Stability Analysis of Frames

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Rigid and semi-rigid frames are often used as important elements of building structures. Therefore, it is necessary to develop an accurate, yet manageable method for effective design of these frames. Numerous studies related to the stability of rigid or semi-rigid frame structures have been performed and described in the form of journal articles. These articles give some insight into how the design theories have changed over time as each one attempts to improve upon the previous standard. The studies deal mainly with the plastic collapse of these structures and the different methods used to determine accurate buckling loads and collapse loads. While some derive simple and efficient design approximations that can be calculated by hand, others explore the shortcomings of such approximations and demonstrate the usefulness of computer programs for a more detailed analysis. This literature review will examine five studies published in the form of journal articles. These studies point out discrepancies in various approximation techniques for the analysis of collapse loads, and some of them propose new approximations that should help prevent large inaccuracies.

When determining the most practical method for analyzing a frame, the engineer must consider the degree of accuracy desired as well as the geometry of the frame. A study completed by D. Anderson and T.S. Lok discusses the shortcomings of the popular Merchant-Rankine approach (eq. 1)
\[ 1/\lambda_{\text{MR}} = 1/\lambda_p + 1/\lambda_c \quad (\text{eq. 1}) \]

- \( \lambda_{\text{MR}} \): Collapse load predicted by Merchant-Rankine formula
- \( \lambda_p \): Rigid-plastic collapse load
- \( \lambda_c \): Lowest elastic critical load

They suggest that the approach is based on contradictory assumptions. At the same time, they admit that the results are simple and accurate enough for most situations. Studies have shown that this is generally a conservative estimation for realistic loading in small and large frames. Their search for a new approximation stems from the idea that one may wish to perform a closer approximation by hand in certain cases [1]. This study refers to an article by R. H. Wood entitled “Effective lengths of columns in multi-storey buildings,” in which he proposed a relationship that would allow strain-hardening and stray composite action to increase allowable loads. This relationship is communicated through (eq. 2).

\[ 1/\lambda_{\text{MWR}} = 0.9/\lambda_p + 1/\lambda_c, \text{ with } \lambda_{\text{MWR}} \text{ no greater than } \lambda_p \quad (\text{eq. 2}) \]

- \( \lambda_{\text{MWR}} \): Collapse load predicted by modified Merchant-Rankine formula [6]

It is only applicable if rigid plastic collapse is by a combined mechanism, which may be difficult to guarantee at the stage where this would be used in design practice. The load given from Wood’s relationship remains within three to four percent of the computer result if the correct criteria are met for this relationship. However, when the plastic collapse mode is not restricted, the approximation may exceed computer results by seven
percent. Anderson and Lok’s study proposes a closer approximation (eq. 3) that is less likely to exceed the results achieved by a computer analysis.

\[
\lambda/\lambda_p = [1-(0.4\lambda_p/\lambda_c)] [1-(\lambda_{det}/\lambda_c)^2] \quad \text{(eq. 3)}
\]

- \( \lambda \) Load factor; collapse load predicted by proposed method
- \( \lambda_{det} \) Deteriorated critical load

Unlike previous approximations, this one does not rely on cladding and strain hardening to account for a higher load than that given by the computer. Thus, the results this method gives are safer than those achieved by Wood’s hand approximation [1].

A computer approach to analyzing elasto-plastic plane frames is introduced in another article, written by J. Creus, P.L. Torres and A.G. Groehs. Their approach uses a matrix method that would be extremely tedious to carry out by hand. However, it can become a relatively simple analysis when a computer program is used. It also takes into account the effect of large displacements, which may be important in elasto-plastic frames. Their method can be used to determine buckling loads with elastic analyses as well as limit and instability loads with elasto-plastic analyses by using the nonlinear tangent stiffness matrix for the elastic case and extending it to the elasto-plastic case. This is achieved by defining a generalized yield criterion that takes moment, shear and normal forces into consideration. The method is still just an approximation since it is an extension of the method of plastic hinges. In certain cases it simplifies to the plastic hinge method. However, the advantages are that it allows consistent modeling of material and section properties and a stability check is included in the elasto-plastic analysis when it is used.
for large displacements. Creus, Torres, and Groehs tested the accuracy of this approach against several previous studies and it produced similar results. The two-story frame in Figure 1 was one of the cases chosen to test the accuracy of the proposed approach. Figure 1 shows a comparison of several different types of analysis that can be performed with this method for a two-story frame [3].

