AIRFOIL SHAPE OPTIMIZATION USING EVOLUTIONARY ALGORITHMS

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Abstract

A new methodology is developed to optimize the shape of airfoils for high aerodynamic performance. A boundary layer panel method coupled solver and an evolutionary algorithm are linked within an automated design loop. A multi-parameter objective function is based on drag to lift ratio of airfoil. The problem is constrained with a minimum allowable lift coefficient, a maximum allowable flow angle of attack, and a moment coefficient range.

1 INTRODUCTION

Airfoil performance is an important parameter in aircraft flight mechanics. Maximum lift, maximum lift to drag ratio, its behavior after stall affects the overall performance of the aircraft. In addition to this, pitching moment generated by the airfoil also has a strong effect on the tail size. Therefore, good airfoil selection and design still remain to be an important problem. There are two basic approaches to designing airfoils. The first, the direct method consists of selecting a known airfoil shape with performance similar to that required by the new application and making slight modifications to the shape to achieve the required performance. The other approach is known as the inverse method. Here, the designer specifies the performance characteristics required by the airfoil and uses computer programs to compute the airfoil geometry that produces that performance. The advantage of direct approach is its simplicity. But classical direct approaches mostly could do a local search. The inverse approach is far more powerful since the designer has much more precise control over the final performance of the airfoil. However, while every airfoil shape produces a particular set of performance characteristics, not every set of performance characteristics can be used to generate a realistic airfoil shape. The designer must be aware of what is practical, the trade-offs required between different types of performance, and physical constraints. As a consequence, classical design methods mainly adopt “trial and error” approach and strongly rely on designer’s future experience rather than the current needs. A global optimization method based on evolutionary algorithms is expected to shorten and simplify the iterative design process and improve the design output. A similar approach has been previously used for aerodynamic optimization of turbomachinery cascades[1], and very good results were obtained.

In this project the flow solutions are obtained using a boundary layer coupled panel method based on source and vortex panels. Boundary layer part of the code is responsible for accurate drag prediction. Method can be easily implemented and gives fast and reliable results at moderate angles of attack.

Optimization part is performed using Differential Evolution algorithm originally developed by Storn and Price[2]. The algorithm is a real coded direct search method for minimizing continuous space functions. This way it is similar to Deb's G3 model[3,4].

In the solutions maximum number of generations is taken to be 50. The algorithm is tested for several population sizes and algorithm parameters. Results are obtained using the optimum program parameters with randomly generated initial population and an initial population consisting of known NACA profiles.

2 FLOW SOLVER

The flow solution is obtained using a panel method, which assumes the flow is inviscid, incompressible, and irrotational. Panel method is based on modeling the airfoil surface with source and vortex panels with unknown strengths and then solving for those strengths to satisfy the solid wall boundary conditions, i.e., flow can not penetrate through the airfoil surface. Panel method can be easily implemented and gives fast and reliable results for moderate angles of attack. The biggest drawback of panel method is; it is based on inviscid flow equations therefore it can not predict aerodynamic drag accurately. To overcome the problem, the method is coupled with a boundary layer correction method. The method solves parabolized Navier-Stokes equations using Falkner-Skan[5] transformation. The transformation reduces the system of partial differential equations to an ordinary differential equation of order 3. The method is capable of solving boundary layer equations for both laminar and turbulent cases. For turbulent flows, Baldwin-Lomax (1978) algebraic turbulence model is employed. The model is very reliable and is sometimes employed for flow solutions around complex three-dimensional bodies[6], such as full aircraft configurations.

3 DIFFERENTIAL EVOLUTION

Problems which involve global optimization over continuous spaces are ubiquitous throughout the scientific community. In general, the task is to optimize certain properties of a system by pertinently choosing the system parameters. The standard approach to an optimization problem begins by designing an objective function that can model the problem's objectives while incorporating any constraints. For some problems, the objective function defines the optimization problem as a minimization task. For such problems, the objective function is more accurately called a "cost" function. When the cost function is non-linear and non-differentiable direct search methods are the methods of choice.

Differential Evolution (DE) is a direct search method which utilizes NP D-dimensional parameter vectors as population for each generation[2]. DE generates new parameter vectors by adding the weighted difference between two population vectors to a third vector. This operation can be called mutation. The mutated vector's parameters are then mixed with the parameters of another predetermined vector, the target vector, to yield a so called trial vector. Parameter mixing can be named crossover. If the trial vector yields a lower cost function value than the target vector, the trial vector replaces the target vector in the following generation. This operation is called selection. The details of these operations are explained in the following subsections.

