Phased Linear Stochastic Array Synthesis via hybrid Particle Swarm Optimization

Zikri Bayraktar

Electrical Engineering Department
The Pennsylvania State University
University Park, PA 16802

Abstract

A hybrid Particle Swarm Optimization (PSO) with uniform mutation and arithmetic crossover applied to optimization of four and six elements stochastic aperiodic phased antenna arrays. Comparison against the conventional arrays has shown that hybrid particle swarm can make the resistance curve flatter over the 30º to 90º scan angles range while keeping the Voltage Standing Wave Ratio (VSWR) of each array element below 2:1 by generating stochastic element structures and perturbing the element spacing.

1 INTRODUCTION

In the design of the phased arrays, designers commonly encounter the problem that the driving-point impedances of the array elements considerably change with the scan angle. A phased array is an n-element antenna array, which has independently powered elements. Each element can be driven by different source amplitude and phase. This allows the designer to steer the beam by driving each element with different phase.

The driving point impedance of a phased array element is a function of the self-impedance, the mutual impedance from the other array elements and the array excitation currents. This can be formulated as follows:

$$Z_n = Z_{n1}\left(\frac{I_1}{I_n}\right) + Z_{n2}\left(\frac{I_2}{I_n}\right) + \ldots + Z_{nN}\left(\frac{I_N}{I_n}\right) \qquad where\ \ n = 1,2,\ldots,N$$

Where Z_n is the driving point impedance of the n^{th} array element, Z_{nN} is the mutual impedance between two elements and I_n is the element excitation current. Here, the mutual impedance depends on the distance from the adjacent array elements and self impedance depends on the shape of the individual element.

In many optimization applications, only the pattern of the antenna is considered without regard to the resulting driving point impedances. This leads to impractical values, which cannot be implemented. Bray et al. (2002) has utilized a Genetic Algorithm (GA) to implement driving point restriction on each element. They took eight elements phased linear dipole arrays and perturb the element distances. One should note that the real part of the driving point impedance is related to the radiated energy and the imaginary part is related to the stored energy on the antenna. Minimizing the imaginary part of impedance reduces the stored energy, and increases the radiation, which is the main task of an antenna. Also, Voltage Standing Wave Ratio (VSWR) is a measure of how well the antenna matches to the transmission line. VSWR 1:1 means that all the power from the transmission line is received by the antenna and radiated. VSWR 2:1 means only 10% of the power is reflected back to the transmission line, which is a good percentage for practical applications.

Bray et al. managed to restrict the real part of the impedance (resistance) to be in the range of 30 and 130 Ω over the entire desired scanning range. From the matching network point of view, placing restrictions on the resistance over the scan range simplifies the design of matching network for each element so that VSWR is below 2:1. In this paper, hybrid PSO is first given the task to find stochastic aperiodic arrays, which has resistance restriction from 40 to 80 Ω. Then, second fitness function, independent from the first fitness function, is set to optimize the VSWR to be less than 2:1. Various configurations are implemented with hybrid PSO.

This paper will continue with a short analysis of a conventional four elements periodic phased dipole array and a random seed analysis on hybrid PSO. In the third section, the stochastic array structure, Particle Swam Optimization and hybridization will be explained. Section four will be split into two parts. In the first part, a parallel grid similar elements configuration results on only resistance optimization will be presented. Second part of section four will talk about 2:1 VSWR optimization, in which the reactance part of the input impedance is included into the optimization. Section five will present various configurations. First, planar grids similar elements; second, parallel grids different elements; and third, planar grids different elements configuration results will be presented. In the second and third cases, the dimension of the problem is four times of the first configuration due to different element structures. This section will conclude with six elements array optimization results. This paper will end with conclusions and future work section.

2 CONVENTIONAL FOUR ELEMENTS PHASED DIPOLE ARRAY

One first should look at the driving point impedance characteristics of four elements phased dipole array to have a fair comparison between the stochastic structures. Below figure displays the four elements periodic phased array where element spacing is half wave length.

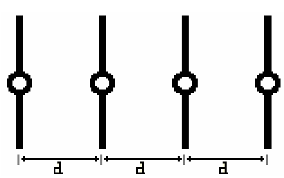

Figure 1: Periodic (d=λ/2) phased dipole array

Figure 2 plots the resistance verses scan angle curves of this conventional periodic (d=0.5λ) phased linear dipole array and Figure 3 plots the VSWR verses scan angle curves of the same conventional array.

