# Kenneth G. Langone Athletic and Recreation Center

Bucknell University Lewiburg, Pa



## **Kyle Oberdorf – Structural Option**

Senior Thesis – April 2007 The Department of Architectural Engineering The Pennsylvania State University

### **EXECUTIVE SUMMARY:**

Sojka Pavilion and the Kinney Natatorium are 122,000+ Sqft addition to the Kenneth Langone Recreation and Athletic center built to house a 4000 seat basketball arena and NCAA regulation size swimming pool on the campus of Bucknell University in Lewisburg Pennsylvania.

The existing system is comprised of large steel truss roof beams constructed of W shape and angles. The roof is supported by W shape columns. The floor system of the gymnasium is a composite steel system. And both buildings lateral system are X- braced frames.

This thesis project is an in depth study of an alternative structural system. The goal of the alternative system is to design a wood system capable of carrying the applied loading as well as providing an economical alternative. This study investigates a circular arched glulam roof system, glulam columns, glulam floor system and glulam lateral force resisting system. The results show that the use of circular arched glulam beams and glulam columns provide a viable architectural and structural alternative to the existing system. Due to the an increase in member size it was determine that the wood lateral force resisting system and floor system are not an adequate alternative to their existing counterparts.

In addition to the depth study, two breadth studies were also performed. A study into LEED certification was performed, and it can be concluded that a level of certification could have been achieved if properly incorporated into the original design as well as an added benefit could be achieved using the proposed wood system. A redesign of the HVAC delivery system in the natatorium was also performed using fabric duct in order to reduce the transfer of mechanical noise transfer.

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Introduction: Building Introduction Building Systems



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## **INTRODUCTION:**



Sojka Pavilion and the Kinney Natatorium are 122,000+ Sqft addition to the Robert Langone Recreation and Athletic center built to house a 4000 seat basketball arena and NCAA regulation size swimming pool on the campus of Bucknell University in Lewisburg Pennsylvania. Because of their functions, both buildings require clear spans of over 100'. The exterior of the two buildings combine the architectural features of the existing buildings and that of the rest of the buildings on campus.

## **Building Envelope:**

The majority of exterior walls are brick façade with faux white stone accents. In addition to the brick, large portions of the front of the natatorium and Gymnasium lobby use glass to provide a view of the interior. Insulated metal panels are also used on the façade in areas that are not highly visible to the public such as the eastern end of the gymnasium and then north ends of both the gymnasium and natatorium.

The roof system on the natatorium is a flat system comprised of 18 gauge steel decking covered by 5 inches of rigid insulation and finished off with EDPM. The roof system on the gymnasium has a similar flat top but has pitched roofing around the perimeter. This perimeter system is comprised of an 18 gauge steel decking base, 5 inches of rigid insulation and finished off by coated 20 gauge architectural steel roofing.

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## **CONSTRUCTION:**



The project began in May 2001 and took a little over a year for completion. Portions of the project were opened for student use in the fall semester of 2002 and the first basketball game in the Sojka Pavilion was held on January 13, 2003. The cost of the entire Kenneth Langone Athletic and Recreation center was approximately \$27 million. The construction, being done in the middle of campus, was to occur while interrupting as little as possible, including activities

in the existing field house and gymnasium, campus activities and the surrounding vegetation that was to remain. Deliveries to the site were scheduled so as to minimize the amount of space used for storage as well as the amount of time required for storage of materials and equipment.

## **ELECTRICAL:**

Kenneth Langone Athletic and Recreation Center is powered by both 480/277V and 208/120V. The larger power supply is needed to run certain lighting applications as well as whirlpools, dryers and stoves in the lounges while the smaller source is needed for outlets and the majority of lighting. The elevator is powered by 480 volt, 60 Hz, 3 phase, wye delta starting. Backup power is supplied by a diesel engine generator that will run at 350 kW for 17 hours.

## **LIGHTING:**

As with any multiuse facility, various types of lights and lighting fixtures were used. Means of egress and office spaces are lit using a variety of fluorescent ceiling and wall fixtures. The gymnasium and natatorium are lit using both high-pressure sodium and metal-halide lamps in HID fixtures. Emergency lighting is installed with batteries and chargers.

## **MECHANICAL:**

Both the Kinney Natatorium and the Sojka Pavilion are serviced by Constant Air Volume Units. The Natatorium receives 75600 cubic feet per minute from 4 units located on an elevated platform off the north wall. Sojka Pavilion is supplied 90800 cubic feet per minute by 3 units also located on elevated platforms located off the north wall.

## **FIRE PROTECTION:**

Both buildings are equipped with automatic sprinkler systems. The sprinkler systems are both wet-pipe and dry-pipe type systems. Spray on fireproofing is used on the beams and columns. The fire protection system includes manual pull stations, audio devices, smoke detectors, heat detectors and duct detectors.

## Existing Structural System



## **EXISTING STRUCTURAL SYSTEM:**

The ground floor of both the Sojka Pavilion and Kinney Natatorium is a 5" deep concrete slab on grade, reinforced by 6x6 W2.9x W2.9 welded wire fabric. All of the floors above the ground floor are a composite steel system. The system is comprised of beams spanning in the long direction and girders spanning in the short direction. The composite deck used is a 2" – 20 gage composite deck with 4  $\frac{1}{2}$ " normal weight concrete, having a total slab depth of 6  $\frac{1}{2}$ ". The beams are cambered  $\frac{3}{4}$ " at center to counteract deflection. The system uses  $\frac{3}{4}$ " diameter by 5" long steel shear studs welded on the center line of the top flange of the beams.

Bearing walls are composed of W shape steel columns of various sizes. The typical bay sizes are 31' 3" wide by 32' 6" tall in the natatorium and 38' wide by 18' tall in the Gymnasium. These bays support curtain walls of glass, brick, CMU and or framed walls with drywall. The columns support a EDPM roof system on the natatorium and a combination of EDPM and architectural metal roofing on the gymnasium.

The foundation is comprised of strip footings. The strip footings range from 1'6" to 8' wide and 1'6" to 4' deep. They are reinforced with continuous #8 or 9 bars with additional reinforcement as required. These strip footings carry concentrated loads from columns and distributed loads from walls. The footings were designed with a soil bearing capacity of 2500 PSF

The existing lateral force resisting systems consists of X braced frames. The frames are located on both the long and short sides. In the natatorium on the long side of the building there are two identical side by side frames. These frames are each 31' 3" wide by 32' 6" tall and are braced by 4" diameter extra strong steel pipe made of ASTM 501A steel. The braces are divided into four sections and are connected at the center and to the frame by  $\frac{1}{2}$ " thick steel gussets. The short sides of the building have only one braced frame. These frames are 36' wide by 32' 6" tall and are braced by 5" diameter extra strong steel pipe. This bracing is divided into four just as the long side was and is connected in the same manor. In the gymnasium, the long side is braced by a truss at the top and one located half way up the column. The middle truss is 5' deep and is composed of W10x26 horizontal members and diagonal members are 2 4x4x3/8 angles back to back. The short ends use cross braced frames with 5" diameter extra strong steel pipe made of ASTM 501A steel. The braces are divided into four sections and are connected at the center and to the frame by  $\frac{1}{2}$ " thick steel gussets.

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Figure 1







Figure 3 Section of Composite Bay

## Problem Statement & Problem Solution



## **PROBLEM STATEMENT:**

A building must be designed to resist all applied forces in accordance with the International Building Code. This includes gravity loads and lateral loads. The gravity loads are determined from the dead loads of the building and the live loads established in Table 1607.1 of the IBC. The lateral forces take into take into account the effects of wind and seismic. These forces are also calculated in accordance with IBC with references to ASCE 7. Because of load combinations set forth in the IBC, the building does not have to resist both wind and seismic concurrently.

After a review of Technical Assignment #2 it was observed that several alternate floor systems were worth further investigation. The most viable alternative floor system is a wood beam system. This was determined because it has the most advantages. It was concluded in Technical Assignment #3 that wind forces control the design of the lateral force resisting system as was the case in the original design. It was however determined that some of the members in the X braced frames were inadequate and therefore alternative frame and or members will need to be designed.

One of the most important things to consider when designing a building is to make sure it is as economical as possible. In addition to this another important consideration is how that building is going to fit into its environment. As such the history of an area needs to be considered. Central Pennsylvania has a very long history of wood production that continues to this day. Because of this many of the buildings use wood construction and have exposed wood structural members. As such, it is important to consider alternative solutions and alternative architectural approaches.

## **PROBLEM SOLUTION:**

The solution to this is to design an alternate system and compare it to the original design. The alternative system being considered in this proposal is a glulam wood system. This system will be comprised of arched LVL roof beams supported by LVL columns. The floor system will be comprised of LVL beams and girders and a wood panel decking. This new system will then be compared to the original design to determine whether it is a considerable alternative.

## Depth Study: Alternative Structural System Design



## **CIRCULAR ARCHED GLULAM ROOF BEAM:**

The roof system of the building must be designed to resist the gravity loads applied from both the dead and snow load. The dead load consists of the total weight of the materials as well as the self weight of the beam itself. A circular arch was selected because it was the only feasible way of spanning the desired length using a glulam product. The arch helps to reduce the maximum bending moment as well as transfers a portion of the vertical reactions into horizontal reactions, there by reducing the load that is transferred down through the rest of the building. The horizontal load is accounted for through a tensioning cable used to keep the arch from spreading.

