tr>
<td>15</td>
<td>0.0687</td>
<td>0.0575</td>
<td>0.0076</td>
<td>0.0073</td>
<td>0.0084</td>
</tr>
<tr>
<td>16</td>
<td>0.0229</td>
<td>0.0185</td>
<td>0.0027</td>
<td>0.0024</td>
<td>0.0029</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.1841</td>
<td>0.1471</td>
<td>0.0233</td>
<td>0.0210</td>
<td>0.0273</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.0229</td>
<td>0.0185</td>
<td>0.0027</td>
<td>0.0024</td>
<td>0.0029</td>
</tr>
</tbody>
</table>

*See Tables 1 and 2 for parameter values.
Linear Regression Models

Testing of variable transformations indicated that a square root transformation reduced the magnitude of two-factor effects, and gave smaller residuals for the linear regression model. Therefore, linear regression was applied to the transformed responses, for example \((\text{ELH}/R_w)^{1/2}\). Effects obtained using the square root variable transformation on the five response variables are shown in Table 4. Use of this transformation eliminated ambiguity associated with interaction effects, reducing these to no more than 10% of the larger effects.

The normal probability plot of effects on the transformed response variable \((\text{ELH}/R_w)^{1/2}\) is shown in Figure 3. Significant effects are shown as solid symbols, while neglected effects are shown by open symbols. The primary effects \(R_i\), \(R_{f}/h_i\), and \(R_{f}/R_w\) are clearly significant because they do not fall along the straight line formed by the other twelve effects. Because only these three effects are significant, coefficients for higher order interactions and the first order coefficient for \(R_{f}\) were set to zero in the regression model [Equation (7)]. The resulting first-order regression is:

\[
\frac{\text{ELH}}{R_w} = \left( 0.12436 - 0.0035133R_i - 0.0066719 \frac{R_D}{h_i} + 0.191073 \frac{R_D}{R_w} \right)^2
\]  (8)

When plotted on normal probability coordinates (Figure 4), the residuals of Equation (8) are small and fall near a straight line, indicating that they tend to be normally distributed and that the model represents the data well.
The regression models for the response variables can be algebraically rearranged into dimensional predictors of thermocline thickness and equivalent lost height:

\[ h_{t, 10\%} = R_w \left( 0.31123 - 0.0076724R_i - 0.015624 \frac{R_D}{h_i} + 0.48658 \frac{R_D}{R_w} \right)^2 \]  
(9)

\[ h_{t, 15\%} = R_w \left( 0.26712 - 0.0068355R_i - 0.012964 \frac{R_D}{h_i} + 0.43602 \frac{R_D}{R_w} \right)^2 \]  
(10)

\[ ELH_{t, 10\%} = R_w \left( 0.11652 - 0.0030074R_i - 0.005995 \frac{R_D}{h_i} + 0.16032 \frac{R_D}{R_w} \right)^2 \]  
(11)

\[ ELH_{t, 15\%} = R_w \left( 0.10369 - 0.0028358R_i - 0.0050979 \frac{R_D}{h_i} + 0.15723 \frac{R_D}{R_w} \right)^2 \]  
(12)

\[ ELH = R_w \left( 0.12436 - 0.0035133R_i - 0.0066719 \frac{R_D}{h_i} + 0.191073 \frac{R_D}{R_w} \right)^2 \]  
(13)

These expressions are valid for the following parameter ranges:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lower Limit</th>
<th>High Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ri</td>
<td>1</td>
<td>11.1</td>
</tr>
<tr>
<td>R_d/h_i</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>R_d/R_w</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>Re_i</td>
<td>1,000</td>
<td>12,000</td>
</tr>
</tbody>
</table>

**Figure 4. Normal probability plot of residuals from linear regression of \( \sqrt{ELH/R_w} \) and CFD model predictions**
Linear regression model predictions were compared with values computed from results of CFD simulations for all sixteen cases in the test plan. Table 5 presents representative results dimensionalized for the case of a representative tank of 35 ft (10.7 m) radius. Thermocline thickness \( h_t \) varies from roughly 0.8 ft (0.24 m) to 6.5 ft (1.98 m). The corresponding ELH values associated with these thermoclines are 0.1 ft (0.03 m) and 0.9 ft (0.27 m), respectively. Such performance would be considered quite acceptable in a real system. This is not surprising, since the parameter ranges employed in this study are representative of actual systems that perform satisfactorily.