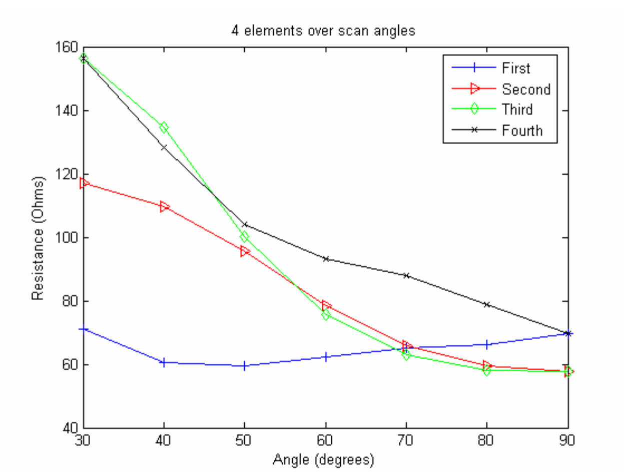

Figure 2: Resistance of periodic (d=λ/2) four elements phased linear dipole array vs. scan angle

As it is seen from the above figure, the resistance (real part of the impedance) of each array element varies drastically as the scan angle changes. The range of the resistance is between 55 and 160 Ω. As the impedance varies, the VSWR changes accordingly.

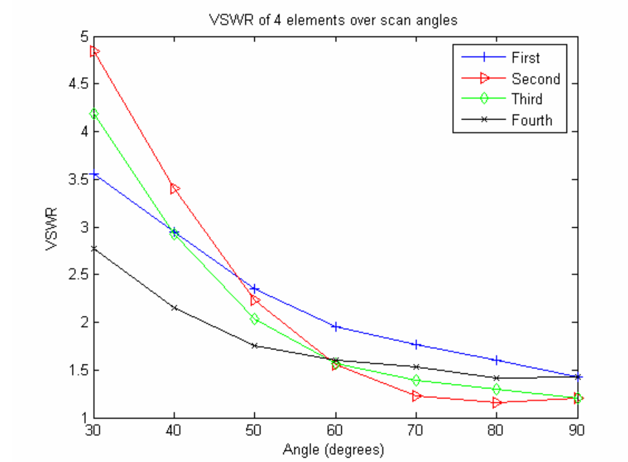

Figure 3: VSWR of periodic (d=λ/2) four elements phased linear dipole array vs. scan angle

A random seed analysis is conducted in order to account for the seed effect in the hybrid PSO. A population of 30 particles was allowed to run for 5,000 generations (150,000 fitness evaluations) with 15 different seeds. The results are plotted below in Figure 4. Three different optima points are observed; two of them were dominated by the third optimum. Seven random seeds out of fifteen have converged to this optimum, which has better fitness than other two optima. Value of X on Figure 4 is the number of function evaluations and Y is the fitness value at that optimum. Figure 4 shows that the results presented here will be seed dependent, the fitness values are not global best optima and if there is a better seed, better results may be found.

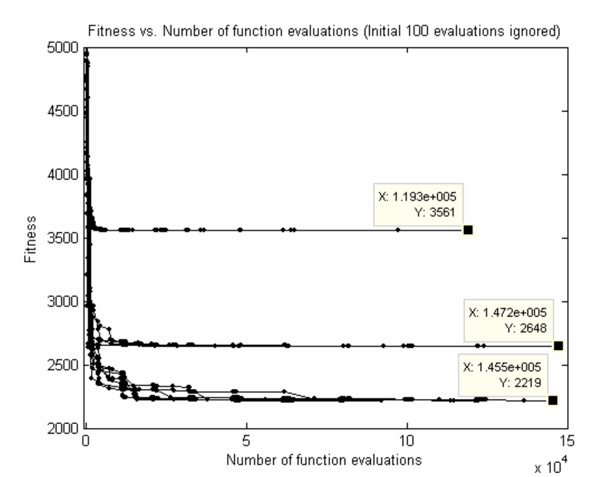

Figure 4: Random seed analysis

3 STOCHASTIC STRUCTURE AND HYBRADIZATION

3.1 STOCHASTIC ARRAY STRUCTURE

Stochastic grid antenna element structure is invented and investigated in Computational Electromagnetics and

Antenna Research Laboratory at PennState. Previously, single element dipoles, crossed dipoles (Werner, 2002) and three elements Yagi-Uda arrays (Bayraktar, 2004) were successfully optimized for performance and miniaturization.