3.1 MUTATION

For each target vector $x_{i,G+1}, i = 1,2,..,NP$, a mutant vector is generated according to

$$v_{i,G+1} = x_{r_1,G} + F(x_{r_2,G} - x_{r_3,G}) \qquad (1)$$

Where r_1, r_2, r_3 are random indexes which are mutually different. *F* is an amplification factor whose value changes between 0 and 2. The indexes r_1, r_2, r_3 are also chosen to be different than the running index *i*. Therefore, *NP* must be greater than or equal to four to allow this condition.

3.2 CROSSOVER

In order to increase the diversity of the perturbed parameter vectors, crossover is introduced. So a trial vector

$$u_{i,G+1} = (u_{1i,G+1}, u_{1i,G+1}, \ldots\ldots, u_{Di,G+1}) \qquad (2)$$

is formed such that

$$u_{ji,G+1} = \begin{cases} v_{ji,G+1} & \text{if } (ran(j) \le CR) \text{ or } j = rnbr(i) \\ x_{ji,G} & \text{if } (ran(j) \rangle CR) \text{ and } j \ne rnbr(i) \end{cases} \qquad (3)$$

where *ran(j)* is a uniform random number, *CR* is the crossover probability, and *rnbr(i)* is a randomly chosen index which ensures that the trial vector gets at least one parameter from the mutated vector.

3.3 SELECTION

To decide whether it should become a member of generation *G*+1, the cost trial vector is compared to the cost of the target vector. If trial vector yields a smaller cost than the target vector, it replaces the target vector in the next generation.

4 PROBLEM FORMULATION

In aerodynamics, lift to drag ratio is called the aerodynamic efficiency. The objective of many airfoil design approaches is to maximize the ratio. But since drag is informally named as the price that one pays to obtain lift, the reciprocal of aerodynamic efficiency can be called a “cost” function and the maximization problem can be turned to a minimization problem. In a typical flow lift to drag ratio of an airfoil is related to flow angle of attack, Reynolds number, Mach number and the shape. For constant low speed flows where density fluctuations are relatively small, Mach number effects can be neglected and Reynolds number can be fixed to a certain value. This approximation leaves only two independent variables: Angle of attack and body shape. The shape of an airfoil is defined using many parameters; maximum thickness, camber distribution, leading edge radius, etc… A typical airfoil with these parameters can be seen in Figure 1.

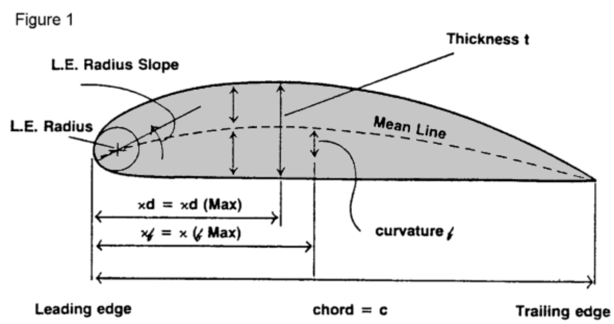

Figure 1. Airfoil profile with shape parameters.

After the Second World War NACA scientists developed a theory of combining mean lines and thickness distribution to obtain desired airfoil shapes. This theory made it possible to obtain airfoil shapes using only three parameters.

1. Maximum thickness,
2. Maximum camber
3. Location of maximum camber.

Knowledge of these three parameters along with the angle of attack is sufficient to obtain the lift the drag ratio of a particular airfoil. Therefore, the problem reduces to a four-parameter single objective optimization problem with drag to lift ratio (D/L) being the objective to be minimized.

Optimization process starts with an initial population of airfoils. The shape parameters are either randomly selected or picked from a known set of airfoils. Then population members are evaluated using the flow solver described above. Shape optimization procedure is performed using the Differential Evolution method. For this purpose a FORTAN code is developed consisting of two parts; DE and flow solver parts. The DE part of the code is developed based on the theory described by Storn and Price[2]. The latter part is developed by combining a first order panel code and a boundary layer code which solves Falkner-Skan equation. The DE part of the code initializes the population and evaluates them by calling the flow solver part. After that it generates the trial vectors are generated using the mutation and crossover operators. Later on the trial vectors are evaluated and a greedy selection operation performed between the trial and target vectors to generate the next generation. The loop continues until the maximum number of generations is reached.

