Video Objectives
You will apply the average shear stress equation for a beam and use the concept of shear flow to determine the minimum:
1. adhesive shear strength between components in a built-up beam section
2. spacing between fasteners in a built-up beam section

Given: A Douglas Fir T-beam is to be made from two nominal 2"x6" boards. The beam must span 10' and will be loaded by 800 lbs. The load could either be concentrated at the center of the beam (F) or uniformly distributed along its length (w).

\[ F = 800 \text{ lbs} \]
\[ L = 10' \]

\[ w = \frac{F}{L} = 80 \text{ lb/ft} = 6.67 \text{ lb/in} \]

Find:
1. the minimum required shear strength of adhesive to bond the 2x6's together
2. the maximum spacing of 3/8" lag screws required to join the 2x6's together; the shear force at failure of a 3/8" diameter lag screw is 1990 lbs
3. The spacing of the lag screws such that the joint will never fail, regardless of the loading

\[ 2) \text{ spacing between fasteners in a built-up beam section} \]
Shear Stress in Beams: Adhesive Strength and Fastener Spacing

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**Find:**
- a) the minimum required shear strength of adhesive to bond the 2x6's together
- b) the maximum spacing of 3/8" lag screws required to join the 2x6's together; the shear force at failure of a 3/8" diameter lag screw is 1990 lbs
- c) The spacing of the lag screws such that the joint will never fail, regardless of the loading

- Use a factor of safety of 1.5 in all cases.
- The failure strengths of Douglas Fir are: 2000 psi in flexure and 90 psi in shear.

**Solution Strategy:**

1. Determine which loading is most severe and governs the design

   - F = 800 lbs
   - w = F/L = 80 lb/ft = 6.67 lb/in
   - L = 10'

2. Calculate cross-sectional properties

   - Centroid location, y
   - First moment, Q
   - Moment of inertia, I
Shear Stress in Beams: Adhesive Strength and Fastener Spacing

Solution Strategy:
1. Determine which loading is more severe and governs the design
2. Calculate cross-sectional properties
3. Apply the factor of safety to find adhesive strength

\[ FS = \frac{\text{Failure Quantity}}{\text{Actual Quantity}} \]

FS > 1.0 !

Solution Strategy:
1. Determine which loading is more severe and governs the design
2. Calculate cross-sectional properties
3. Apply the factor of safety to find adhesive strength
4. Define shear flow to find the lag screw spacing

\[ q = \frac{VQ}{T} \quad q \cdot x = V \]

Solution Strategy:
1. Determine which loading is more severe and governs the design
2. Calculate cross-sectional properties
3. Apply the factor of safety to find adhesive strength
4. Define shear flow to find the lag screw spacing
5. Re-design the joint such that it will never fail
1. Internal loads analysis by drawing V & M diagrams

- \( F = 800 \text{ lbs} \)
- \( L = 10' \)
- \( w = \frac{F}{L} = 80 \text{ lb/ft} = 6.67 \text{ lb/in} \)
- Maximum Values: \( V = 400 \text{ lb}, M = 24,000 \text{ lb-in} \)

2. Geometric properties

- Average shear stress, \( \tau = \frac{VQ}{b} \)

- Actual size of 2x6: 1.5" x 5.5"
  - \( A = 8.25 \text{ in}^2 \)

- Centroid location:
  - \( y' = \frac{\sum A_i y_i}{\sum A_i} = \frac{8.25(5.5/2) + 8.25(5 + 5.5/2)}{2(8.25)} = 2.5" \)
  - \( y = [5.5 + 5.5] - 2.5 = 4.5" \)

- \( Q \) @ joint between 1 & 2:
  - \( Q = \tau A' = (2.5 - 1.5/2)(8.25) = 14.44 \text{ in}^3 \)
2. Geometric properties

Average shear stress, \( \tau = \frac{VQ}{b} \)

Actual size of 2x6: 1.5" x 5.5"
A = 8.25 in\(^2\)

Centroid location:
\[ \gamma = \frac{\sum A \times y_i}{\sum A} = \frac{8.25(1.5\times2) + 8.25(5.5\times1.5/2)}{28.25} = 2.5" \]
\[ y_i = (0.5\times5.5) - 2.5 = 4.5" \]