The residuals of linear regression models are generally small when predicting the CFD results from which they were derived. The largest ratio of residual to CFD result occurs for the thinnest thermoclines. For the various measures of thermocline thickness and equivalent lost tank height, these are within about 20%. On average, linear regression models predicted CFD model results to within less than 10%. Efforts to improve the performance of the model by adding the neglected effects in order of significance failed unless most of the neglected effects were added. Including all effects merely guarantees agreement at the sixteen test cases values and does not ensure significantly better accuracy for arbitrary combinations of parameter values. Therefore, the simpler correlations [Equations (9) through (13)] are preferable as first-order models.

### DISCUSSION

The results presented show that, to first order, charge inlet thermocline thickness or equivalent lost tank height in a cylindrical tank with radial diffusers is governed by \( R_i \), \( R_f/h_t \) and \( R_f/R_w \) for the range of parameters considered. The strongest primary effect is associated with \( R_f/R_w \), however the effects of \( R_i \) and \( R_f/h_t \) are 80 to 95% as large. Viewed as isolated effects, increasing \( R_i \) and \( R_f/h_t \) improve performance while increasing \( R_f/R_w \) cause deterioration in performance.

The inverse proportion between thermal performance and \( F_{ri} \) is well documented (Wildin and Truman 1989, Hussain 1989). Since \( R_i = Fr_i^{-2} \), an increase in Richardson number is equivalent to a decrease in Froude number and the results of the present investigation are in agreement with earlier studies.

The characteristics of the gravity wave leaving an inlet diffuser are related to inlet height (Yoo 1986). Therefore, the height of the inlet diffuser, represented non-dimensionally by the ratio \( R_f/h_t \), should influence mixing. An increase in this parameter corresponds to a decrease in inlet height, all other factors being equal. For sufficiently large \( R_i \), it is reasonable to expect that a thinner inlet jet will result in a thinner thermocline.

The significance of \( R_f/R_w \) effects has not been discussed in the literature in quantitative terms, although, Mackie and Reeves (1988) recommended over a decade ago that the plan area of radial diffusers in cylindrical tanks should not exceed approximately half the area of the tank.

<table>
<thead>
<tr>
<th></th>
<th>( h_t,10% ), ft (m)</th>
<th>( h_t,15% ), ft (m)</th>
<th>( ELH_t,10% ), ft (m)</th>
<th>( ELH_t,15% ), ft (m)</th>
<th>ELH, ft (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum CFD result</td>
<td>6.44 (1.96)</td>
<td>5.15 (1.57)</td>
<td>0.82 (0.25)</td>
<td>0.74 (0.22)</td>
<td>0.95 (0.29)</td>
</tr>
<tr>
<td>Minimum CFD result</td>
<td>0.80 (0.24)</td>
<td>0.65 (0.20)</td>
<td>0.09 (0.03)</td>
<td>0.09 (0.03)</td>
<td>0.10 (0.03)</td>
</tr>
<tr>
<td>Maximum residual</td>
<td>0.18 (0.055)</td>
<td>0.11 (0.032)</td>
<td>0.035 (0.011)</td>
<td>0.018 (0.005)</td>
<td>0.042 (0.013)</td>
</tr>
</tbody>
</table>

Residuals, % CFD value

<table>
<thead>
<tr>
<th></th>
<th>Maximum</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum</td>
<td>22</td>
<td>8</td>
</tr>
<tr>
<td>Average</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

Maximum residual
An equivalent statement is that $RD/R_w$ for a radial diffuser should not be greater than 0.707. The present work shows sensitivity to this parameter for substantially smaller values of $RD/R_w$. The optimal value of $RD/R_w$ may vary from case to case depending upon interactions with other system parameters. This finding does not contradict published guidance, which simply sets an outer limit on $RD/R_w$. The gist of the discussion in the literature is that when $RD/R_w$ exceeds 0.707 performance will deteriorate, not that optimal performance will occur at this value.