Figure 5 displays the predetermined grid planes (grey) and four elements stochastic phased planar array structure (black). Each grid consists of nine columns and 21 rows. The three rows in the middle are used to place a voltage source. This leaves nine rows above the source and nine rows below the source. Due to symmetry respect to the source, only one side will be parameterized and included in the optimization. Thus, each grid has nine parameters to optimize. From the center of the first grid to the center of the second grid is the first distance (d_1) and other distances between adjacent elements are named similar to the first one as d_n. For a four different elements configuration the number of parameters (number of dimension of the search space would be 9x4 + 3 = 39).

The same grid structure is used in parallel grid configuration too. In this case though, grids are aligned in such a way that they look like parallel surfaces to each other. The distance between each adjacent grid is represented by d_n.

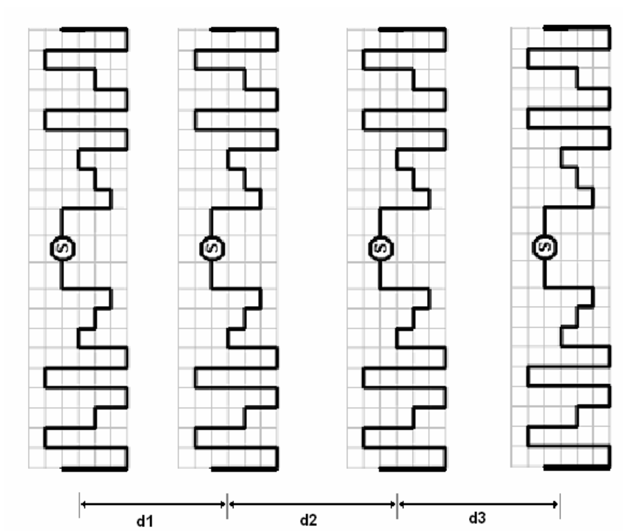

Figure 5: Four elements stochastic phased planar array

3.2 PARTICLE SWARM OPTIMIZATION (PSO)

PSO is one of the newest population based evolutionary optimization techniques which was introduced to the literature in 1995 (Kennedy et al. 2001). Unlike the Genetic Algorithms, PSO does not utilize genetic operators such as crossover, mutation or selection. PSO has position and velocity operators, which are simply algebraic equations. The members of the population are called particles. Initially, each particle is assigned random position and random velocity on each dimension of the problem. Then the position of the each particle is evaluated and fitness is assigned based on this evaluation. After finding the personal best positions (pbest) and population's best position (gbest), positions and velocities are updated based on the below operators:

$$V_{i,d}^{next} = (w \times V_d^{now}) + c1 \times (p_{best,d} - X_d^{now}) + c2 \times (g_{best,d} - X_d^{now})$$

where, $V^{next}_{i,d}$ is the updated velocity for i^{th} particle in the d dimension, ω is the nostalgia constant, V^{now}_{d} is the current velocity in the d dimension, c1 and c2 are randomly generated constants in the range of [0,1], X^{now}_{d} is the current position in the d dimension, $p_{best,d}$ is the personal best of the i^{th} particle in the d dimension and $g_{best,d}$ is the global best of the whole swarm in the same dimension. The position update equation is as follows:

$$X_d^{next} = X_d^{now} + \Delta t \times V_d^{next}$$

where X^{next}_{d} is the updated position in the d dimension, and the time step (Δt) is usually chosen to be one for simplicity. The value of ω is linearly decreased from 0.9 to 0.4 throughout the search by this equation:

$$\omega = 0.5 \times \frac{(\max \# \; of \; iterations - iteration \; \#)}{iteration \; \#} + 0.4$$