Q @ joint between 1 & 2:

\[ Q = \tau A = (2.5 - 1.5/2)(8.25) = 14.44 \text{ in}^3 \]

Moment of Inertia:
\[ I = \sum (I_y + A d_y^2) \]
\[ = \frac{1}{12} (0.5\times5.5)^3 + 8.25(2.5 - 1.5/2)^2 + \frac{1}{12} (0.5\times5.5)^3 + 8.25(4.5 - 5.5/2)^2 \]
\[ = 1.55 + 25.26 + 20.80 + 25.26 = 72.87 \text{ in}^4 \]

3. Adhesive strength

Factor of Safety, \( FS \)

\[ \tau_{\text{actual}} < FS \tau_{\text{allowable}} \]
\[ \tau_{\text{actual}} = \frac{VQ}{b} = 1.5 \left( \frac{400 - 44.44}{72.87 + 5} \right) = 79.3 \text{ psi} \]

\[ \therefore a) \text{ Use an adhesive strength of 80 psi or more} \]

4. Lag screw spacing

Define shear flow, \( q = \frac{VQ}{L} \) internal shear force per unit length

Factor of Safety, \( FS \)

\[ \frac{V_{\text{allowable}}}{V_{\text{actual}}} \leq \frac{V_{\text{allowable}}}{V_{\text{actual}}} \leq \frac{V_{\text{allowable}}}{V_{\text{actual}}} \]
\[ = \frac{1.5 \left( \frac{400 - 14.44}{72.87} \right) = 16.74" \]

\[ \therefore b) \text{ Use a spacing of 16"} \]

Q1: Suppose smaller lag screws were used, would the spacing need to be larger or smaller?
4. Lag screw spacing

Define shear flow
\[ q = \frac{VQ}{T} \] internal shear force per unit length

Factor of Safety, \( FS > \)
Failure Load
\[ \frac{V_s}{q} = \frac{400}{14.44} = 27.83 \]
Actual Load
\[ \frac{V_s}{q} = \frac{72.87}{1.5(79.26)} = 880 \]

\[ s \leq \frac{V_{perm}}{FS q} = \frac{1990}{1.5(79.26)} = 16.74" \]

b) Use a spacing of 16"

If we use smaller screws (1/4" diameter) with 880 lb capacity:

\[ s = \frac{V_{perm}}{FS q} = \frac{880}{1.5(79.26)} = 7.40" \]

5. Design joint not to fail

For the screw joint to never fail, we must have one of the following:
1. No loading \( x \)
2. Screw having infinite shear strength \( y \)
3. The beam fails before the joint \( \checkmark \)

Determine load carrying capacity of the beam

\[ F = \frac{M c}{I} = \frac{444}{1280} \text{ ft-lb} \]

Flexural strength
\[ \sigma = \frac{F l}{I c} \geq \sigma F \]

Maximum Q occurs at neutral surface
\[ Q = \frac{t}{2}(3.0)^2 = 4.5 \text{ in}^2 \]

Q2: Why is it easier to compute Q from the area below the plane rather than the area above the plane?
5. Design joint not to fail

For the screw joint to never fail, we must have one of the following:
1. No loading
2. Screw having infinite shear strength
3. The beam fails before the joint

Determine load carrying capacity of the beam

Flexure: worst case is concentrated force at center of beam, $M = FL/4$

$$\sigma = \frac{Mc}{I} \implies F = 4I\sigma \frac{4}{Lc} \cdot \frac{72.87 \cdot 2000}{120 \cdot 4.5} = 1080 \text{ lbs}$$

Shear: $V = F/2$

Maximum $Q_3$ occurs at neutral surface

$$Q = \frac{V}{(I + J)} = (4.5 / 2)(4.5 \cdot 1.5) = 15.19 \text{ in}^3$$

$$\tau = \frac{VQ}{F} \implies F = 2\tau \frac{Q}{\frac{15.19}{2}} = 1295 \text{ lbs}$$

Q3: why 540 lbs?

Repeating the lag screw spacing analysis of part 4...

Repeating the lag screw spacing analysis of part 4...

c) Using a 12" screw spacing forces the beam to fail in flexure

\[ \text{The End} \]