Although $Re_i$ has been identified as an important stratification parameter (Yoo 1986, Wildin and Truman 1989), it did not significantly affect thermocline thickness over the range of parameters considered in this analysis. Effects attributed to Reynolds number in prior studies may be explicable in terms of variation of other geometric parameters or could have resulted from coupling of $Ri$ and $Re_i$. For example, if $Re_i$ increases due to increased flow rate through a given diffuser, $Ri$ will also increase. Increase in $Ri$ should degrade performance independent of any $Re_i$ effect.

Wildin (1989) compared the performance of a small radial diffuser, a large radial diffuser, and an octagonal diffuser at equal $Fr_i$ and found that performance improved with decreasing $Re_i$. Although direct comparisons cannot be made with the present study because the parameters of Wildin’s system were partly outside the ranges considered here, the findings of the two investigations are not qualitatively incompatible. The small and large radial diffusers studied by Wildin had $RD/h_i$ values of 4.5 and 16, respectively. The ratio $RD/R_w$ was 0.15 for the smaller radial diffuser and 0.30 for the larger. The present work has demonstrated a tendency for performance to improve with increasing $R_D/h_i$ and to deteriorate with increasing $R_D/R_w$. For fixed $Ri$ (or $Fr_i$) as in the experiments by Wildin, the relative strength of these effects will determine whether thermal performance improves or deteriorates as both parameters increase. The large $RD/h_i$ value of Wildin’s larger radius/lower $Re_i$ diffuser may be the underlying cause of the reported behavior. This analysis is, however, conjectural.

**Comparison of Model with Measured Data**

Regression model predictions were compared with field data collected by the authors from several full-scale tanks with radial diffusers (Musser 1998, Musser and Bahnfleth 1999). Five charge tests were obtained with $Ri$ in the range of validity of the regression models. Four tests were performed in a university medical center tank, and one was performed in a tank at a county government complex (Musser and Bahnfleth 1999). A slow decrease in charge inlet temperature occurred during each of these tests, which differs from the constant inlet temperature condition applied in the simulations on which the regressions are based. This tends to increase measured thermocline thickness. Given this discrepancy in boundary conditions, the agreement between model and data was reasonably good.

Table 6 compares measured full-scale tank performance with linear regression model predictions. As in validation studies of the CFD model (Musser and Bahnfleth 2000), much better agreement is obtained for performance measures that discard larger portions of the thermocline tail. The worst agreement occurs for ELH, with measured values exceeding the predictions by a factor of 1.7 to 3.4. It is expected that ELH would show the poorest agreement, since the entire tail of the temperature distribution is included in its calculation. When the tails of the temperature profile are discarded at a dimensionless temperature of 0.1, agreement improves, with measured thermocline thickness ($h_{t,10%}$) exceeding the prediction by an average factor of 1.9 and with measured ELH$_{t,10%}$ exceeding the prediction by an average factor of 1.6. The better agreement between ELH$_{t,10%}$ as compared to $h_{t,10%}$ is also expected based on the validation study.

When measured thermocline thicknesses exceed predicted values because of an inlet temperature variation that lengthens the thermocline tail, thermocline thickness and equivalent lost tank height should more closely match their predicted values for larger dimensionless cutoff
temperatures. If the cutoff dimensionless temperature is increased to 0.15, measured $h_{t,15\%}$ exceeds predictions by an average factor of 1.3, and $ELH_{t,15\%}$ exceeds its predicted value by an average factor of 1.1. For three of the five tests, $ELH_{t,15\%}$ is within 10% of its predicted value.

The observed inlet temperature variation was larger for the two tests in which poor correlation was achieved.