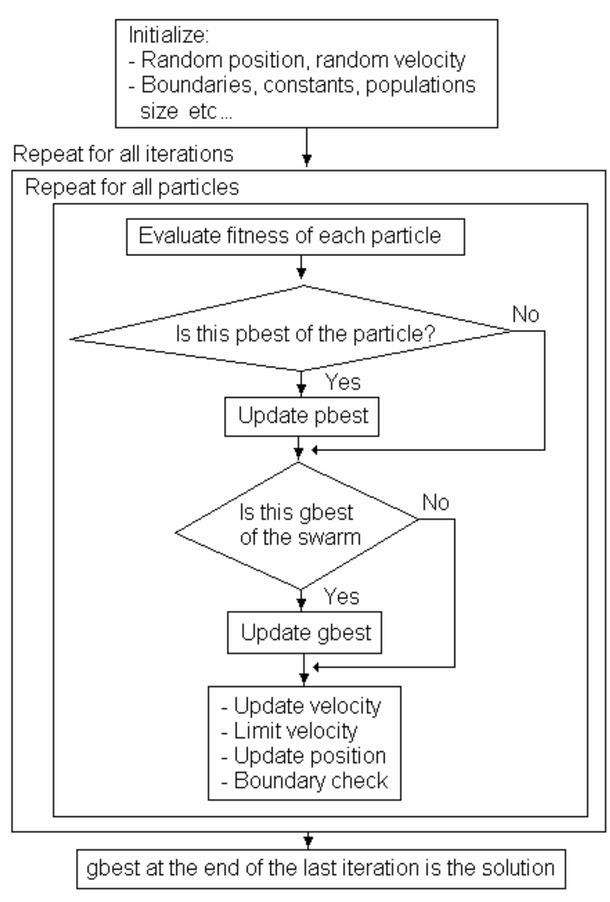

Figure 6: Particle Swarm Algorithm Flowchart

The fitness evaluation and operator update loop continues as shown in Figure 6. After every 15 generations, either arithmetic crossover or mutation (in turns) is applied to the swarm in order to increase the diversity among the particles. Both the position and the velocity are undergone boundary limiting. The position is limited to be in the range of [0, 1] and the velocity to be in the range of [-1, 1]. Yet, these boundaries interpolated to the discrete grid structure and to the continuous element spacing. If the particle goes out of the search space, its position is set to the nearest boundary. If the velocity is increased too much a simple velocity limit rule is applied as shown below:

$$\text{If } \left(\left|V^{next}{}_{i,d}\right| > V_{\max}\right) \text{ then } \quad V^{next}{}_{i,d} = \frac{V_{\max} \times V^{next}{}_{i,d}}{\left|V^{next}{}_{i,d}\right|}$$

With an ideal population size of 30 particles, the code is allowed to run minimum of 300,000 fitness evaluations before it terminates.

3.3 HYBRIDIZATION

Due to the greedy search habit of PSO, genetic operators are introduced to the solution vector. In order to encourage diversity over the search, arithmetic crossover (Tsai et al. 2004) and uniform mutation are implemented as explained below.

3.3.1 Arithmetic Crossover

The crossover operators used here are taken from (Tsai et al. 2004) in which the one-cut-point crossover integrated with an arithmetical operator derived from convex set theory. This operator randomly picks two particles as parents and selects one cut-point. Then it swaps the right parts of two parents after the cut-point, and calculates the linear combinations at the cut-point to generate new offspring particles. For example, let two parents be $x = (x_1, x_2, ..., x_N)$ and $y = (y_1, y_2, ..., y_N)$. If they are crossed after the m^th position, the resulting offspring (children) are

$$x^c = (x_1, x_2, ..., x_M^c, y_{M+1}, y_{M+2}, ..., y_N)$$
$$y^c = (y_1, y_2, ..., y_M^c, x_{M+1}, x_{M+2}, ..., x_N)$$

Where $x_M^c = x_M + \beta(y_M - x_M)$ and $y_M^c = L_M + \beta(U_M - L_M)$ L_M and U_M are the domain of y_M and β is a random value.

3.3.2 Uniform Mutation

Another operator that used in turns with arithmetic crossover is a uniform mutation operator. This operator takes the position vector of each particle and introduces a uniformly generated random noise. The range of the generated random noise is limited by [-*U/2 U/2*] where the search spaces is bounded by [0 *U*].

4 PARALLEL GRIDS SIMILAR ELEMENTS CONFIGURATION

This configuration is constructed simply taking the structure in Figure 5 and rotating the grids 90 degrees, so that each grid looks like parallel surfaces to each other. This configuration allows the PSO to place the elements as close as 0.021λ, if the array performs well.

The impedance of an array element is represented as $Z_n=R_n+jI_n$, where R_n is the real part and I_n is the imaginary part of the impedance. With this configuration two different fitness functions are considered. In the first fitness function, PSO was to optimize the real part of the impedance (resistance) between 40 and 80 Ω, without taking the imaginary part into account. In this fitness function it is assumed that the imaginary part of the impedance can be taken care of with a matching network.