Using the NDS and AITC design specifications, an arch was designed for both the roof of the natatorium and gymnasium. Each arch had a different span, radius, and tributary area.

The natatorium beams were designed to span 123'8" at a spacing of 10'. The radius of the arch was determined to be 135' in order to both limit the affects on the architecture and to maximize the beneficial effects of the horizontal reaction on reducing the total vertical reaction. The beam was designed using dead load, wind load, balance and unbalanced snow loads. In addition to being designed to withstand gravity load, the beams were designed with additional capacity to meet the required 2 hour fire rating for roof systems. It was determined that a 12" x 54" Southern Pine 30F-E2 was required to meet both the two hour fire rating and carry the applied gravity loads. A 7/8" diameter 6 x 7FC structural steel cable, supported by additional cable hung from the beam itself was required to carry the developed horizontal forces.



Original Natatorium Truss Outline

Figure 4



Figure 5

Though the arch adds an additional 15' to the building height over the original design, it also provides an 7'  $5 \frac{1}{2}$ " of interior height. In addition to the added interior height, the use of the tensioning cable has reduced the vertical reaction by almost 40 percent.

The gymnasium beams were designed to span 158' 10" at a spacing of 11'. The radius of the arch was determined to be 138' 8" to again limit the affects on the architecture and to maximize the beneficial effects of the horizontal reaction on reducing the total vertical reaction. Using the same standards and loads as the natatorium beam, it was determined that a 12" x 68" Southern Pine 30F-E2 was required to meet both the two hour fire rating and carry the applied gravity loads. As with the natatorium a 7/8" diameter 6 x 7FC structural steel cable, supported by additional cable hung from the beam itself was required to carry the developed horizontal forces.







Figure 6



Figure 7

Though the arch adds an additional 25' to the building height over the original design, it also provides an 2'  $5\frac{1}{2}$ " of interior height. In addition to the added interior height, the use of the tensioning cable has reduced the vertical reaction by almost 40 percent.

## **GLULAM COLUMNS:**

The columns in both the gymnasium and natatorium must be designed to support the gravity loads that are transferred from the roof beams. The columns are designed to NDS standards using southern pine grade N1D14. The natatorium columns are 40 feet tall and unbraced in both the strong and weak axis's. The gymnasium columns are 60 feet tall, unbraced in the strong axis and supported at two different places in the weak axis with the greatest unbraced length being 30 feet. The reduction of vertical forces attributed to using arched roof beams allows the column sizes in both situation to remain small.





Using arch geometry both columns were designed to the same size. The reduction in vertical forces allow the columns to remain small as seen in the figure above. Both columns are 12" x 18". The major controlling factor of the design was to keep the column the same width as the beam to keep the connection cheap and easy to assemble. In addition the column was designed with enough capacity to meet the required two hour fire rating. The previous two reason in addition to a reduction of cost due to quantity and easy of production made making all of the columns in the building the same size the most economical solution. A major benefit of the wood column is that unlike the steel no additional fire proofing is required. The steel columns require either a fire proofing or are encased in bricks and or CMU, both of which require an addition cost and increase the columns size.

### **GLULAM FLOOR SYSTEM:**

The floor system of the building must be designed to resist the gravity loads applied from both the dead load and live load. The dead load consists of the total weight of the materials and the live load is 100 psf in accordance with the IBC for gathering areas. The wood system was designed to follow the same spacing as the steel system in order to minimize the effects on mechanical and electrical systems.



#### Figure 9

Due to the lack of a feasible and available wood decking system that is capable of providing the desired span and fire protection, the same concrete decking system that was used with the steel system. The combination of this heavy decking solution and the long spans, which could not be changed with out jeopardizing the architects vision of the building, created an average floor system depth that was greater then that of the steel system. The wood system is also incapable of the composite action that was used in the steel system. As such, the wood system averages almost 12" deeper then the steel system as is seen in Figure 9 above. The average wood beam required for this system is a 14" x 29" southern pine 30F-E2 and is supported on average by an 18" x 36" girder of the same material as can be seen in the plans in the appendix. The smallest allowable member based on fire rating is a 12" x 16".

## **GLULAM LATERAL SYSTEM:**

The lateral force resisting system for this building must be designed to handle the controlling lateral force. The controlling lateral force for this building is wind loading. The cross braced frames were redesigned using southern pine grade N1D14 material and were designed to a two hour fire rating. The frames in both buildings required a 9.5" x 12" to carry the tension side of the frame in addition to lower 14" x 22" member frame, designed to work just in tension to eliminate the possibility of buckling, in the gymnasium.

Due to the increased sizing for fire protection, both buildings have a greatly reduced drift from the original system. Under normal loading, the natatorium sees a total drift of 0.5" which is less then half of the allowable drift of 1.2", calculated using L/400. The gymnasium has total drift of 1.1" which is 0.7" less then the allowable drift of 1.8". With both drifts so far below the allowable, the economics of using wood over steel in this application come into question.

## **COST/CONSTRUCTION TIME ANALYSIS:**

While time did not allow for an in depth analysis of the cost and construction time a quick analysis was performed in order to make a comparison to the existing system. As seen in the table below, the approximate cost of an individual roof beam and column in the natatorium are compared in both steel and wood.

	Steel	Wood
Beam	\$14500.00	\$15900.00
Column	\$3700.00	\$2100.00
Table 1		

As can clearly be seen above, both systems have a positive and a negative but when the entire system is put together, there is very little difference therefore financially justifying both systems.

When construction times are compared, both systems utilize bolted connections and similar erection procedures. As such there is very little difference in the time of construction of the two systems. However, when production times are considered, steel as a definitive advantage as the wood components can take up to twice the time to produce. In addition to production time considerations when using wood, storage and delivery issues must also be considered as wood is affected far more by environmental factors then steel.

## Breadth Studies: LEED Certification HVAC Delivery System



## **BREADTH STUDIES:**

Two breadth topics were studied in addition to the structural depth work. The first was topic is an investigation into what could have been incorporated into the design and construction process in order to receive a LEED certification. The second will be a redesign of the existing HVAC delivery system in the natatorium using cloth duct to reduce the possible mechanical noise transfer.

## **LEED CERTIFICATION STUDY:**

The Kenneth G. Langone Athletic and Recreation Center was not originally designed as a green building. This study was performed to see what changes could have been implemented to achieve certification on the LEED checklist. This was done using the US Green Building Council's Green Building Rating System for New Construction and Major Renovations, Version 2.2. In order for a building to achieve certification, twenty six points must be achieved on this checklist. Time did not allow for a complete study of all sixty nine possible points, so the sustainable sites category was chosen for a more in depth study. In addition, the possible points associated with changing the structural system to wood will also be studied. There are fourteen possible points that can be achieved in the sustainable sites category, as well as one required point.

#### **Prerequisite 1: Construction Activity Pollution Prevention**

In order to meet the prerequisite, construction activity pollution must be reduced. This is accomplished by creating an erosion and sedimentation control plan for the project. The goal of this credit is to prevent loss of topsoil during construction, to prevent sedimentation of storm sewer or receiving streams, and to prevent polluting the air with dust. To prevent the loss of topsoil, it will be stockpiled in the location shown on the plan below. In addition, the stockpile will be temporarily seeded to prevent erosion until it is needed. The silt fence will help prevent run off from reaching the sewer or the stream that runs in front of the construction site. More care will be taken during the construction process to help reduce polluting the air with dust and other particles.



#### Figure 10

#### Credit 1: Site Selection

The building must be constructed in a manner that results in the least environmental impact. The building cannot be constructed on a site that meets any of the following conditions:

- Prime farmland as defined by the United States Department of Agriculture in the United States Code of Federal Regulations, Title 7, Volume 6, Parts 400 to 699, Section 657.5 (citation 7CFR657.5)
- Previously undeveloped land whose elevation is lower than 5 feet above the elevation of the 100-year flood as defined by FEMA
- Land that is specifically identified as habitat for any species on Federal or State threatened or endangered lists
- Within 100 feet of any wetlands as defined by United States Code of Federal Regulations 40 CFR, Parts 230-233 and Part 22, and isolated wetlands or areas of special concern identified by state or local rule, OR within setback distances from wetlands prescribed in state or local regulations, as defined by local or state rule or law, whichever is more stringent
- Previously undeveloped land that is within 50 feet of a water body, defined as seas, lakes, rivers, streams and tributaries which support or could support fish, recreation or industrial use, consistent with the terminology of the Clean Water Act
- Land which prior to acquisition for the project was public parkland, unless land of equal or greater value as parkland is accepted in trade by the public landowner (Park Authority projects are exempt)

The site does not meet any of these criteria and therefore, this point can be attained.

#### Credit 2: Development Density and Community Connectivity

This credit is achieved by constructing on a previously developed site and within <sup>1</sup>/<sub>2</sub> mile of a residential zone with an average density of ten units per acre net as well as within <sup>1</sup>/<sub>2</sub> mile of at least ten basic services. A few examples of basic services include banks, places of worship, fire stations, beauty salons, libraries, restaurants, and schools. Being located in Lewisburg, this point is easily met.