The approximations described in the studies discussed to this point assume the formation concentrated plastic hinges. However, a study by U. Andreaus and P. D’Asdia at the University of Rome suggests that this can be an unsafe assumption. In some cases, the plastic collapse load of a frame is reduced due to full plasticity not being reached and the spreading of plastic hinges. Plastic hinges are useful for simple structures, but complex frames are unlikely to collapse in a regular mechanism. Multiple plastic hinges generally will form before the number is sufficient for a mechanism. This causes elastic

Figure 1

<table>
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<tr>
<th>Line</th>
<th>Description</th>
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<tbody>
<tr>
<td>A</td>
<td>Linear elastic analysis</td>
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<tr>
<td>B</td>
<td>Geometrically nonlinear elastic analysis</td>
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<tr>
<td>C</td>
<td>Elasto-plastic analysis without consideration of changes in geometry</td>
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<tr>
<td>D</td>
<td>Elasto-plastic analysis with consideration of changes in geometry</td>
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plastic instability and can induce the spread of plastic hinges. Their study suggests a cyclic analysis method that accounts for the effect of spreading yield zones. It uses a set of independent elastic-plastic kinematical compatibility equations and a six-step procedure. In this procedure, steps three through six are repeated until the system of equations becomes singular and the collapse mechanism is achieved. This allows for the same four types of analysis that were compared in Creus, Torres, and Groehs’ study, but it does not require the assumption of concentrated plastic hinges. Therefore, this cyclic method makes it possible to evaluate the descending branch of the equilibrium path until a plastic collapse mechanism is formed. While it provides more accurate and safer results for the analysis of complex frames, this procedure requires a larger computational expense than the previous methods [2].

All of the methods described thus far provide suggestions for determining plastic collapse loads. However, plastic collapse is not the only criteria in determining what load to use for design. A study by Gunther Lacher from Budapest, Hungary points out that many approximations ignore the influence of rotation capacity at the joints of a frame. Buckling loads are governed by initial stiffness, but collapse loads are a factor of plastic level as well as rotation capacity, which is often neglected. The study points out that even the most flexible connections can cause a considerable increase in the collapse load when compared to a hinge. The Merchant-Rankine formula (eq. 1) provides acceptable results for ideal pinned and jointed frames, but according to the calculations from this study, the results for semi-rigid frames are often unsafe. This is related to the fact that $K_{ki}$, the buckling factor, takes initial stiffness into account but ignores rotational capacity. The worst cases for overestimation of collapse load are those with connections of
medium and low stiffness. The limiting cases, fully-rigid and ideal hinged connections, are the only cases where the Merchant-Rankine formula produces accurate results. The difference between these limit states and the true stiffness of connections becomes increasingly important in buildings with a large number of stories or bays. Both $K_{kr}$, the collapse factor, and $K_{ki}$, depend on the number of stories and bays in the structure. Figure 2a and 2b show the collapse load factor and the buckling load factor, respectively, as a function of story and bay number. The loss of capacity occurs much more rapidly when flexible connections are used than rigid connections. As the figures show, the collapse load decreases with increasing stories and decreasing bays. The buckling load decreases with the number of stories and bays [4].

Figure 2a
Collapse load factor, $K_{ki}$, as a function of story and bay number

Figure 2b
Buckling load factor, $K_{kr}$, as a function of story and bay number
Additional assumptions used in simple approximations are questioned in yet another study. Gabor Domokos and Peter Nedli of Budapest analyzed second order bending assumptions in asymmetrical frames. Their study analyzes the frame in Figure 3 where the geometry can vary and is described by the parameters $a$ and $b$ [5].

For this frame, regardless of the parameters, second order analysis assumes that the structure will bend to the left unless it is symmetrical ($a=1$ or $b=0$). However, through a third order analysis, Domokos and Nedli found a third line in the parameter plane. At each point of this line the structure has a bifurcation point instead of a limit point. The points are asymmetrical and correspond to a special class of fold catastrophe points.

Figure 4 shows the shape of equilibrium paths near the critical point for several shapes.
It is simple to imagine how the frame under consideration would bend to the left, especially if the parameter $a = 0$ is used. However, this study found that under certain geometries the frame will bend to the right. The authors of the article developed an intuitive explanation for why some of these frames may bend to the left instead of right. This is illustrated in Figure 5.

Initially points C and E deflect equally, but since E is higher, the frame begins to bend to the left. Once joint D has rotated counterclockwise, C is above E, causing the frame to bend to the right. The direction in which the frame bends can have a significant effect on the post-critical behavior of the frame. The Domokos and Gabor were not able to determine the required level of accuracy needed to predict the direction that a frame will
bend. Therefore, they question how far it is safe for engineers to trust these approximations [5].

By examining the purpose and findings of each of these articles, a relationship may be made between design approaches. This relationship can be used to determine the strengths and weaknesses of various methods of rigid or semi-rigid frame analysis. For many cases, the simple hand approximations, such as those achieved with the Merchant-Rankine formula are sufficient. Anderson and Lok’s study proposed another hand approximation is almost as simple and may be slightly more accurate. Computer programs are capable of producing more accurate results but at a greater computational cost. However, the problem with all approximations is that they require assumptions that may not always be true. When these assumptions prove to be incorrect, the approximation becomes inaccurate. Some risky assumptions include the formation of discrete plastic hinges to form mechanisms, ignoring the rotational capacity of joints in determining collapse loads, and assuming a frame is going to collapse in a particular direction. It is important to know the limitations of any approximation before using it to design a building. All of approximations presented may be useful in certain cases, but engineers should understand the basis for such approximations before using them in order to avoid dangerous mistakes.
Reference List:


Articles:


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