Figures 5 and 6 are "radar" plots comparing regression model prediction and field data agreement for the same five cases summarized in Table 6. The comparison is done in terms of magnitude of relative error. Each axis represents one case and values of error in thermocline thickness or equivalent lost tank height for the case are plotted along the axis. Values for each metric for the five cases are connected to form polygons, the area of which is indicative of the aggregate error. Note that in Figure 5 the area enclosed by the $h_{t,10\%}$ polygon is much larger than that enclosed by the $h_{t,15\%}$ polygon. In similar fashion, the $ELH_{15\%}$ is nested within $ELH_{10\%}$, which lies within $ELH$ in Figure 6. This clearly shows the improved correlation that occurs when more of the thermocline tail is discarded.

Regression Model Linearity

It is implicit in Equations (9) through (13) that behavior of square root transformed response variables is linear over the range of validity of the regressions. These relationships may, in fact, be significantly nonlinear, something that can be assessed rigorously only by expanding the factorial experiment to include tests at three or more values of each parameter. However, preliminary observations can be made by comparing linear regression models with the results of CFD model validation simulations. Two constant inlet temperature simulations used for validation (Musser and Bahnfleth 2001) have input parameters that fall within the range of validity of the parametric study [c.f. Equation (14). $Ri$ is out of range for the others]. Results of these simulations are compared with the predictions of the linear regression model for identical conditions in Table 7.

The first order relationships provide a reasonable estimate of each of the performance measures. For each metric and both cases, the performance measure predicted by the simplified equation is within 18% of the actual CFD result and most are within 10%. For both cases, the values of the two geometric parameters are close to the limit of the range tested: therefore, these effects might be better predicted by the linear regression model than they would be at values near the center of the range. For the Medical Center validation case, however, the Richardson

<table>
<thead>
<tr>
<th>Site</th>
<th>$Ri$</th>
<th>$h_{t,10%}$</th>
<th>$h_{t,15%}$</th>
<th>$ELH_{t,10%}$</th>
<th>$ELH_{t,15%}$</th>
<th>$h_{t,10%}$</th>
<th>$h_{t,15%}$</th>
<th>$ELH_{t,10%}$</th>
<th>$ELH_{t,15%}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medical Center</td>
<td>7.22</td>
<td>2.92</td>
<td>2.24</td>
<td>0.41</td>
<td>0.36</td>
<td>0.31</td>
<td>3.59</td>
<td>1.95</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.89)</td>
<td>(0.68)</td>
<td>(0.12)</td>
<td>(0.11)</td>
<td>(0.09)</td>
<td>(1.09)</td>
<td>(0.59)</td>
<td>(0.24)</td>
</tr>
<tr>
<td>($RD/hi=5.56$, $R_D/R_w=0.23$)</td>
<td>5.66</td>
<td>3.18</td>
<td>2.44</td>
<td>0.46</td>
<td>0.39</td>
<td>0.34</td>
<td>5.57</td>
<td>3.58</td>
<td>1.79</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.97)</td>
<td>(0.74)</td>
<td>(0.14)</td>
<td>(0.12)</td>
<td>(0.10)</td>
<td>(1.70)</td>
<td>(1.09)</td>
<td>(0.55)</td>
</tr>
<tr>
<td></td>
<td>11.16</td>
<td>2.33</td>
<td>1.78</td>
<td>0.31</td>
<td>0.28</td>
<td>0.24</td>
<td>4.82</td>
<td>2.44</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.71)</td>
<td>(0.54)</td>
<td>(0.09)</td>
<td>(0.09)</td>
<td>(0.07)</td>
<td>(1.47)</td>
<td>(0.74)</td>
<td>(0.22)</td>
</tr>
<tr>
<td></td>
<td>5.56</td>
<td>3.20</td>
<td>2.45</td>
<td>0.46</td>
<td>0.40</td>
<td>0.34</td>
<td>7.56</td>
<td>4.57</td>
<td>1.40</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.98)</td>
<td>(0.75)</td>
<td>(0.14)</td>
<td>(0.12)</td>
<td>(0.10)</td>
<td>(2.30)</td>
<td>(1.39)</td>
<td>(0.43)</td>
</tr>
<tr>
<td>County Gov’t.</td>
<td>2.16</td>
<td>3.30</td>
<td>2.54</td>
<td>0.49</td>
<td>0.42</td>
<td>0.36</td>
<td>7.37</td>
<td>5.19</td>
<td>1.40</td>
</tr>
<tr>
<td>($RD/hi=5.70$, $R_D/R_w=0.20$)</td>
<td>(1.01)</td>
<td>(0.77)</td>
<td>(0.15)</td>
<td>(0.13)</td>
<td>(0.11)</td>
<td>(2.25)</td>
<td>(1.58)</td>
<td>(0.43)</td>
<td>(0.21)</td>
</tr>
</tbody>
</table>
The performance of lower radial parallel plate diffuser charge inlet performance was investigated using a combination of computational fluid dynamic (CFD) modeling, factorial experimental design theory, dimensional analysis, and field data. A CFD model was developed and