#### Credit 3: Brownfield Redevelopment

The purpose of this point is to rehabilitate a contaminated site. The Kenneth G. Langone Athletic and Recreation Center was not constructed on a contaminated site; therefore, this point cannot be earned.

#### Credit 4.1: Alternative Transportation: Public Transportation Access

This credit is intended to reduce pollution from automobiles. It can be achieved by locating the project within <sup>1</sup>/<sub>4</sub> mile of one or more stops for at least two campus bus lines. Bucknell is a small walking campus with no need for transportation so this point is not applicable.

#### Credit 4.2: Alternative Transportation: Bicycle Storage

For institutional buildings, this credit is earned by providing covered storage facilities for bicycles for 5% of the buildings occupants as well as providing shower and changing facilities. As a natatorium and gymnasium, shower facilities are already part of the design and ample bike storage is easily obtained. As such, this credit can be easily attained.

#### Credit 4.3: Alt. Transportation: Low Emitting & Fuel Efficient Vehicles

There are three options that meet this credit. The first option is to provide low emitting, fuel efficient vehicles for 3% of the building occupants as well as providing preferred parking for these vehicles. The second option is to provide preferred parking for low emitting, fuel efficient vehicles for 5% of the building occupants. The third option is to install alternative fuel refueling stations for 3% of the total vehicle parking capacity of the building. As a sports facility, transportation vehicles are already provided for the teams to use. As such it would be simple to provide the proper amount of vehicles and a place for them to park in order to achieve this credit.

#### Credit 4.4: Alternative Transportation: Parking Capacity

For this section, four options exist. The easiest of these options to achieve credit is to provide 5% of the total available parking as carpool and vanpool parking.

#### Credit 5.1: Site Development: Protect or Restore Habitat

The purpose of this credit is to conserve existing natural areas and also restore damaged areas. This is done by restoring or protecting 50% of the site area with native vegetation. This area does not include the building footprint. After construction was completed, the area that originally was a soccer field was reseed and plant around the perimeter with trees to provide a recreation area. Thus this point is already attained.

#### Credit 5.2: Site Development: Maximize Open Space

This credit is similar to 5.1. The amount of vegetated open space must be equal to the building footprint. The building footprint is approximately 122,000 square feet. The area of the site is approximately 300,000 square feet. The majority of the area adjacent to the building is vegetated. Therefore, this point can be achieved.

#### Credit 6.1; Stormwater Design: Quantity Control

The requirement for this credit is to implement a storm water management plan that prevents the discharge rate of storm water after construction from being higher than the discharge rate before construction for the one and two year twenty four hour storm. The easiest way to keep the storm water from running off the site is to promote infiltration. This can be done by allowing the storm water to discharge onto vegetated areas instead of impervious areas. Once discharged onto vegetated areas, the water can slowly perk into the ground. This prevents the storm water from leaving the site. Therefore, this credit can be earned.

#### Credit 6.2: Stormwater Design: Quality Control

The intention of this point is to limit the disruption and pollution of natural water flows by managing the storm water runoff. This is accomplished by treating the storm water runoff. The purpose of treating the water is to remove the suspended solids. Certain types of vegetation are able to treat the runoff. However, this requires in field monitoring to determine if the treatment level is sufficient to meet the requirements of this credit. Because of this, earning this credit requires careful monitoring, making it impractical.

#### Credit 7.1: Heat Island Effect: Non-roof

In order to meet the requirements of credit 7.1, 50% of the site hardscape must be a combination of shaded, paving materials with a solar reflectance index of 29, or an open grid pavement system. The use of Portland cement concrete meets the required solar reflectance value. Thus, to meet this requirement, all sidewalks should be constructed of this type of concrete. Additionally, the trees planted on the site will provide shade within five years. Therefore, with the use of Portland cement concrete, this credit can be earned.

#### Credit 7.2: Heat Island Effect; Roof

This intent of this credit is to reduce heat islands on the roof surface. This can be done by using roofing materials having a Solar Reflectance Index value of 29 or higher, installing a green roof or a combination of both of a minimum amount of the roof area. The Solar Reflectance Index is a measure of the surface's ability to reflect solar heat. Because of the curvature of the roof, installing a green roof might be quite difficult. The current metal roofing needs a coating to be applied in order to meet the required Solar Reflectance Index. These coatings come in a variety of colors which allow for a level of architectural freedom. With this coating, this credit can be earned.

#### Credit 8: Light Pollution Reduction

The requirements for light pollution reduction include interior and exterior lighting. For interior lighting, the angle of maximum candela must intersect opaque building surfaces instead of exiting out through windows. For exterior light, site and building mounted luminaries cannot produce an initial illuminance value higher than 0.20 horizontal and vertical footcandles at the site boundary and 0.01 horizontal footcandles fifteen feet beyond the site boundary. A lot of the luminaries in the building are indirect type light fixtures. This meets the requirements for interior lighting. However, in areas with different types of lights, as well as exterior lights, this credit can be earned by simply selecting appropriate fixtures and laying them out so that light does not escape through the windows.

#### **LEED Benefits of Wood System:**

## MR Credit 5.1: Regional Materials: 10% Extracted, Processed & Manufactured Regionally

The requirements for this section are that the building materials used be extracted, harvested or recovered as well as manufactured within 500 miles of the project site for a minimum of 10% (based on cost) of the total materials value. With a large number of glulam manufacturers and forest products producers this should be easy to achieve with the redesign encompassing so much of the structural system.

#### MR Credit 5.2: Regional Materials: 20% Extracted, Processed & Manufactured Regionally

This requirement is the same as above however, 20% is required for an additional point. As with above, the redesign encompasses enough of the building to earn this point.

#### MR Credit 6: Rapidly Renewable Materials

The requirement for this section states that 2.5% of the total value of all building materials and products used in the project be from rapidly renewable materials. Wood meets this requirement and the scope of the redesign encompasses well over 2.5% and as such this point would be earned.

#### MR Credit 7: Certified Wood

The requirements for this section state that a minimum of 50% of the building materials be wood-based materials and products. As with the sections above, the redesign encompasses enough of the material to earn this point.

#### **LEED Summary;**

In summary, eleven out of fourteen of the credits in the sustainable sites category can feasibly be incorporated into the design of the Kenneth G. Langone Athletic and

Recreation Center. In addition, the use of the wood redesign attains another four credits from the Materials category. This is a very good indication that at least twenty six credits can be earned, and the building can be LEED certified. An important aspect to consider is the cost of this process. A cost analysis was not performed on this study, but it is evident that additional costs may be incurred from the use of different materials. Additional costs may also come from items such as bike lockers. However, these initial additional costs would be offset by the increased efficiency of the green systems. Overall, if properly incorporated into the design, this process could be done and would allow the building to make less of an environmental impact.

## HVAC DELIVERY SYSTEM:

For my second breadth, I choose to redesign the natatorium HVAC delivery system using DuctSox to reduce the transfer of mechanical noise from the air handling units to natatorium. The natatorium has very few surfaces that are not sound reflective, so even the slightest mechanical sound can cause a disturbance. This problem is magnified by the size of the natatorium, which requires 3 air handling units to supply the needed CFM to maintain the desired comfort levels.

The pool area itself is broken it three areas, each supplied by an individual air handling unit. The three zone are the north end of the pool, south end of the pool and the pool seating area. The two pool zones are identical, therefore requiring only one design while the seating area requires a design of its own.

In addition to reducing the transfer of mechanical noise, DuctSox also works to reduce additional ambient sound. The soft flexible fabric acts as a baffle along the ceiling, reducing sound by breaking up small amounts of reflective noise.

The initial conditions for each of the 3 systems are the same. It was determined that the cylindrical series would best each system. The cordona fabric was chosen for its antimicrobial treatment for use in a humid environment. Finally, the suspended track system was chosen to accommodate the arched roof design.

Both the north and south zones of the pool are designed to provide a constant air volume of 27300 CFM. An even dispersion side air handling unit configuration was chosen to best distribute the air evenly. An inlet velocity of 1200fpm was chosen to provide the least air noise giving a duct size of diameter of 66 inches. With the ducts being located over 30 feet above the desired comfort area, a high throw hole pattern was chosen. 2" holes were chosen to best distribute the air over the given area.

The Pool seating area is designed to provide a constant air volume of 12000 CFM. A straight line configuration was chosen to best distribute the air evenly over a long narrow area. An inlet velocity of 1200fpm was chosen to provide the least air noise giving a duct size of diameter of 44 inches. With the ducts being located less then 20 feet above the desired comfort area, a comfort throw hole pattern was chosen. 2" holes were chosen to best distribute the air over the given area.

This system has been used successfully in natatoriums across the country for both its noise reduction and corrosion resistance. And while the antimicrobial treatment is too new for significant data to show its benefits, initial installations have shown the ability to reduce the risk of airborne contaminates.

## Conclusions and Recommendations



### **CONCLUSIONS & RECOMMENDATIONS:**

The goal of this thesis project was to explore the possibility of a more effect and economical structural system. Futhermore, the architectural aesthetics were also taken into account to better fit the history and architecture of the surrounding area.

