Answers to Quiz 1

Math 230. Friday, 9/8/6

Problem 1 (75%) Evaluate: \( \langle 1, 0, -2 \rangle + |\overrightarrow{a}| \langle 2, 1, -1 \rangle = ?, \) where \( \overrightarrow{a} = \langle -1, -1, \sqrt{2} \rangle. \)

\[ |\overrightarrow{a}| \text{ is a real number, the length of the vector } \overrightarrow{a}. \text{ The length of a vector } \langle x, y, z \rangle \text{ is } \sqrt{x^2 + y^2 + z^2}. \text{ Therefore, } |\overrightarrow{a}| = \sqrt{(-1)^2 + (-1)^2 + (\sqrt{2})^2} = \sqrt{4} = 2. \text{ So } |\overrightarrow{a}| \langle 2, 1, -1 \rangle = 2 \langle 2, 1, -1 \rangle = \langle 4, 2, -2 \rangle, \text{ and therefore } \langle 1, 0, -2 \rangle + |\overrightarrow{a}| \langle 2, 1, -1 \rangle = \langle 1, 0, -2 \rangle + \langle 4, 2, -2 \rangle = \langle 5, 2, -4 \rangle. \]

Problem 2 (25%) Which coordinate planes and which coordinate axes are intersected by the segment with endpoints \((1, 2, 2)\) and \((1, -2, -2)\)?

There are three coordinate axes (the \(x\)-axis, the \(y\)-axis and the \(z\)-axis) and three coordinate planes (the \(xy\)-plane, the \(xz\)-plane and the \(yz\)-plane).

Answer: the \(x\)-axis and the \(xy\)- and \(xz\)-coordinate planes.

All of them are intersected by this segment in the point \((1, 0, 0)\), which is the midpoint of the segment, because \((1, 0, 0) = \left( \frac{1+1}{2}, \frac{2+(-2)}{2}, \frac{2+(-2)}{2} \right)\). The \(x\)-axis consists precisely of the points \((x, y, z)\) satisfying \(y = z = 0\), and thus contains the point \((1, 0, 0)\). The \(xy\)-coordinate plane is given by the equation \(z = 0\), and the \(xz\)-coordinate plane is given by the equation \(y = 0\), so both of them contain the point \((1, 0, 0)\).