Superprompt-Critical Excursion Following a Step Reactivity Insertion

Flux Transient During a Superprompt-Critical Excursion

&

Post-Burst Flux Transient
Superprompt-Critical Excursion Following a Step Reactivity Insertion

Assumptions:

1. Prompt Kinetics Approximation (PKA)

\[ \dot{p} = \frac{\rho - \beta}{\Lambda} p \]

\[ p(0) = p^0 = \frac{\rho_1}{\rho_1 - \beta} p_0 \]

2. The reactivity contains the Doppler feedback as described by an energy coefficient \( \gamma \)

\[ C_{QTR} \left( \frac{\partial \rho}{\partial T} \right)_{Dop} = \gamma^D_e \left[ \frac{\delta \rho}{MW \cdot s} \right] = \text{energy coefficient} \]

3. Linear energy model with neglected stationary cooling:

\[ \rho(t) = \rho_1 + \gamma \int_0^t p(t')dt' \]

\[ \gamma \int_0^t (-p_0)dt' = -\gamma p_0 t = 0 \]
Outcomes:

1. The prompt reactivity vanishes at the same time when the flux has a maximum.

2. Consequently, the prompt part of the inserted reactivity is compensated when the flux has reached its maximum.

3. The energy release is independent of the neutron generation time.

4. Below $\rho = \beta$, the PKA only describes the prompt die-away of an existing neutron population - it cannot account for the behavior of the delayed neutron source, except in the initial condition.
Superprompt-Critical Excursion Following a Step Reactivity Insertion

Outcomes:

5. The prompt feedback compensates the prompt reactivity twice during the transient - the reactivity swing is two times the initial prompt reactivity

Superprompt-critical transient \( (\rho_i = 1.1$, $p_0 = 0.1p_n) \) from the PKA with linear energy feedback model
Flux Transient During a Superprompt-Critical Excursion

The PKA together with the linear energy feedback model describes the flux transient during a superprompt-critical excursion by:

\[
p(t) = \frac{p_m}{\left\{ \cosh \left[ \frac{\rho_b}{2\Lambda} (t - t_m) \right] \right\}^2}
\]

Thus, the PKA together with the linear energy feedback model gives a transient that is symmetrical around \((t-t_m)\)
Burst width:

The simplest formula for the "width" of the flux pulse (or burst) is obtained when the width $\Delta t$ is defined by the flux integral divided by the maximum value:

$$\Delta t = \frac{4\Lambda}{\rho_{p1}} \left( 1 - \frac{p^0}{p_m} \right)$$

for $p_m >> p^0$

$\Lambda$-independent energy production

Case 2:

$$t_m \approx \frac{\Lambda}{\rho_{p1}} \left( \ln 4 \frac{p_m}{p^0} \right)$$

for $p^0 < p_m$

The maximum is quickly reached after the onset of the burst
Post-Burst Flux Transient

Applicability of the prompt jump approximation?

Superprompt-critical transient ( $\rho_1 = 1.1$, $p_0 = 0.1p_n$ ) with reactivity feedback
**Post-Burst Flux Transient**

The temperature increase during the entire transient is composed of

1. The part produced during the prompt burst &
2. The part produced during the subsequent relatively gradual power decrease:

\[
\Delta T_{\text{total}} = \Delta T_{\text{burst}} + \Delta T_{\text{post-burst}}
\]

\[
\Delta T_{\text{total}} = -\frac{1}{\gamma_T} [2(\rho_1 - \beta)] - \frac{1}{\gamma_T} [\rho_1 - 2(\rho_1 - \beta)] = -\frac{\rho_1}{\gamma_T}
\]
Lecture 16

Superprompt-Critical Transients Induced by Reactivity Ramps

Dr. Maria Avramova
The Pennsylvania State University
Superprompt-Critical Transients Induced by Reactivity Ramps

Note:

- Reactivity steps in the superprompt reactivity domain of reactors are only an idealization.

- Realistic simulation of superprompt-critical reactivity insertions in a reactor must account for the reactivity insertion rate.