First fitness function is as follows:

$$Fitness = \sum_{\theta=30}^{90} \sum_{n=1}^{N} \{\alpha \,|\, R_n(\theta) - 60 \,|+ \beta \,|\, R_n(\theta) - R_n(\theta - 10) \,|\}$$

where $R_n(\theta)$ is the driving point resistance of the n^th array element when the beam is scanned to an angle of θ from endfire, α and β are parameters that have to be experimentally determined so as to produce the best driving point resistance curves.

In the second fitness function, PSO was to optimize VSWR to be below 2:1. VSWR calculation is given below and a ratio of 2:1 means that 90% of the power received from the transmission line is radiated properly by the antenna. VSWR optimization either simplifies the matching network or removes the need for it.

Second fitness function is as follows:

```
if (VSW(J).GT.2) then
       VSWR(K) = VSWR(K) + (100x(VSW(J)-1)^2)
else
       VSWR(K) = VSWR(K) + |VSW(J)-1|
end if
```

VSWR can be calculated as follows:

$$\Gamma(J) = \frac{Z_n(J) - 60}{Z_n(J) + 60} \qquad where \quad Z_n = R_n + jI_n$$

$$VSWR = \frac{1 + |\Gamma(J)|}{1 - |\Gamma(J)|}$$

Figure 7 plots the resistance verses scan angle curves for each element for the first fitness function. Hybrid PSO had found a parallel similar elements configuration as shown below, which has resistance values bounded by 40 and 80 Ω.

Common coax transmission line impedances are 50 and 75 Ω. Thus, in the VSWR calculations, the value of 60 Ω is used because it is a good approximation to both these impedance values.

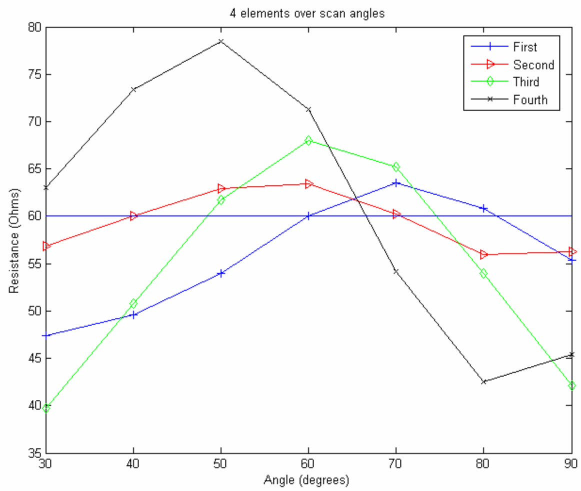

Figure 7: Resistance (R_n) optimization of four elements parallel grids configuration

When the second fitness function is implemented with the same configuration, hybrid PSO optimized the structure to make the VSWR below 2:1 for all elements at all scan angles. Figure 8 displays the VSWR optimization results, which is better than the conventional periodic (d=λ/2) phased dipole array shown in Figure 3.

VSWR is optimized in the expense of a wider range of resistance when it is compared to only resistance optimization in the first fitness function. The resistance values for the VSWR optimized array (Figure 8) are shown in Figure 9 and it can be seen that the values fluctuate between 35 and 110 Ω. Although the range of resistances is worse than the only resistance optimized case (range of [40 to 80 Ω]), it performs better than periodic phased dipole which was analyzed in section 2.

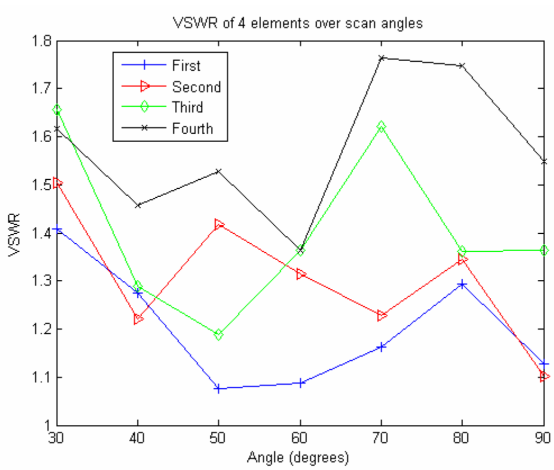

Figure 8: VSWR optimization of four elements parallel grids configuration

Unlike the half-wave dipole arrays, stochastic structure that presented above is reduced 16% in projected length. The projected length of each element is 0.42 λ, which helps to reduce the resistance values. The element distances are restricted to be minimum 0.2 λ and maximum 1.8 λ. This allowed the PSO, to perturb distances freely and reduce mutual coupling between elements in order to meet the design objectives.