In conclusion, the circular arched glulam beam and glulam columns present a very reasonable alternative to the existing system. However, the glulam flooring system and lateral force resisting system failed to show any significant benefits over the existing system. I would be just as economical to increase the size of the steel lateral force resisting system to accommodate the proper lateral loads. While the increased depth of the wood flooring system makes it a less efficient use of both space and materials when compared to the current system.

A quick cost comparison of the existing system with the new system showed that there were no significant cost benefits of either. In addition to the cost analysis, a construction time comparison also showed no real advantage to either system. This study shows that use of the wood arched beam and columns provide a feasible alternative the existing gravity force resisting system.

It can also be concluded that a level of LEED certification could have been achieved if appropriately incorporated into the original design. This would allow the building to make as little environmental impact as possible. Any addition initial costs of this aspect of the project would be offset by the long term cost savings of the green systems.

The final recommendation made from this thesis is to incorporate these results into future projects. Wood construction should be given more consideration for projects of this size as it is the only renewable building material avialible to us at this time. LEED certification should also be given more consideration as the combination of green building procedures and the use of renewable resources will go a long way in protecting the environment.

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And finally I would especially like to thank my family and friend for keeping me both grounded and on track throughout this entire project. Without their influence I wouldn't be where I am today.

### **REFERENCES:**

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AISC Manual of Steel Construction 3rd Edition. Load and Resistance Factor Design. 1999.

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ASCE 7-98 American Society of Civil Engineers. *ASCE Standard* 7-98 – *Minimum Design Loads for Buildings and Other Structures.* 

*Green Building Rating System for New Construction & Major Renovations Version 2.2. U.S. Green Building Council. 2005.* 

International Building Code 2000.

*RS Means Building Construction Cost Data 2006*. 64th Annual Edition. 2006.





\* Calculations not shown in this appendix are available upon request



Gymnasium Roof



Gymnasium First Floor


E-W Gymnasium Frame



N-S Gymnasium Frame

# Natatorium Roof



217'-

0	0		D			D	0	0
0								0
0								0
0								0
0	D	٥	٥	٥	٥	٥	0	0

# Natatorium First Floor



E-W Natatorium Frame



N-S Natatorium Frame

## Wind Loading

Using ASCE7-05

Basic Wind Speed 90mph (ASTM 6.1 Figure 6.1C)

Exposure B (ASCE 7-05 6.4.6.3)

Importance Factor I = 1.15 (ASCE 7-05 Table 1-1& Table 6-1)

 $K_{zt} =$ 

1

0.85 ASCE 7-05 Table 6-4

ft	K <sub>z</sub> ASC	E 7-05 Table 6-3
0-15	0.57	
20	0.62	
25	0.66	
30	0.7	
40	0.76	
50	0.81	
60	0.85	
ft	qz=0.00256K <sub>z</sub> K <sub>z</sub>	<sub>rt</sub> K <sub>d</sub> V <sup>2</sup> I ASCE 7-05 6.5.10
0-15	11.55358	
20	12.56705	
25	13.37783	
30	14.18861	
40	15.40477	
50	16.41825	
60	17.22902	
-		
G	0.85	ASCE 7-05 6.5.8.1
Cp (windward)	0.8	ASCE 7-05 Fig 6-6
Cp (leeward)	-0.5	ABOE 1-03 Hg 0 0
op (iceward)	0.0	
Gcpi	0.18	ASCE 7-05 Fig 6-5
GCpi	-0.18	
F -		

$P=q_z(GC_p)-q_h(GC_{pi})$	ASCE 7-05 Eq 6-23	$P=q_h(GC_p)-q_h(GC_{pi})$	ASCE 7-05 Eq 6-23
ft	Windward	ft	Leeward
0-15	10.95766	0-15	-10.4236
20	11.64682	20	-10.4236
25	12.19815	25	-10.4236

30	12.74948	30	-10.4236
40	13.57647	40	-10.4236
50	14.26563	50	-10.4236
60	14.81696	60	-10.4236

21.4
22.1
22.6
23.2
24.0
24.7
25.2

Seismic Design Category	В
S <sub>s</sub> =	0.182
S <sub>1</sub> =	0.0628
S <sub>ms</sub> =	0.182
S <sub>m1</sub> =	0.0628
S <sub>ds</sub> =	0.08
S <sub>d1</sub> =	0.666
l <sub>e</sub> =	1
No. Stories=	1
Cd=	3.25
R=	3.25
h <sub>n</sub> =	42
C <sub>T</sub> =	0.02
x=	0.75
$T_a = C_T (h_n)^x$	0.329964
T <sub>max</sub> =C <sub>u</sub> T <sub>a</sub>	1.072384
T=	0.3358
$C_s = S_{Ds}/(R/I_E)$	0.024615
C <sub>smax</sub> =S <sub>D1</sub> /(R/I <sub>E</sub> )/T	0.610253
C <sub>smin</sub> =0.044S <sub>Ds</sub> I <sub>E</sub>	0.00352
C <sub>smin</sub> =0.5S <sub>1</sub> /(R/I <sub>E</sub> )	0.009662
C <sub>s</sub> =	0.016

Structure Weight	W=	3589147 lbs
Base Shear	V=CsW	57426.35 lbs

F <sub>x</sub> =C <sub>vx</sub> V		K=1.00								
C <sub>vx</sub> =W <sub>x</sub> h <sub>x</sub>	<sup>k</sup> /Sum(W <sub>i</sub> h	<sup>k</sup> )								
x=1	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.043021	F <sub>x</sub> =	2470.56
x=2	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.043021	F <sub>x</sub> =	2470.56
x=3	W <sub>x</sub> =	362035	h <sub>x</sub> =	3	$W_x h_x^k =$	1086105	C <sub>vx</sub> =	0.100869	F <sub>x</sub> =	5792.56
x=4	W <sub>x</sub> =	362035	h <sub>x</sub> =	3	$W_x h_x^k =$	1086105	C <sub>vx</sub> =	0.100869	F <sub>x</sub> =	5792.56
x=5	W <sub>x</sub> =	419741	h <sub>x</sub> =	3	$W_x h_x^k =$	1259223	C <sub>vx</sub> =	0.116947	F <sub>x</sub> =	6715.856
x=6	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.043021	F <sub>x</sub> =	2470.56
x=7	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.043021	F <sub>x</sub> =	2470.56
x=8	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.043021	F <sub>x</sub> =	2470.56
x=9	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.043021	F <sub>x</sub> =	2470.56
x=10	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.043021	F <sub>x</sub> =	2470.56
x=11	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.043021	F <sub>x</sub> =	2470.56
x=12	W <sub>x</sub> =	208216	h <sub>x</sub> =	3	$W_x h_x^k =$	624648	C <sub>vx</sub> =	0.058013	F <sub>x</sub> =	3331.456
x=13	W <sub>x</sub> =	342216	h <sub>x</sub> =	3	$W_x h_x^k =$	1026648	C <sub>vx</sub> =	0.095347	F <sub>x</sub> =	5475.456
x=14	W <sub>x</sub> =	659624	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	1978872	C <sub>vx</sub> =	0.183783	F <sub>x</sub> =	10553.98
		3589147			Sum	10767441			Sum	57426.35

6 7411.68 9 26066.52 12 34755.36 15 50368.92 18 22235.04 21 25940.88 24 29646.72 27 33352.56 30 37058.4 33 40764.24 36 59966.21 39 106771.4 42 221633.7 Total Mom. 699677.4 Fx @ top 16658.99 PLF 134.716

3 3705.84

# **Natatorium Columns**

Using Sothern Pine		Adjustmer	nt Factors
Table 5B Visually Graded Southern Pine		C <sub>D</sub> := 1.15	Table 2.3.2
Identification Number 50 Grade N1D14		С <sub>М</sub> := 1.0	Table 5B
$E := 1.9 \cdot 10^6 \text{psi}$		C <sub>t</sub> := 1.0	Table 2.3.3
$E_{\min} := 0.98 \cdot 10^6 psi$			
4 or More Laminations			
F <sub>C</sub> := 2300psi			
Force = P Column Length = L			
		•	
L;;= 40ft	Mem b := :	ber Size 5.125in	
P := 59940lbf	d :=	1 1 in	
$f_c := \frac{P}{b \cdot d}$			
$f_c = 1.063 \times 10^3 \text{ psi}$			
K <sub>e</sub> := 1.0			
$l_e := K_e \cdot L$ 3.7.1.2	SR <sub>1</sub> := −	l <sub>e</sub> d	
·	SR <sub>2</sub> :=	$\frac{l_e}{b}$	
$\mathbf{E'}_{\min} \coloneqq \mathbf{E}_{\min} \cdot \mathbf{C}_{\mathbf{M}} \cdot \mathbf{C}_{\mathbf{t}}$	SR :=	SR <sub>1</sub> if SF	$R_1 > SR_2$
$F_{Cstar} := F_{C} \cdot C_{D} \cdot C_{M} \cdot C_{t}$		SR <sub>2</sub> other	wise

$$F'_{C} := F_{C} \cdot C_{D} \cdot C_{M} \cdot C_{t} \cdot C_{P}$$
$$F'_{C} = 1.267 \times 10^{3} \text{ psi}$$