- In fully realistic situations, ramps are always terminated at some maximum available reactivity.
The PKA for reactivity ramps is applied only after the time, $t_p$, when the reactor became superprompt critical:

$$\dot{p}(\tau) = \frac{\rho_p(\tau)}{\Lambda} p(\tau) \quad \text{with} \quad \tau = t - t_p$$

which is solved with $p(0) = p^0$, and $\rho_p(0) = 0$ for $\tau = 0$

For reactivity ramps, the pseudo-initial flux, $p^0$, is approximately:

$$p^0 \approx p_0 \beta \left( \frac{2\pi}{\Lambda a} \right)^{1/2}$$
Investigation of the Differential Equation

The prompt reactivity, with the adiabatic approximation applied to the energy above $p_0$ is given by

$$ \rho_p(\tau) = a\tau + \gamma \int_0^\tau [p(\tau') - p_0] d\tau' $$

or

$$ \rho_p(\tau) = a'\tau + \gamma \int_0^\tau p(\tau') d\tau' $$

with

$$ a' = a - \gamma p_0 $$
The stationary cooling (−ρ₀) only modifies the ramp rate → it is included in the feedback reactivity:

\[ \rho_p(\tau) = a\tau + \gamma \int_0^\tau [p(\tau') - p_0] d\tau' \]

\( \dot{p}(\tau) \) and \( \rho_p(\tau) \) pass through zero simultaneously:

\[ \dot{p}(\tau) = (\rho_p(\tau) / \Lambda)p(\tau) \quad \rightarrow \quad \dot{p}(\tau) = 0 \quad \text{when} \quad \rho_p(\tau) = 0 \]
Investigation of the Differential Equation

The flux, $p(t)$, and the prompt reactivity $\rho_p(t)$, are shown for a reactivity-ramp-induced transient with temperature-independent energy coefficient where $p_0 = p_n$, $a = 50$/s, $\gamma = -0.5$/fp $- s$, and $p^0 = 28.0 \ p_0$. 
Investigation of the Differential Equation

Since $\rho_p(\tau)$ is zero initially, $p(\tau)$ in the PKA starts with a zero slope and has a minimum at $\tau = 0$

Subsequently, $\rho_p(\tau)$ and $p(\tau)$ both increase

With increasing accumulation of energy, the reactivity feedback overcomes the ramp reactivity and $\rho_p(\tau)$ is again reduced to zero; say at $\tau = \tau_{m1}$

The flux has its first maximum at this time

After passing through zero, $\rho_p$ approaches a negative value, corresponding to $-\rho_{p1}$ in the case of a step-induced transient
Investigation of the Differential Equation

- By this time, the high flux during the prompt burst is reduced so that further reactivity reduction through feedback occurs relatively slowly.

- A similar reduction of the reactivity is to be expected in the case of a ramp reactivity insertion.

- However, as the ramp continues to increase the reactivity, it again becomes equal to $\beta$.

- At this time, a second flux minimum occurs and the cycle repeats.
The discussion of maxima and minima showed that the ramp-induced transient treated in the PKA consists of a *repetition of superprompt flux bursts*

The maxima and minima of the flux occur when $\rho(t)$ passes through $\beta$:

$$p(t) = p_{\text{max}} \text{ or } p_{\text{min}} \quad \text{for } \rho_p(t_{ml}) = 0$$
The succession of bursts can be characterized by an averaged power that can be determined as follows:

1. Let $\Delta t_i$ be the duration of the $i^{\text{th}}$ burst and $\Delta Q_i$ the corresponding energy release:

$$\Delta Q_i = \int_{\Delta t_i} [P(t') - P_0] dt'$$

2. Since $\rho_p(t)$ is zero at the beginning and at the end of each burst, the relation between the reactivity insertion and the feedback reactivity during a burst can be given by:

$$0 = a\Delta t_i + \gamma_e \int_{\Delta t_i} [P(t') - P_0] dt' = a\Delta t_i + \gamma_e \Delta Q_i$$
3. The energy release per burst time, i.e., the "average" power increase during the burst, is obtained as:

\[
\frac{\Delta Q_i}{\Delta t_i} = \Delta \bar{P} = \bar{P} - P_0 = -\frac{a}{\gamma_e}
\]

4. The average power is independent of the generation time and of the initial power → it is the same for all bursts under the conditions of the feedback model

5. Using an effective ramp rate, the flux amplitude becomes:

\[
\bar{p} = -\frac{a'}{\gamma} \text{ where } a' = a - \gamma P_0
\]
Note:

For a *step-induced* supercritical transient, information on the energy release was obtained by investigating the transition of $\rho$ through $\beta$.