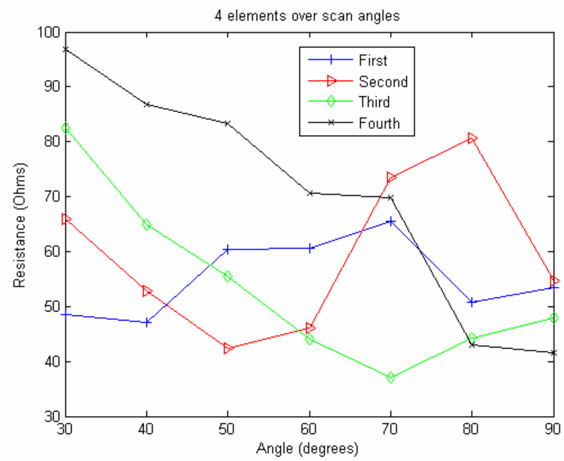

Figure 9: Resistance vs. Scan Angle plot of VSWR optimized four elements parallel grids configuration

5 OTHER CONFIGURATIONS

In addition to the configurations presented so far, three other configurations are inspected for four element arrays and two for six element arrays in this paper.

First of these three structures is planar grids similar elements configuration in which each element has the same shape and grids are placed on the same plane as it is in Figure 5. The projected length of this array is 0.42 λ and the boom length is 5.12 λ which is longer than half-wave spaced dipole arrays (1.5 λ). VSWR plot on Figure 10 shows that all elements have a VSWR of less than 1.5:1 at all scan angles. Significance of having VSWR close to 1:1 ratio is that power delivered by the transmission line is not reflected back to the power source. The resistance curves are more flat compared to the periodic phased dipole in the range of 40 and 75 Ω.

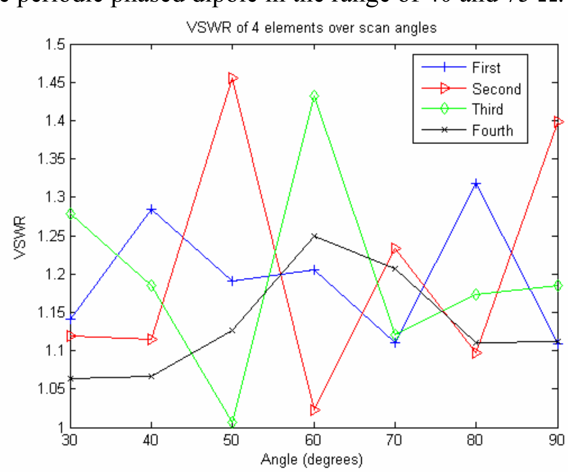

Figure 10: VSWR optimization of four element planar grid configuration

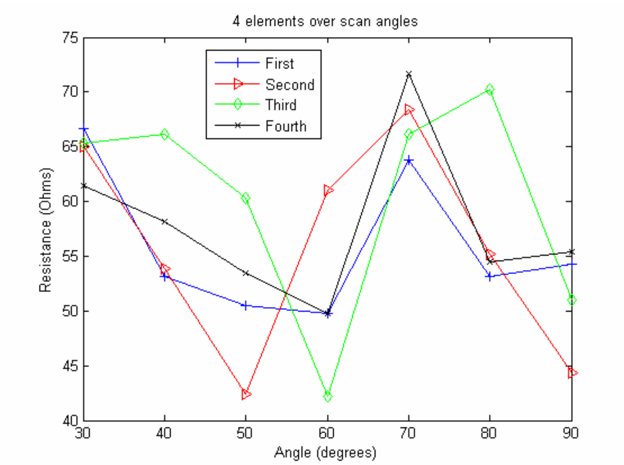

Figure 11: Resistance vs. Scan Angle plot of VSWR optimized four elements planar grid configuration

In another configuration, grids are aligned parallel, in which each element has a different shape. The projected length of each element is 0.42 λ with a boom length of 3.428 λ. Boom length simply is the length of the array from the tip of the first array element to the tip of the last array element. The array is shown in Figure 12. Different element shapes lead to different self-inductances and different element spacing leads to different mutual couplings between elements which may lead to better optimization. One should point out that different element configurations are more difficult optimization problems. While in similar elements array optimization the number of dimensions is 12, this number rises to 39 for different elements array optimization.