Member := 
$$|"ok" \text{ if } f_c \leq F'_C$$
  
"Fail" otherwise

$$Member = "ok"$$

# **Fire Protection**

Ref 1, 16.1

2 hour Required

Nominal Char Rate 
$$\beta_n := 1.5 \cdot \left(\frac{in}{hr}\right)$$

$$\beta_{\text{eff}}(t) \coloneqq \frac{1.2 \cdot \beta_n}{\left(\frac{t}{\text{hr}}\right)^{0.187}}$$

 $t := 2 \cdot hr$ 

$$\beta_{\text{eff}}(t) = 1.581 \left(\frac{\text{in}}{\text{hr}}\right)$$

```
Additional Width b_{add}\coloneqq\beta_{eff}(t){\cdot}t{\cdot}2 b_{add}=6.325~in Additional Depth
```

$$d_{add} := \beta_{eff}(t) \cdot t \cdot 2$$

 $d_{add} = 6.325$  in

Width Needed

 $b_{needed} := b + b_{add}$  $b_{needed} = 11.45$  in

Depth Needed

 $d_{needed} := d + d_{add}$ 

 $d_{needed} = 17.325$  in

# <u>Use 12"x18"</u>

# References

 2001 NDS National Design Specification for Wood Construction. ANSI/AF&PA NDS-2001. AFPA Washington, DC. 2001.
 2001 NDS National Design Specification for Wood Construction Supplement. ANSI/AF&PA NDS-200 AFPA Washington, DC. 2001.

11.

Natatorium Connection Design Using 1/4" ASTM A 36 Steel Plate and 1" Bolts Z<sub>parallel</sub> := 2150lb Planes<sub>shear</sub> := 4

$$\theta := 62.742 deg$$

$$Z_{\theta} \coloneqq \frac{Z_{\text{parallel}} \cdot Z_{\text{perpendicular}}}{Z_{\text{parallel}} \cdot \sin(\theta)^2 + Z_{\text{perpendicular}} \cdot \cos(\theta)^2}$$

$$Z_{\theta} = 910.859 \text{ lb}$$

Using 11.3.8

$$Z_{\Theta \text{Total}} \coloneqq Z_{\Theta} \cdot \text{Planes}_{\text{shear}}$$

$$Z_{\Theta Total} = 3.643 \times 10^{5} \, \text{lb}$$

Number<sub>Bolts</sub> := 
$$\frac{P}{Z_{\theta Total}}$$

Number<sub>Bolts</sub> = 8.089

## Try 8 Bolts

C<sub>D</sub> := 1.15

 $n\coloneqq 8$ 

$$R_{EA} := .703$$

$$C_{g} \coloneqq \left[\frac{m \cdot \left(1 - m^{2n}\right)}{n \cdot \left[\left[\left(1 + R_{EA}\right) \cdot m^{n}\right](1 + m) - 1 + m^{2 \cdot n}\right]}\right] \cdot \left(\frac{1 + R_{EA}}{1 - m}\right)$$

$$C_g = 6.105$$

Use Cg=1.0

 $\begin{aligned} Z'_{\theta Total} &\coloneqq Z_{\theta Total} \cdot C_D \\ Z'_{\theta Total} &= 4.19 \times 10^3 \, \text{lb} \\ \text{Connection}_{Capacity} &\coloneqq Z'_{\theta Total} \cdot n \\ \text{Connection}_{Capacity} &= 3.352 \times 10^4 \, \text{lb} \\ \text{Connection} &\coloneqq \text{"ok" if Connection}_{Capacity} > P \\ &\text{"no" otherwise} \\ \text{Connection} &= \text{"ok"} \end{aligned}$ 



Number of steps to subdivide the arch n := 10

i := 1 .. n

s= length of the segment with the angle  $\Delta_{\alpha}$ =A/n. Use radians for angles, this simplifies calculations.

$$\Delta_{\alpha} := \frac{A}{n}$$
 start :=  $A - \frac{\Delta_{\alpha}}{2}$  start = 25.9 deg s:=  $\frac{A \cdot R}{n}$  s = 6.423 ft

 $\beta_i := start - (i - 1) \cdot \Delta_{\alpha}$ 

 $X_i := x(\beta_i) \qquad Y_i := y(\beta_i)$ 

## Dead load vertical reaction

The DL is distributed over the arch length.

$$R_{VDL} := W_{DL} \cdot R \cdot A$$

Express moments as the function of the angle

$$\begin{array}{l} \alpha.\\ m1(\alpha) := \left[ R_{\text{VDL}} \cdot \mathbf{x}(\alpha) - w_{\text{DL}} \cdot R \cdot \mathbf{y}(\alpha) + \left( w_{\text{DL}} \cdot R^2 \cdot \sin(\alpha) \right) \cdot (\mathbf{A} - \alpha) \right] \\ a_1 := \int_0^A m1(\alpha) \cdot \mathbf{y}(\alpha) \, d\alpha \\ a_2 := \int_0^A \mathbf{y}(\alpha)^2 \, d\alpha \end{array}$$

$$\begin{array}{l} \text{Dead load horizontal reaction} \\ R_{\text{HDL}} := \frac{a_1}{a_1} \end{array}$$

Dead load horizontal reaction

$$R_{\text{HDL}} := \frac{a_1}{a_2}$$

 $R_{\text{HDL}} = 2.196 \times 10^4 \, \text{lbf}$ 

## Dead load moment

$$M_{DL}(\alpha) := -R_{HDL} \cdot y(\alpha) + m1(\alpha)$$











$$\begin{split} M_{us\_L}(\alpha) &\coloneqq -R_{HUSL} \cdot y(\alpha) + m_{usl\_L}(\alpha) &< ----- \text{ Moment from unbalanced snow load} \\ M_{us\_R}(\alpha) &\coloneqq -R_{HUSL} \cdot y(\alpha) + m_{usl\_R}(\alpha) \\ M_{USL\_left} &\coloneqq M_{us\_L}(\beta_i) & M_{USL\_right} &\coloneqq M_{us\_L}(\beta_i) \\ M_{USL\_left} &= \bullet & M_{USL\_right} &= \bullet & M_{USL\_right} &\coloneqq & M_{us\_R}(\alpha) & \text{if } \alpha \leq A \\ M_{us\_R}(\alpha - A) & \text{if } \alpha > A \end{split}$$









# Axial thrust at the max moment location

$$P := \left[ R_{VSL} + R_{VDL} - \left( w_{SL} \cdot x(\beta_{ii}) + w_{DL} \cdot R \cdot \beta_{ii} \right) \right] \cdot \sin(\beta_{ii}) + \left( R_{HDL} + R_{HSL} \right) \cdot \cos(\beta_{ii})$$

 $P=\,61826\,lbf$ 

## Shear at support

$$\underbrace{V}_{W} := (R_{VSL} + R_{VDL}) \cdot \cos(A) + (R_{HDL} + R_{HSL}) \cdot \sin(A) \qquad F'_{vx} := F_{vx} \cdot C_D \cdot C_M \cdot C_t \\
F'_{vx} = 345 \text{ psi} \\
V = 5.362 \times 10^4 \text{ lbf} \qquad f_v := \frac{1.5 \cdot V}{b \cdot d} \qquad f_v = 341.177 \text{ psi} \\
Shear := \begin{bmatrix} \text{"ok"} & \text{if } f_v < F'_{vx} \\ \text{"fail"} & \text{otherwise} \end{bmatrix}$$

## MAX moment

$$\begin{split} \text{MIN} &= -7.386 \times 10^4 \, \text{lbf} \cdot \text{ft} \\ \text{MAX} &= 6.984 \times 10^4 \, \text{lbf} \cdot \text{ft} \\ \text{P1}(\beta_1) &\coloneqq \left[ \text{R}_{\text{VSL}} + \text{R}_{\text{VDL}} - \left( \text{w}_{\text{SL}} \cdot \text{x}(\beta_1) + \text{w}_{\text{DL}} \cdot \text{R} \cdot \beta_1 \right) \right] \cdot \sin(\beta_1) + \left( \text{R}_{\text{HDL}} + \text{R}_{\text{HSL}} \right) \cdot \cos(\beta_1) \\ \text{P1}(\beta_{11}) &= 6.183 \times 10^4 \, \text{lbf} \\ \text{T}_{-a} &\coloneqq \text{P1}(\beta_{11}) \\ \text{G}(\beta_1) &\coloneqq \text{T}_{-a} \end{split}$$
  $\begin{aligned} \textbf{Variation of axial thrust along the left half of arch} \end{split}$ 





$$F_{T_{L}prime} \coloneqq \frac{1}{3} F_{YX}$$

$$\& \left\{ \begin{array}{l} {}^{P}_{T_{L}} \prod_{j=1}^{P}_{T_{L}} \prod_{j=$$

	Bending := "ok" if f <sub>bx</sub> < F <sub>b_prime</sub> "fail" otherwise Bending = "ok"
According to the AITC (Ref 3), the b	eam stability factor "is not customarily applied to arches".
$R_{VTotal} \coloneqq R_{VDL} + R_{VSL}$	
$R_{VTotal} = 2.947 \times 10^4 lbf$	
$R_{HTotal} := R_{HDL} + R_{HSL}$	
$R_{HTotal} = 5.988 \times 10^4  lbf$	
Fire Protection	
	Ref 1, 16.1
2 hour Required	
Nominal Char Rate	$\beta_{n} := 1.5 \cdot \left(\frac{in}{hr}\right)$
$\beta_{\text{eff}}(t) \coloneqq \frac{1.2 \cdot \beta_{\text{n}}}{\left(\frac{t}{\text{hr}}\right)^{0.187}}$	$t := 2 \cdot hr$
$\beta_{\text{eff}}(t) = 1.581 \left(\frac{\text{in}}{\text{hr}}\right)$	

