Lecture 13

Transients in Subprompt-Critical or Subcritical Reactivity Domain - Asymptotic Transients

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Asymptotic Transients

In addition to the expansion at $t = 0$,

$$p(t) = p^0 + p't + \cdots;$$

$$\rho(t) = \rho_1 + \rho't + \cdots;$$

$$\zeta_k(t) = \zeta_{k0} + \zeta'_k t + \cdots$$

we can derive an asymptotic expansion of

$$0 = \left\{ \rho_1 - \beta + \gamma \int_0^t [p(t') - p_0] dt' \right\} p + \sum_k \lambda_k \zeta_k.$$
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With $\dot{p} = 0$, the kinetic equation in the PJA, assumes the form

$$0 = \frac{\lambda \rho_{as} + \dot{\rho}_{as}}{\beta - \rho_{as}}$$

This requires that the numerator vanish, i.e.,

$$\lambda \rho_{as} + \dot{\rho}_{as} = 0$$

The asymptotic time-independent solution of above equation is

$$\rho_{as} = 0 \text{ since } \dot{\rho}_{as} = 0$$

The approach to the asymptotic solution above is describes by the time-dependent solution, i.e.,

$$\rho_{as} (t) \propto \exp(-\lambda t)$$
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Since $\rho_{as} = 0$, the inserted reactivity is asymptotically compensated for by the feedback:

$$-\gamma \int_{0}^{\infty} [p(t') - p_0] dt' = \rho_1$$

In this equation, the integral has to converge, which requires $p(t)$ to converge sufficiently fast toward $p_0$

Thus, a feedback reactivity given by the left side of this equation requires that $\rho_{as} = p_0$

Under the assumptions, on which the approximate kinetics equation and its initial condition are based; the prompt feedback forces the power back to its initial value
The asymptotic flux, \( p_{as} \), with feedback described by

\[ -\gamma \int_0^\infty [p(t') - p_0] dt' \]

cannot be smaller than \( p_0 \)

- If \( p(t) < p_0 \) in a certain time interval, the contribution of this interval to the integral is negative

- The fuel temperature is decreasing

- Decreasing fuel temperature together with a negative temperature coefficient leads to a reactivity increase, which is followed by an increase in the flux
**Asymptotic Transients**

**Example 1:** The highest of the curves in the figure

According to

\[-\gamma \int_{0}^{\infty} [p(t') - p_0] dt' = \rho_1 \quad \text{and} \quad p_{as} = p_0\]

the transient is supposed to converge to \(p_0\).

Its integral is approximately given by

\[
\int_{0}^{\infty} [p(t') - p_0] dt' = -\frac{\rho_1}{\gamma} = \left( \frac{0.949}{2.15 \times 10^{-3}} \right) \text{MW} \cdot \text{s} \cong 430 \text{MW} \cdot \text{s}
\]
**Asymptotic Transients**

**Example 2:** Subprompt-critical transients with $\rho_1=0.95$ (top) and 0.5 (bottom), $p_0=1$

Energy coefficient of: $\gamma = -0.8$/fp-s

Reactivity feedback by:

$$0 = \left( \rho_1 - \beta + \gamma \int_0^t [p(t') - p_0] dt' \right) p + \sum_k \lambda_k \zeta_k$$

According to

$$\int_0^\infty [p(t') - p_0] dt' = -\frac{\rho_1}{\gamma}$$

the energy releases is:

1.19 fp-s if $\rho_1=0.95$

and

0.625 fp-s if $\rho_1=0.5$
Asymptotic Transients

Example 2: (cont.)
The dashed lines cross the initial flux levels at ~3 s

The overshooting of the final flux level is due to:

The stationary flux level is given by $p_0 = \frac{s_{d0}}{\beta}$

This is fulfilled initially and it must hold asymptotically since $p_{as} = p_0$

If $p(t) \geq p_0$ and $\rho(t) \geq 0$ during the entire transient, the reactor would be permanently "loaded" with additional precursors

Then the asymptotic flux should be larger than $p_0$ in contradiction to $p_{as} = p_0$

Thus, the flux must be less than $p_0$ for some period of time to eliminate the additional precursors that are produced in the early part of the transient in order to achieve asymptotically $s_d = s_{d0}$