For a *ramp-induced* transient, information on the *average power* is obtained instead.

This difference occurs because only the *rate* of reactivity insertion is given for ramp-induced transients, which then determines the *rate* of energy release:

$$-\frac{\rho_{pl}}{\gamma_e} \Rightarrow \text{energy, for reactivity steps}$$

$$-\frac{a}{\gamma_e} \Rightarrow \text{average power, for reactivity ramps}$$
Investigation of the Differential Equation

Additional information is obtained by investigating the extrema of $\rho_p(\tau)$:

1) setting the time derivative of the prompt reactivity equation equal to zero and 2) applying the adiabatic approximation applied to the energy above $p_0$ gives

$$\dot{\rho}_p(\tau) = a + \gamma(p - p_0) = a' + \gamma p = 0$$

Thus, the flux at the extrema of the reactivity is equal to the average flux:

$$p(\rho_p^{\text{max,min}}) = p_0 - \frac{a}{\gamma} = -\frac{a'}{\gamma} = \bar{p}$$

i.e., the extrema of the reactivity coincide with $p(t)$ passing through $\bar{p}$
Investigation of the Differential Equation

The extrema of the reactivity determine the inflection points of the flux transient in a semi-logarithmic presentation:

\[
\frac{1}{\Lambda} \frac{d}{d\tau} \rho_p(\tau) = \frac{d}{d\tau} \ln p(\tau)
\]

Thus, the second derivative of the logarithm of the flux is zero at the extrema of the reactivity
Investigation of the First Integral

Re-writing the kinetics equation:

\[ \dot{p}(\tau) = \frac{\rho_p(\tau)}{\Lambda} p(\tau) \]

\[ \tau = t - t_p \]

Multiplying by \( \dot{\rho}_p \):

\[ \dot{\rho}_p(\tau) = a' + \gamma p \]

\[ a' = a + \gamma p_0 \]

\[ (a' + \gamma p) \dot{p} = \frac{1}{\Lambda} \rho_p \dot{\rho}_p p \]

Dividing by \( p \):

\[ \left( \frac{a'}{p} + \gamma \right) \dot{p} = \frac{1}{\Lambda} \rho_p \dot{\rho}_p \]

Integrating:

\[ \int_0^\tau \left( \frac{a'}{p} + \gamma \right) \dot{p} d\tau' = \frac{1}{\Lambda} \int_0^\tau \rho_p \dot{\rho}_p d\tau' \]
**Investigation of the First Integral**

The derivatives of $p$ and $\rho_p$ may be conveniently combined with $d\tau'$.

Then, the two integrals are carried out over $p$ and $\rho_p$, respectively:

\[
\int_{p(0)}^{p(\tau)} \left( \frac{\alpha'}{p} + \gamma \right) dp = \frac{1}{\Lambda} \int_{\rho_p(0)}^{\rho_p(\tau)} \rho_p d\rho_p
\]

Inserting $p(0) = p^0$ and $\rho_p(0) = 0$ as the lower limits gives the *first integral,* which represents a relation between the flux and the reactivity:

\[
\alpha' \ln \frac{p}{p^0} + \gamma(p - p^0) = \frac{1}{2\Lambda} \rho_p^2
\]
Investigation of the First Integral

The first integral is used to find the maximum and minimum values of both the reactivity and the flux transients:

\[
\ln \frac{p}{p^0} + \gamma(p - p^0) = \frac{1}{2\Lambda} \rho_p^2
\]

\[
\bar{p} = -a'\gamma
\]

\[
(\rho_p^{\text{max,min}})^2 = 2\Lambda a' \left( \ln \frac{p}{p^0} - 1 + \frac{p^0}{\bar{p}} \right)
\]

\[
\frac{\rho_p^{\text{max,min}}}{\beta} = \pm \frac{\sqrt{2\Lambda a'}}{\beta} \sqrt{\ln \frac{p}{p^0} - 1 + \frac{p^0}{\bar{p}}}
\]

1. Positive root represents the reactivity maxima
2. Negative root represents the reactivity minima
3. Maxima and minima of \( \rho \) are located symmetrically about \( \beta \)
Investigation of the First Integral

Maximum reactivity values versus $\frac{\bar{p}}{p^0}$ for a reactivity-ramp-induced transient where $a = 50\$/s$ and $\gamma = -0.5\$/fp - s$

The abscissa covers all transients with realistic ramp rates and feedback coefficients starting at (or around) nominal power.