Additional W	$\mathbf{b}_{add}\coloneqq \boldsymbol{\beta}_{eff}(t){\cdot}t{\cdot}2$	
	b <sub>add</sub> = 6.325 in	
Additional Dept	h	
	$d_{add} := \beta_{eff}(t) \cdot t \cdot 2$	
	$d_{add} = 6.325$ in	
Width Needec	ł.	
	$b_{needed} := b + b_{add}$	
	$b_{needed} = 11.45 \text{ in}$	
Depth Needed		
	$d_{needed} := d + d_{add}$	
	$d_{needed} = 52.325 \text{ in}$	
<u>Use</u>	<u>12"x54"</u>	

# References

1. 2001 NDS National Design Specification for Wood Construction. ANSI/AF&PA NDS-2001. AFPA Washington, DC. 2001.

2. 2001 NDS National Design Specification for Wood Construction Supplement. ANSI/AF&PA NDS-2001. AFPA Washington, DC. 2001.

3. AITC, Timber Construction Manual. Fifth Edition. American Institute of Timber Construction (AITC). John Villey and Sons. New York, NY 2004.

# Lateral Force Resistance Member

Using Sothern Pine	Adjustment Factors	
Table 5B Visually Graded Southern Pine	_	<b>T</b>     000
Identification Number 50 Grade N1D14	C <sub>D</sub> := 1.15	Table 2.3.2
	C <sub>M</sub> ≔ 1.0	Table 5B
$E := 1.9 \cdot 10^6 \text{psi}$	C <sub>t</sub> := 1.0	Table 2.3.3
$E_{\min} := 0.98 \cdot 10^6 psi$		
4 or More Laminations		
F <sub>C</sub> := 2300psi		
$F_T := 1350 psi$		
Force = P	I	
Column Length = L	Member Size	
L:= 48.5ft	b := 3in	
P := 6761lbf	d := 5.5in	

Member in Tension

$$f_t := \frac{P}{b \cdot d}$$
  $f_t = 409.758 \text{ psi}$ 

$$\mathbf{F'}_{\mathbf{T}} \coloneqq \mathbf{F}_{\mathbf{T}} \cdot \mathbf{C}_{\mathbf{D}} \cdot \mathbf{C}_{\mathbf{M}} \cdot \mathbf{C}_{\mathbf{t}} \qquad \mathbf{F'}_{\mathbf{T}} = 1.552 \times 10^3 \, \mathrm{psi}$$

$$Member_T := \begin{array}{ll} "ok" & \text{if } f_t \leq F'_T \\ "Fail" & \text{otherwise} \end{array}$$

 $Member_T = "ok"$ 

Member in Compression

$$f_{c} := \frac{P}{b \cdot d}$$

$$f_{c} = 409.758 \text{ psi}$$

$$K_{e} := 1.0$$

$$l_{e} := K_{e} \cdot L \qquad 3.7.1.2$$

$$SR_{1} := \frac{l_{e}}{d}$$

$$SR_{2} := \frac{l_{e}}{b}$$

$$E'_{min} := E_{min} \cdot C_{M} \cdot C_{t}$$

$$SR := |SR_{1}| \text{ if } SR_{1} > 0$$

$$E'_{\min} := E_{\min} \cdot C_{M} \cdot C_{t}$$

$$SR := \begin{bmatrix} SR_{1} & \text{if } SR_{1} > SR_{2} \\ SR_{2} & \text{otherwise} \end{bmatrix}$$

$$F_{Cstar} := F_{C} \cdot C_{D} \cdot C_{M} \cdot C_{t}$$

$$F'_{C} = 1.197 \times 10^{3} \text{ psi}$$

Member := 
$$"ok"$$
 if  $f_c \le F'_C$   
"Fail" otherwise

# **Fire Protection**

Ref 1, 16.1

2 hour Required

Nominal Char Rate 
$$\beta_n := 1.5 \cdot \left(\frac{in}{hr}\right)$$

$$\beta_{eff}(t) \coloneqq \frac{1.2 \cdot \beta_n}{\left(\frac{t}{hr}\right)^{0.187}} \qquad t \coloneqq 2 \cdot hr$$

$$\beta_{eff}(t) = 1.581 \bigg( \frac{in}{hr} \bigg)$$
 Additional Width

$$b_{add} \coloneqq \beta_{eff}(t) \cdot t \cdot 2$$

$$b_{add} = 6.325 \text{ in}$$

Additional Depth

$$d_{add} := \beta_{eff}(t) \cdot t \cdot 2$$

$$d_{add} = 6.325$$
 in

Width Needed

 $b_{needed} := b + b_{add}$ b, 0 325 ii

$$p_{\text{needed}} = 9.325 \text{ in}$$

Depth Needed

 $d_{needed} := d + d_{add}$ 

$$d_{needed} = 11.825$$
 in
# Use 9.5"x12"

# References

 2001 NDS National Design Specification for Wood Construction. ANSI/AF&PA NDS-2001. AFPA Washington, DC. 2001.
 2001 NDS National Design Specification for Wood Construction Supplement. ANSI/AF&PA NDS-2001.
 AFPA Washington, DC. 2001.



#### 1 of 18

Number of steps to subdivide the arch n := 10

i := 1 .. n

s= length of the segment with the angle  $\Delta_{\alpha}$ =A/n. Use radians for angles, this simplifies calculations.

$$\Delta_{\alpha} := \frac{A}{n} \qquad \text{start} := A - \frac{\Delta_{\alpha}}{2} \qquad \text{start} = 33.2 \text{ deg} \qquad \underset{\mathcal{N}}{\text{start}} := \frac{A \cdot R}{n} \qquad \text{s} = 8.457 \text{ ft}$$

 $\beta_i := start - (i - 1) \cdot \Delta_{\alpha}$ 

 $X_i := x(\beta_i) \qquad Y_i := y(\beta_i)$ 

## Dead load vertical reaction

The DL is distributed over the arch length.

$$R_{VDL} := W_{DL} \cdot R \cdot A$$

Express moments as the function of the angle

$$\begin{array}{ll} \alpha.\\ m1(\alpha) := \left[ R_{\text{VDL}} \cdot \mathbf{x}(\alpha) - w_{\text{DL}} \cdot R \cdot \mathbf{y}(\alpha) + \left( w_{\text{DL}} \cdot R^2 \cdot \sin(\alpha) \right) \cdot (\mathbf{A} - \alpha) \right] \\ a_1 := \int_0^A m1(\alpha) \cdot \mathbf{y}(\alpha) \, d\alpha \\ a_2 := \int_0^A \mathbf{y}(\alpha)^2 \, d\alpha \end{array}$$

$$\begin{array}{ll} \text{Dead load horizontal reaction} \\ R_{\text{HDL}} := \frac{a_1}{a_1} \end{array}$$

ead load horizontal reaction

$$R_{\text{HDL}} \coloneqq \frac{a_1}{a_2}$$

 $R_{\text{HDL}} = 2.356 \times 10^4 \, \text{lbf}$ 

## Dead load moment

$$M_{DL}(\alpha) := -R_{HDL} \cdot y(\alpha) + m1(\alpha)$$











$$\begin{split} M_{us\_L}(\alpha) &\coloneqq -R_{HUSL} \cdot y(\alpha) + m_{usl\_L}(\alpha) \\ M_{us\_R}(\alpha) &\coloneqq -R_{HUSL} \cdot y(\alpha) + m_{usl\_R}(\alpha) \\ M_{USL\_left_i} &\coloneqq M_{us\_L}(\beta_i) \\ M_{USL\_left_i} &\coloneqq M_{us\_L}(\beta_i) \\ M_{USL\_left_i} &\coloneqq M_{us\_L}(\alpha) & \text{if } \alpha \leq A \\ M_{us\_R}(\alpha - A) & \text{if } \alpha > A \end{split}$$







Thesis  

$$\begin{array}{c|c} MAX := & MAX & \text{if } |MAX| > |MIN & \text{otherwise} \\ \hline \\ \hline \\ \hline \\ Load combination math \\ \hline \\ DL & SL & (DL+SL) & (DL+USLleft) & (DL+USLright) \\ \hline \\ M4 = & & \text{Ibf-ft} \\ \hline \\ MAX = 1.251 \times 10^{5} \text{ lbf-ft} \\ \hline \\ MAX = 1.251 \times 10^{5} \text{ lbf-ft} \\ \hline \\ \hline \\ Ii := & \text{for } i \in 1...5 \\ & \text{is } i \in i \\ Ii = 6 \\ \hline \\ M4_{Ii, JI} = 1.251 \times 10^{5} \text{ lbf-ft} \\ \hline \\ \\ Slope at the point with maximum moment \\ & JI := 5 \\ \hline \\ \\ \hline \\ M1N = -1.376 \times 10^{5} \text{ lbf-ft} \\ \hline \\ \end{array}$$