As it has been mentioned above, the problem is subject to three constraints: Minimum allowable lift coefficient, minimum and maximum moment coefficients, and maximum allowable angle of attack. First and second constraints are handled by penalizing the infeasible solutions using linear penalty functions. For these cases the minimum allowable lift coefficient is chosen to be 0.3, and upper and lower bounds of the moment coefficient are selected as 0.0 and -0.1, respectively. For the third constraint the flow angle of attack is not allowed to exceed 5 degrees to prevent any possible flow separation which will increase drag and decrease lift.

5 RESULTS AND DISCUSSION

The algorithm is tested for different population sizes, amplification factors, *F*, and crossover rates, *CR*. First, all the crossover rate is set to 0.9 and solutions with different amplification factors and population sizes are compared and displayed in Figure 1. Here the initial population is generated by randomly selecting the decision variables.

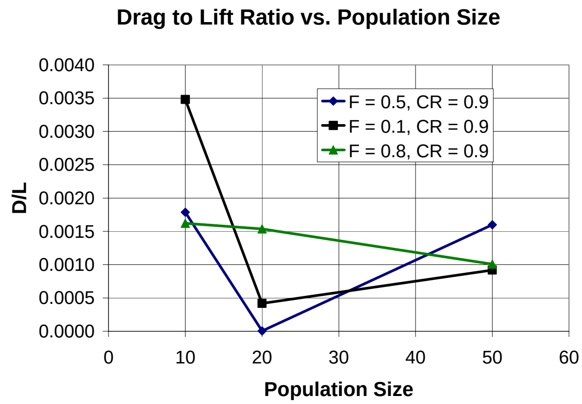

Figure 1. Comparison of population sizes for different amplification factors. ($CR = 0.9$)

It is clear from Figure 1 that the best solution is obtained when $F = 0.5$ and $NP = 20$. According to Figure 1 and reference 2, the optimum population size and DE parameters were selected to be:

$$NP = 20$$

$$F = 0.5$$

$$CR = 0.9$$

Based on these settings, the evolution of D/L is shown in Figure 2.

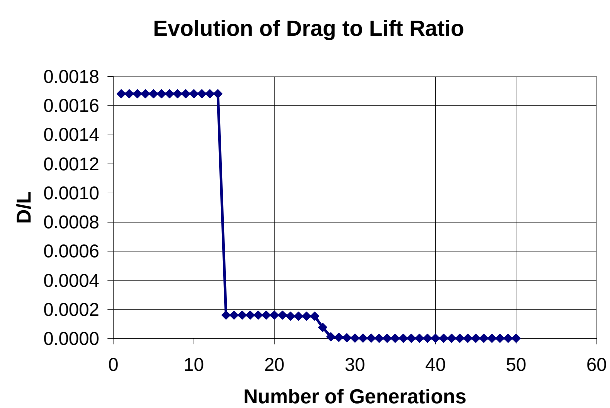

Figure 2. Evolution of D/L

Drag to lift ratio, lift coefficient and moment coefficient of the best airfoil are listed in Table1.

Table 1. Properties of the best airfoil

D/L	C_l	C_m
$3.25x10^{-6}$	0.823477	-0.0964

According to Figure 2, the evolutionary algorithm successfully decreased D/L by 99.8 %. It is also evident from Table 1 that the constraints were also satisfied. Since the algorithm employs a greedy selection, the best solution keeps surviving until been beaten. This leads to a piecewise continuous evolution curve which can be seen in Figure 2.

The best members of the initial and the final populations and their pressure distributions are displayed in figures 3 and 4.

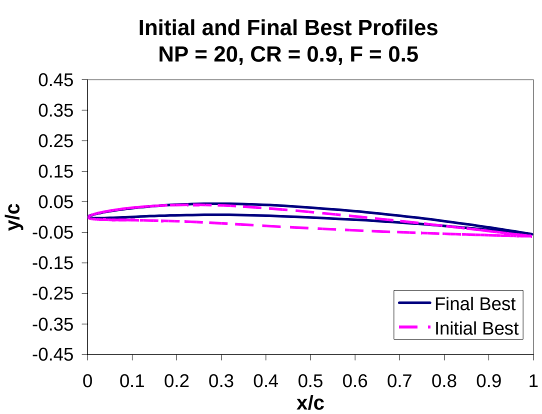

Figure 3. Best airfoils of the initial and final population.