The lower end of the scale is $\frac{\bar{p}}{p^0} = 1$ since the Equation has real roots only if $\frac{\bar{p}}{p^0} \geq 1$.

The upper end of the scale in the figure may be exceeded only at very low starting power.

If the Equation has no real roots, i.e., if $\bar{p} < p^0$ with $\bar{p} = -a' / \gamma$, then the PKA is fundamentally inapplicable.
The inequality $\bar{p} < p^0$ may be physically realized for small ramp rates, large feedback coefficients (small $\bar{p}$), or a large pseudo-initial flux (large $p^0$).

Investigation of the First Integral

In such cases, the feedback effect during the subprompt-critical part of the transient is strong enough that the reactivity does not reach the superprompt-critical domain.

The figure shows that a ramp rate of 50$/s$ can only push a fast reactor beyond prompt critical by a small fraction of a dollar.

$a = 50$/s $\gamma = -0.5$/fp - s
Investigation of the First Integral

Results for different ramp rates may be obtained from this figure by changing the factor in front of the square root

\[
\frac{\rho_p^{\text{max, min}}}{\beta} = \pm \frac{\sqrt{2a'}}{\beta} \sqrt{\ln \frac{p}{p^0} - 1 + \frac{p^0}{p}}
\]

Example:
An increase of the ramp rate from 50 to 100$/s increases the maximum of \( \rho_p \) by a factor of \(~1.4\)
The ratio of the flux maxima to the average flux, $p_{\text{max}}/\bar{p}$, as a function of $\bar{p}/p^0$

The figure shows that $p_{\text{max}}$ does not become very much larger than $\bar{p}$.

$p_{\text{max}}/\bar{p}$ increases with increasing ramp rate since $\bar{p}/p^0$, is approximately proportional to $(a)^{3/2}$.

$a = 50$/s  \hspace{1cm} \gamma = -0.5$/fp$s$
Investigation of the First Integral

Example 1:

Reactivity-ramp-induced transient with temperature-independent energy coefficient where $p_0 = p_n$, $a = 50$/s, $\gamma = -0.5$/fp – s, and $p^0 = 28.0$ $p_0$.

$p_0 = p_n$
$p^0 = 28p_0$
$\bar{p} = 101p_n$

$\frac{p_{\text{max}}}{\bar{p}} \approx 2.4$ and $\rho_p^{\text{max}} = 9.5 \xi$
Investigation of the First Integral

The investigation of the first integral for ramp-induced transients provided important information such as $\bar{\rho}$, $\rho_{\text{max}}$, and $\rho_{\text{max}}$

The pseudo-initial flux $p^0$ decreases and the average flux increases with increasing ramp rate

\[
p^0 = p_0 \beta \sqrt{\frac{2\pi}{\Lambda a}} \quad  \bar{\rho} = - \frac{a^4}{\gamma}
\]
Maximum reactivity is calculated as a function of the ramp rate from:

\[
\frac{\rho_{p}^{\text{max,min}}}{\beta} = \pm \frac{\sqrt{2\Lambda a'}}{\beta} \sqrt{\ln \left(\frac{\bar{p}}{p^0}\right)} - 1 + \frac{p^0}{\bar{p}}
\]

\[
\bar{p} = -\frac{\alpha'}{\gamma}
\]

\[
p^0 = p_0\beta \sqrt{\frac{2\pi}{\Lambda a}}
\]

Since \(\rho_{p,\text{max}}\) is set equal to \(\rho_{p1}\), the figure yields directly the ramp rate equivalent to the step reactivity \(\rho_{p1}\).
Discussion of the Flux Transient

In class work: …
Next Class: *Moderator / Coolant Feedback Effects*