Subprompt-critical transients ($\rho_1 = 0.95$ (top) and 0.5$ (bottom), $p_0=1$) with reactivity feedback including first-order heat transient (solid line) and no heat transfer (dashed line)
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The kinetic equation in the PJA are based on three assumptions:

1. PJA
   - Should be accurate asymptotically since it is violated only in the immediate vicinity of fast changes $p_{as} = p_0$

2. One-delay-group kinetics
   - Might be questionable since the kinetics parameters cannot be lumped uniquely into a one-delay-group $\lambda$
   - $\lambda$ not even appear in the asymptotic relations: the asymptotic state is a stationary state - the flux level is independent of the precursor decay constant altogether

3. No transient heat transfer
   - Violated during the transient and even more so asymptotically
**Assumption for no transient heat transfer:**

The influence of heat transfer on the asymptotic flux can be semi-quantitatively investigated by using

\[ \delta \rho(t) = \gamma \int_0^t [p(t') - p_0] \exp[-\lambda_H(t-t')] dt' \]

with \( \rho_{as} = 0 \), the following equation is obtained

\[ \lim_{t \to \infty} - \gamma \int_0^t [p(t') - p_0] \exp[-\lambda_H(t-t')] dt' = \rho_1 \]

Asymptotically, \( \rho_{as} \) can be inserted as the value of the flux amplitude in performing the limit to large \( t \)

The exponential function practically eliminates the contribution of the integrand, which is far away from the upper limit.
Assumption for no transient heat transfer:

Asymptotically, the lower limit “0” is infinitely far away from “t”; it can therefore be replaced by -∞

This gives

$$-γ \int_{-∞}^{t} [p_{as} - p_0] \exp[-λ_H (t - t')] dt' = ρ_1$$

or

$$- \frac{γ}{λ_H} (p_{as} - p_0) = ρ_1$$

Solving for the asymptotic flux amplitude yields:

$$p_{as} = p_0 - ρ_1 \frac{λ_H}{γ} > p_0$$
**Asymptotic Transients**

**Assumption for no transient heat transfer:**

Thus:

- The prompt negative feedback does not shut the reactor down in sense of making it subcritical with an asymptotically vanishing flux.

- The control or shutdown system must be employed to restore the initial power or shut the reactor down.

- Only if $p_0$ is practically zero and $\lambda_H$ is very small, i.e., if there is practically no heat release, can $p_{as}$ be very small.
Asymptotic Transients

The basic effect of heat transfer is that the flux is not forced back to its initial value → it assumes asymptotically a value above the initial flux.

The numerical calculation of the transient yields the same asymptotic flux value as

\[ p_{as} = p_0 - \rho_1 \frac{\lambda_H}{\gamma} > p_0 \]

The formula based on a single delay constant \( \lambda_H \) provides the correct qualitative understanding of why an asymptotically constant flux above the initial flux is obtained.
Asymptotic Transients

Independent of the simplifications employed in describing the heat transfer, the negative reactivity feedback restores criticality.

The inserted reactivity $\rho_1$ is compensated by a temperature rise

$$\Delta T = -\rho_1 \left( \frac{\partial \rho}{\partial T} \right)^{-1}$$

With $\partial \rho / \partial T \approx -0.002 \, \text{S/K}$, a 0.5$ (or 0.95$) reactivity insertion would lead to a temperature rise of 250 K (or 475 K) if the control system did not start its counteraction during the transient.

The temperature rise in the center of the hottest pellet may be substantially larger than that given by the above.
**Asymptotic Transients**

**In summary:**

1. Prompt negative reactivity feedback reduces a subprompt reactivity insertion to zero: i.e., it restores criticality and does not shut the reactor down.

2. A permanently inserted reactivity is compensated by a permanent temperature rise, given by \( \Delta T = -\rho_1 (\partial \rho / \partial T)^{-1} \)

3. The flux approaches asymptotically a constant value. An asymptotic power level higher than the original power is required to sustain a permanent temperature rise.

4. If a core is originally at near zero power and “uncooled”, the Doppler feedback resulting from a “permanent” temperature increase will reduce the neutron flux to near zero and lead to a “shutdown” in this limited sense. However, the asymptotic core would still be critical.
Next Class: Superprompt-Critical Excursion Following a Step Reactivity Insertion