# Axial thrust at the max moment location

$$P := \left[ R_{VSL} + R_{VDL} - \left( w_{SL} \cdot x(\beta_{ii}) + w_{DL} \cdot R \cdot \beta_{ii} \right) \right] \cdot \sin(\beta_{ii}) + \left( R_{HDL} + R_{HSL} \right) \cdot \cos(\beta_{ii})$$

 $P = 68250 \, lbf$ 

# Shear at support

$$\underbrace{W}_{v} := (R_{VSL} + R_{VDL}) \cdot \cos(A) + (R_{HDL} + R_{HSL}) \cdot \sin(A) \qquad F'_{vx} := F_{vx} \cdot C_{D} \cdot C_{M} \cdot C_{t} \\
F'_{vx} = 345 \text{ psi} \\
V = 7.117 \times 10^{4} \text{ lbf} \qquad f_{v} := \frac{1.5 \cdot V}{b \cdot d} \qquad f_{v} = 341.478 \text{ psi} \\
Shear := \begin{bmatrix} \text{"ok"} & \text{if } f_{v} < F'_{vx} \\ \text{"fail"} & \text{otherwise} \end{bmatrix}$$

# MAX moment

$$\begin{split} \text{MIN} &= -1.376 \times 10^5 \, \text{lbf} \cdot \text{ft} \\ \text{MAX} &= 1.251 \times 10^5 \, \text{lbf} \cdot \text{ft} \\ \text{P1}(\beta_1) &\coloneqq \left[ \text{R}_{\text{VSL}} + \text{R}_{\text{VDL}} - \left( \text{w}_{\text{SL}} \cdot \text{x}(\beta_1) + \text{w}_{\text{DL}} \cdot \text{R} \cdot \beta_1 \right) \right] \cdot \sin(\beta_1) + \left( \text{R}_{\text{HDL}} + \text{R}_{\text{HSL}} \right) \cdot \cos(\beta_1) \\ \text{P1}(\beta_{11}) &= 6.825 \times 10^4 \, \text{lbf} \\ \text{T}_a &\coloneqq \text{P1}(\beta_{11}) \\ \text{GG}(\beta_1) &\coloneqq \text{T}_a a \end{split}$$
  $\begin{aligned} \textbf{Variation of axial thrust along the left half of arch} \end{split}$ 







$$F_{T_{L}\text{prime}} \coloneqq \frac{1}{3} F_{VX}$$

$$\& I^{=} \begin{vmatrix} {}^{\circ}OK^{-} & \text{if } f_{T_{L}} \subseteq F_{T_{L}\text{prime}} \\ {}^{\circ}Fall^{*} & \text{oterwise} & a = {}^{\circ}OK^{*} \end{vmatrix}$$
If the radial stress in tension exceeds 15 psi adjusted by  $C_{D} C_{M}$  and  $C_{t}$  then radial reinforcement is required.
  
Reinforce := 
$$\begin{vmatrix} YES^{*} & \text{if } 15\text{psi-}C_{D}C_{M}^{*}C_{t} \leq f_{T} \\ {}^{*}NO^{*} & \text{otherwise} \end{vmatrix}$$
  
Reinforce = "NO"
  
The reinforcement rule has further limitations - see ------->
  
**Ref. 1, Sec.** 5.4.1
  
**Combined bending and axial compression**
  
Axial stress from P
  
 $f_{c} \coloneqq \frac{P}{b \cdot d}$ 
  
 $f_{c} \simeq 218.313 \text{ psi}$ 
  
Bending stress - negative moment
  
 $f_{bx} \coloneqq \frac{|MTN|}{6}$ 
  
 $f_{bx} = 519.605 \text{ psi}$ 
  
(gymbam.xmd) 15 df 18

	Bending := $    ok  $ if $f_{bx} < F_{b_prime}$						
	Bending = "ok"						
According to the AITC (Ref 3), the beam stability factor "is not customarily applied to arches".							
$R_{VTotal} \coloneqq R_{VDL} + R_{VSL}$							
$R_{VTotal} = 4.168 \times 10^4 lbf$							
$R_{HTotal} \coloneqq R_{HDL} + R_{HSL}$							
$R_{HTotal} = 6.46 \times 10^4 lbf$							
Fire Protection	Ref 1, 16.1						
2 hour Required							
Nominal Char Rate	$\beta_{n} \coloneqq 1.5 \cdot \left(\frac{\text{in}}{\text{hr}}\right)$						
$\beta_{\text{eff}}(t) \coloneqq \frac{1.2 \cdot \beta_{\text{n}}}{\left(\frac{t}{\text{hr}}\right)^{0.187}}$	$t := 2 \cdot hr$						
$\beta_{\text{eff}}(t) = 1.581 \left(\frac{\text{in}}{\text{hr}}\right)$							

Additional Width $b_{add} \coloneqq \beta_{eff}(t) \cdot t \cdot 2$
b <sub>add</sub> = 6.325 in
Additional Depth
$d_{add} \coloneqq \beta_{eff}(t) \cdot t \cdot 2$
$d_{add} = 6.325$ in
Width Needed
$b_{needed} := b + b_{add}$
$b_{needed} = 11.45$ in
Depth Needed
$d_{needed} := d + d_{add}$
$d_{needed} = 67.325 \text{ in}$
Use 12"x68"

## References

1. 2001 NDS National Design Specification for Wood Construction. ANSI/AF&PA NDS-2001. AFPA Washington, DC. 2001.

2. 2001 NDS National Design Specification for Wood Construction Supplement. ANSI/AF&PA NDS-2001. AFPA Washington, DC. 2001.

3. AITC, Timber Construction Manual. Fifth Edition. American Institute of Timber Construction (AITC). John Villey and Sons. New York, NY 2004.

# Gym Columns

	Using Sothern Pine	Adjustment Factors			
	Table 5B Visually Graded Southern Pine Identification Number 50 Grade N1D14		C 1 15	Table 2.3.2	
			$C_{\rm D} = 1.15$ $C_{\rm M} \approx 1.0$	Table 5B	
	$E := 1.9 \cdot 10^6 \text{psi}$		C <sub>t</sub> := 1.0	Table 2.3.3	
	$E_{\min} := 0.98 \cdot 10^6 psi$				
	4 or More Laminations				
	F <sub>C</sub> := 2300psi				
	Force = P Column Length = L				
	$ \begin{array}{l} L := 60 \text{ft} \\ P := 64600 \text{lbf} \end{array} $	Member Size b := 5.125in d := 11in			
	$f_c := \frac{P}{b \cdot d}$	u .			
	$f_c = 1.146 \times 10^3  \text{psi}$				
	K <sub>e</sub> := 1.0				
	$l_e := K_e \cdot L$ 3.7.1.2	sr <sub>1</sub> :=	$\frac{l_e}{d}$		
	1	SR <sub>2</sub> :=	$=\frac{l_e}{b}$		
	$\mathbf{E'_{min}} \coloneqq \mathbf{E_{min}} \cdot \mathbf{C_M} \cdot \mathbf{C_t}$	SR :=	SR <sub>1</sub> if SI	$R_1 > SR_2$	
	$F_{Cstar} := F_{C} \cdot C_{D} \cdot C_{M} \cdot C_{t}$		SR <sub>2</sub> other	wise	

$$F'_{C} := F_{C} \cdot C_{D} \cdot C_{M} \cdot C_{t} \cdot C_{P}$$
$$F'_{C} = 1.216 \times 10^{3} \text{ psi}$$

Member := 
$$|"ok" \text{ if } f_c \leq F'_C$$
  
"Fail" otherwise

Member = "ok"

# **Fire Protection**

Ref 1, 16.1

2 hour Required

Nominal Char Rate 
$$\beta_n := 1.5 \cdot \left(\frac{in}{hr}\right)$$

$$\beta_{\text{eff}}(t) \coloneqq \frac{1.2 \cdot \beta_n}{\left(\frac{t}{\text{hr}}\right)^{0.187}}$$

 $t := 2 \cdot hr$ 

$$\beta_{\text{eff}}(t) = 1.581 \left(\frac{\text{in}}{\text{hr}}\right)$$

```
Additional Width b_{add}\coloneqq\beta_{eff}(t){\cdot}t{\cdot}2 b_{add}=6.325~in Additional Depth
```

$$d_{add} := \beta_{eff}(t) \cdot t \cdot 2$$

 $d_{add} = 6.325$  in

Width Needed

 $b_{needed} := b + b_{add}$  $b_{needed} = 11.45$  in

Depth Needed

 $d_{needed} := d + d_{add}$ 

 $d_{needed} = 17.325$  in

# <u>Use 12"x18"</u>

# References

 2001 NDS National Design Specification for Wood Construction. ANSI/AF&PA NDS-2001. AFPA Washington, DC. 2001.
 2001 NDS National Design Specification for Wood Construction Supplement. ANSI/AF&PA NDS-200 AFPA Washington, DC. 2001.