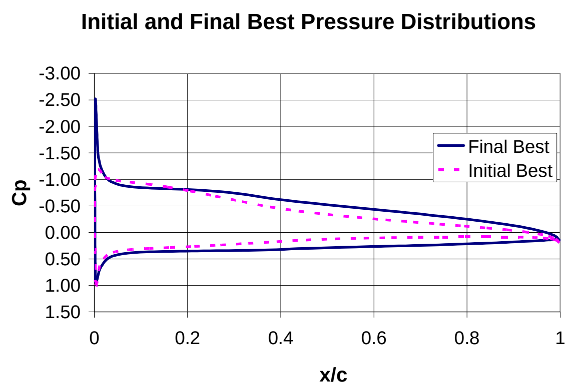

Figure 4. Pressure distributions of the best airfoils of initial and final populations.

In the solutions above, the initial population is generated randomly, in which airfoil shape parameters and flow angle of attack are selected using random numbers. The airfoils of the initial population satisfied the constraints. Therefore, they can be easily considered as a reliable population. But it would also be interesting to see the performance of the algorithm when the initial population is constructed using some known airfoils. For his reason, a population of 20 airfoils is initiated using the NACA profiles listed in Table 2.

Table 2. Initial NACA profiles.

0003	1403	2203	4403
0006	1406	2206	4406
0009	1409	2209	4409
0012	1412	2212	4412
0015	1415	2215	4415

The flow angle of attack is selected randomly just like the previous case.

Figure 5 shows the evolution history of the D/L.

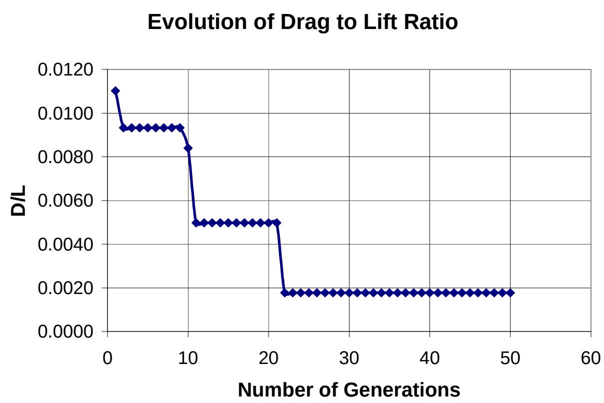

Figure 5. Evolution of D/L

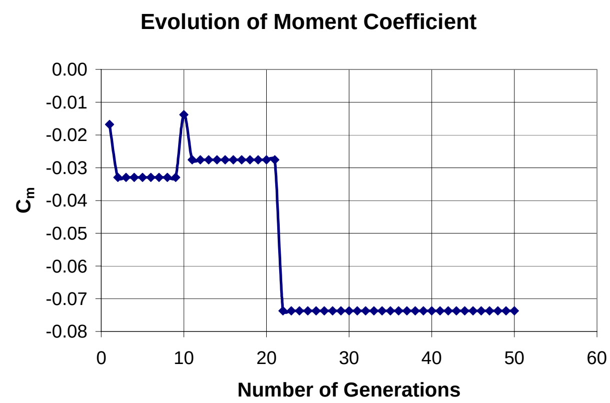

Figure 7. Evolution of moment coefficient.

According to the figure, the algorithm dropped D/L by 83.9 %. Although not as good as the previous case, this result is really satisfactory. The variation of lift and moment coefficients during the evolution are displayed in figures 6 and 7

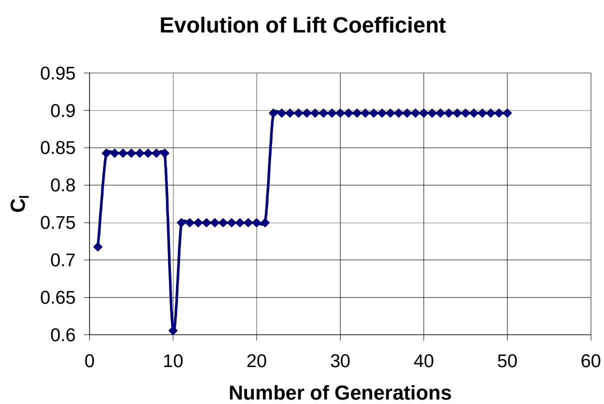

Figure 6. Evolution of lift coefficient.

Figures 6 and 7 clearly showed that the constraints are satisfied all the time. The optimum airfoil found after the evolution had the maximum lift coefficient. It had a slightly larger moment (magnitude wise) but it is still in the acceptable limits.

Figures 8 and 9 show the optimum airfoils at the initial and final populations, respectively.