CONCLUSIONS

The performance of lower radial parallel plate diffuser charge inlet performance was investigated using a combination of computational fluid dynamic (CFD) modeling, factorial experimental design theory, dimensional analysis, and field data. A CFD model was developed and
validated, radial diffuser inlet flow was parameterized using the Buckingham Pi theorem, and a $2^4$ factorial test plan was developed in terms of the five dimensionless parameters resulting from this analysis. Sixteen CFD simulations (in addition to validation cases) were performed. Analysis of these results led to the identification of significant parameters and the development of first-order regression models for thermocline thickness and equivalent lost tank height. These models were tested against both simulated and measured performance.

The results of the study suggest the following conclusions:

- Parameters of first order significance for radial diffuser inlet thermal performance are the inlet Richardson number, the ratio of diffuser diameter to diffuser height, and the ratio of diffuser diameter to tank diameter.

- The Richardson number (or the equivalent inlet Froude number) is reconfirmed as the most significant flow parameter governing stratification for slot-type diffusers. Current design guidance recommends the use of a Richardson number equal to one; however, the present work indicates that the use of larger values (equivalent to Froude numbers less than one) can significantly improve performance.

- The Inlet Reynolds number, which has been identified in the open literature as the second most significant flow parameter, did not strongly influence initial stratification for slot-type diffusers. Current design guidance recommends the use of a Richardson number equal to one; however, the present work indicates that the use of larger values (equivalent to Froude numbers less than one) can significantly improve performance.

- The relationship between diffuser radius and tank radius had a strong effect on performance, with smaller radius diffusers performing better. This effect was stronger than that of Ri within the range investigated. This is evidently due to reduced mixing the increased distance over which the gravity current slows before it impacts the tank wall.

- Smaller inlet height (other parameters remaining fixed) also improved thermal performance, if for no other reason than that the gravity current associated with a shorter inlet is itself shorter than that produced by a taller inlet. Yoo (1986) has previously documented this effect for linear gravity currents with Froude numbers less than 1.0.

- Correlations such as those developed in the present study may be valuable for designing systems. However, they should be applied with due regard for the assumptions on which they rest. In field tests for validation purposes the initial charge inlet temperature routinely

![Table 7. Evaluation of the Linearity of Effects Assumption](image-url)
exceeded the charge set point and fell slowly toward the intended value. As a result, thermocline thickness and equivalent lost tank height were both greater than predicted by a model with constant inlet temperature, although they were well-predicted from actual inlet temperatures.

ACKNOWLEDGMENT

Funding for this work was provided through ASHRAE Research Project 1077 (Bahnfleth and Musser, 1999) with the sponsorship of TC 6.9 Thermal Storage. The authors extend their appreciation to ASHRAE for its financial support and to the members of the Project Monitoring Subcommittee for their guidance throughout the course of the project.

REFERENCES