11.

Title : Dsgnr: Description :

Scope :



Title : Dsgnr: Description :

Scope :



## **Fire Protection**

Gym Floor Girder

 $\beta_n := 1.5 \cdot \left(\frac{in}{hr}\right)$ 

2 hour Required

Nominal Char Rate

$$\beta_{eff}(t) := \frac{1.2 \cdot \beta_n}{\left(\frac{t}{hr}\right)^{0.187}} \qquad t := 2hr$$

$$\beta_{\text{eff}}(t) = 1.581 \left(\frac{\text{in}}{\text{hr}}\right)$$

Additional Width

$$b_{add} := \beta_{eff}(t) \cdot t \cdot 2$$
  
 $b_{add} = 6.325$  in

Additional Depth

$$d_{add} := \beta_{eff}(t) \cdot t \cdot 2$$

$$d_{add} = 6.325$$
 in

Width Needed

$$b_{needed} := b + b_{add}$$
  
 $b_{needed} = 17.075$  in

Depth Needed

 $d_{needed} := d + d_{add}$ 

 $d_{needed} = 28.825$  in

# Use 18"x29"

Ref 1, 16.1

# References

 2001 NDS National Design Specification for Wood Construction. ANSI/AF&PA NDS-2001. AFPA Washington, DC. 2001.
 2001 NDS National Design Specification for Wood Construction Supplement. ANSI/AF&PA NDS-200 AFPA Washington, DC. 2001. )1.

## **Fire Protection**

Gym Floor Beam

 $\beta_n := 1.5 \cdot \left(\frac{in}{hr}\right)$ 

2 hour Required

Nominal Char Rate

$$\beta_{eff}(t) := \frac{1.2 \cdot \beta_n}{\left(\frac{t}{hr}\right)^{0.187}} \qquad t := 2hr$$

$$\beta_{\text{eff}}(t) = 1.581 \left(\frac{\text{in}}{\text{hr}}\right)$$

Additional Width

$$b_{add} \coloneqq \beta_{eff}(t) \cdot t \cdot 2$$
  
 $b_{add} = 6.325$  in

Additional Depth

$$d_{add} := \beta_{eff}(t) \cdot t \cdot 2$$

$$d_{add} = 6.325$$
 in

Width Needed

$$b_{needed} := b + b_{add}$$
  
 $b_{needed} = 18.575 \text{ in}$ 

Depth Needed

 $d_{needed} := d + d_{add}$ 

 $d_{needed} = 28.825$  in

# Use 19"x29"

Ref 1, 16.1

# References

 2001 NDS National Design Specification for Wood Construction. ANSI/AF&PA NDS-2001. AFPA Washington, DC. 2001.
 2001 NDS National Design Specification for Wood Construction Supplement. ANSI/AF&PA NDS-200 AFPA Washington, DC. 2001. )1.

Gymnasium Connection Design Using 1/4" ASTM A 36 Steel Plate and 1" Bolts Z<sub>parallel</sub> := 2150lb Planes<sub>shear</sub> := 4

P := 41680lb

$$\theta := 55.06 \text{deg}$$

$$Z_{\theta} \coloneqq \frac{Z_{\text{parallel}} \cdot Z_{\text{perpendicular}}}{Z_{\text{parallel}} \cdot \sin(\theta)^2 + Z_{\text{perpendicular}} \cdot \cos(\theta)^2}$$

$$Z_{\theta} = 996.824 \text{ lb}$$

Using 11.3.8

$$Z_{\Theta \text{Total}} \coloneqq Z_{\Theta} \cdot \text{Planes}_{\text{shear}}$$
  
 $Z_{\Theta \text{Total}} = 3.987 \times 10^3 \text{ lb}$ 

Number<sub>Bolts</sub> := 
$$\frac{P}{Z_{\theta Total}}$$

Number<sub>Bolts</sub> = 10.453

C<sub>D</sub> := 1.15

 $R_{EA} := .703$ 

$$C_g := \left[\frac{m \cdot \left(1 - m^{2n}\right)}{n \cdot \left[\left[\left(1 + R_{EA}\right) \cdot m^n\right](1 + m) - 1 + m^{2 \cdot n}\right]}\right] \cdot \left(\frac{1 + R_{EA}}{1 - m}\right)$$

$$C_g = -17.776$$

Use Cg=1.0

 $\begin{aligned} Z'_{\theta Total} &\coloneqq Z_{\theta Total} \cdot C_D \\ Z'_{\theta Total} &= 4.585 \times 10^3 \, \text{lb} \\ \text{Connection}_{Capacity} &\coloneqq Z'_{\theta Total} \cdot n \\ \text{Connection}_{Capacity} &= 5.502 \times 10^4 \, \text{lb} \\ \text{Connection} &\coloneqq \text{"ok"} \quad \text{if Connection}_{Capacity} > P \\ &\text{"no"} \quad \text{otherwise} \\ \text{Connection} &= \text{"ok"} \end{aligned}$ 

Title : Dsgnr: Description :

Scope :



Title : Dsgnr: Description :

Scope :


Seismic Design Category	В
S <sub>s</sub> =	0.182
S <sub>1</sub> =	0.0628
S <sub>ms</sub> =	0.182
S <sub>m1</sub> =	0.0628
S <sub>ds</sub> =	0.08
S <sub>d1</sub> =	0.666
l <sub>e</sub> =	1
No. Stories=	1
Cd=	3.25
R=	3.25
h <sub>n</sub> =	42
C <sub>T</sub> =	0.02
x=	0.75
$T_a = C_T (h_n)^x$	0.329964
T <sub>max</sub> =C <sub>u</sub> T <sub>a</sub>	1.072384
T=	0.3358
$C_s = S_{Ds}/(R/I_E)$	0.024615
C <sub>smax</sub> =S <sub>D1</sub> /(R/I <sub>E</sub> )/T	0.610253
C <sub>smin</sub> =0.044S <sub>Ds</sub> I <sub>E</sub>	0.00352
$C_{smin}=0.5S_1/(R/I_E)$	0.009662
C <sub>s</sub> =	0.016

Structure Weight	W=	11864569 lbs
Base Shear	V=CsW	189833.1 lbs

F <sub>x</sub> =C <sub>vx</sub> V		K=1.00										
C <sub>vx</sub> =W <sub>x</sub> h <sub>x</sub>	<sup>k</sup> /Sum(W <sub>i</sub> h	li <sup>k</sup> )										
x=1	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	$W_x h_x^k =$	463230	C <sub>vx</sub> =	0.013014	F <sub>x</sub> =	2470.56	3	3705.84
x=2	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.013014	F <sub>x</sub> =	2470.56	6	7411.68
x=3	W <sub>x</sub> =	362035	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	1086105	C <sub>vx</sub> =	0.030514	F <sub>x</sub> =	5792.56	9	26066.52
x=4	W <sub>x</sub> =	362035	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	1086105	C <sub>vx</sub> =	0.030514	F <sub>x</sub> =	5792.56	12	34755.36
x=5	W <sub>x</sub> =	1648703	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	4946109	C <sub>vx</sub> =	0.13896	F <sub>x</sub> =	26379.25	15	197844.4
x=6	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.013014	F <sub>x</sub> =	2470.56	18	22235.04
x=7	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.013014	F <sub>x</sub> =	2470.56	21	25940.88
x=8	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.013014	F <sub>x</sub> =	2470.56	24	29646.72
x=9	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.013014	F <sub>x</sub> =	2470.56	27	33352.56
x=10	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.013014	F <sub>x</sub> =	2470.56	30	37058.4
x=11	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.013014	F <sub>x</sub> =	2470.56	33	40764.24
x=12	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.013014	F <sub>x</sub> =	2470.56	36	44470.08
x=13	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.013014	F <sub>x</sub> =	2470.56	39	48175.92
x=14	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.013014	F <sub>x</sub> =	2470.56	42	51881.76
x=15	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.013014	F <sub>x</sub> =	2470.56	45	55587.6
x=16	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.013014	F <sub>x</sub> =	2470.56	48	59293.44
x=17	W <sub>x</sub> =	154410	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	463230	C <sub>vx</sub> =	0.013014	F <sub>x</sub> =	2470.56	51	62999.28
x=18	W <sub>x</sub> =	208216	h <sub>x</sub> =	3	W <sub>x</sub> h <sub>x</sub> <sup>k</sup> =	624648	C <sub>vx</sub> =	0.017549	F <sub>x</sub> =	3331.456	54	89949.31
x=19	W <sub>x</sub> =	402216	h <sub>x</sub> =	3	$W_x h_x^k =$	1206648	C <sub>vx</sub> =	0.033901	F <sub>x</sub> =	6435.456	57	183410.5
x=20	W <sub>x</sub> =	6719624	h <sub>x</sub> =	3	$W_x h_x^k =$	20158872	C <sub>vx</sub> =	0.566361	F <sub>x</sub> =	107514	60	3225420
11864569				Sum	35593707	•		Sum	189833.1	Total Mom.	4279969 F	

9 Fx @ top 101904 PLF 824.0662