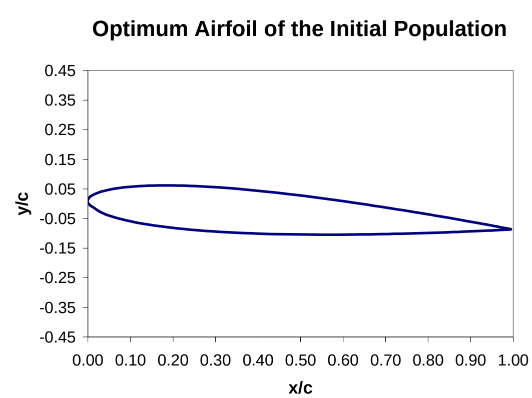

Figure 8. Optimum Airfoil at the initial population

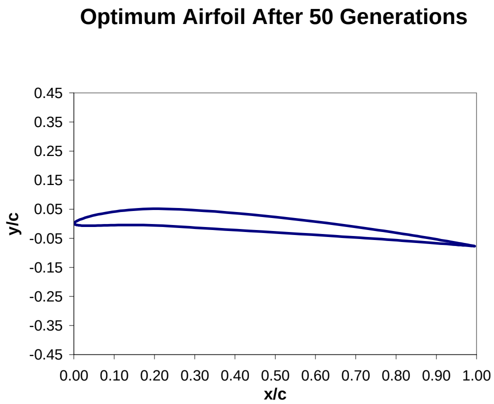

Figure 9. Optimum airfoil after 50 generations.

Since the airfoils are drawn to scale in figures 8 and 9, one can easily realize the changes in the shape parameters and the flow angle of attack. The pressure distributions of these airfoils are available in figures 10 and 11.

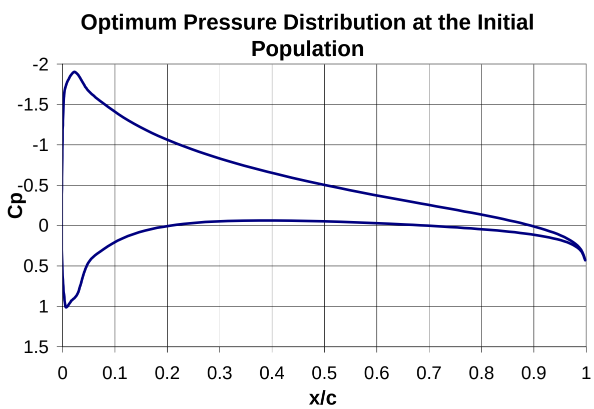

Figure 10. Optimum pressure distribution at the initial population

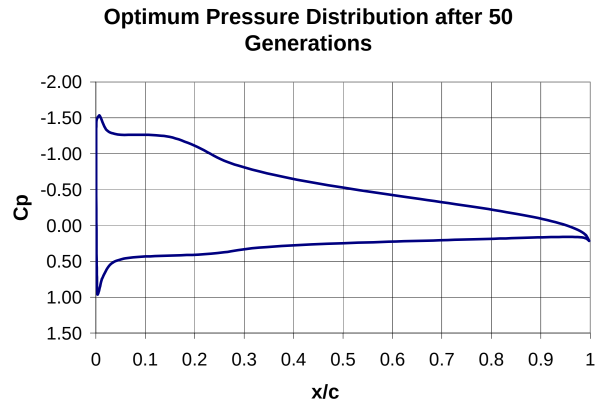

Figure 11. Optimum pressure distribution after 50 generations.

The following conclusions can be made after a detailed examination of figures 10 and 11:

The area between the upper and lower part of the Cp curve gives the lift coefficient of an airfoil. For the optimum airfoil of the initial population most of the lift is generated close to the leading edge, while it is more evenly distributed for the final optimum. This clearly decreases the drag coefficient. Unfortunately, this situation increases the moment generated by the airfoil. But although relatively higher, the moment coefficient of the final airfoil is still in the acceptable range.

6 CONCLUSIONS

Airfoil shape optimization was for high aerodynamic performance is performed using Differential Evolution algorithm. In the solutions drag to lift coefficient is minimized while satisfying the constraints on lift coefficient, moment coefficient and flow angle of attack. In the first part of the computations, initial population was generated randomly. For this case optimum drag to lift ratio was minimized by 99.8 %, which was really amazing. In the second part, initial population was constructed using some well-known NACA profiles. For this case the optimum drag to lift ratio was dropped by 83.9 %. Although not as high as the first case, this is also a very significant decrease.

The results clearly showed that Differential Evolution can be effectively used for airfoil shape optimization. The FORTRAN code developed using this algorithm is very fast and robust. It only takes a couple of minutes to run on Pentium III processors.

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