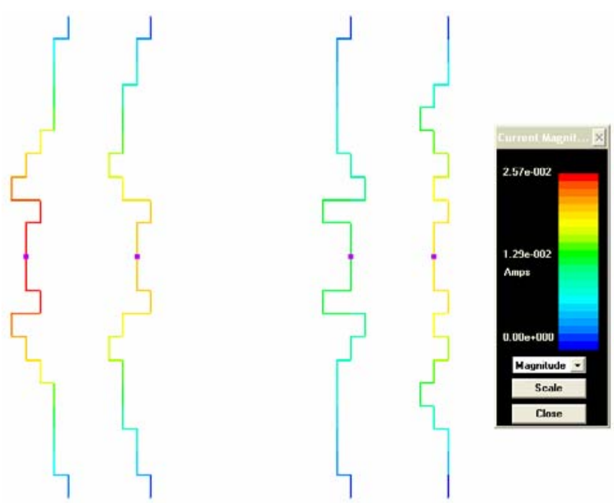

Figure 12: Four elements different shaped parallel grids phased array configuration

Although it is not clear in Figure 12, element grids are parallel to each other. It also displays the magnitude of the currents on each wire. Purple dots on each element represent the voltage sources with uniform magnitude but different phase.

The structure shown in Figure 12 is optimized by the hybrid PSO for 2:1 VSWR ratio and the VSWR verses scan angle plot is shown in Figure 13.

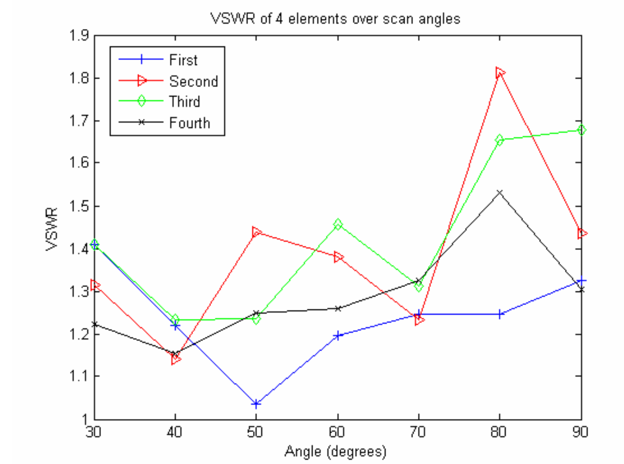

Figure 13: VSWR optimization of four different elements parallel grid configuration

Having VSWR ratio less than 2:1 is important to reduce the reflected power from the antenna, but having a flat resistance curve is as important as VSWR optimization. While PSO keeps the VSWR below 2:1 ratio, the resistance curves are in the range of 40 and 86 Ω shown in Figure 14, which is very close to the target range aimed in this paper (acceptable range is 40 to 80 Ω).

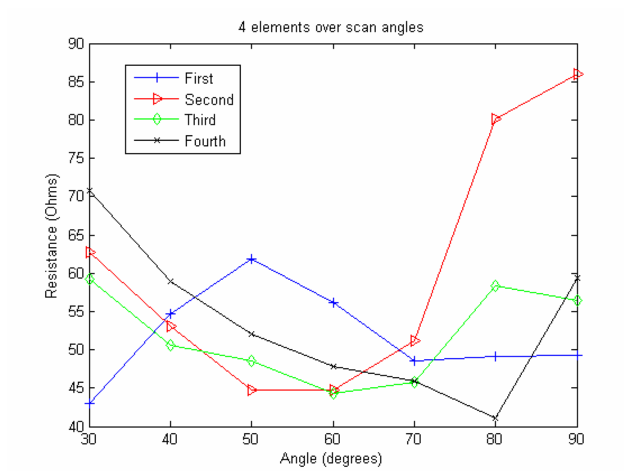

Figure 14: Resistance vs. Scan Angle plot of VSWR optimized four different elements parallel grids

Last two configurations will be on six elements stochastic phased arrays. As the number of the elements increase, the mutual coupling among the array elements increases also. The first six elements configuration is similar element parallel grids configuration. This configuration is only a 14 dimensional problem: 9 discrete valued dimensions for the shape of the similar elements and 5 continuous valued for the element distances. Before presenting the stochastic array results, one should look at the six elements phased periodic dipole array VSWR and resistance plots.

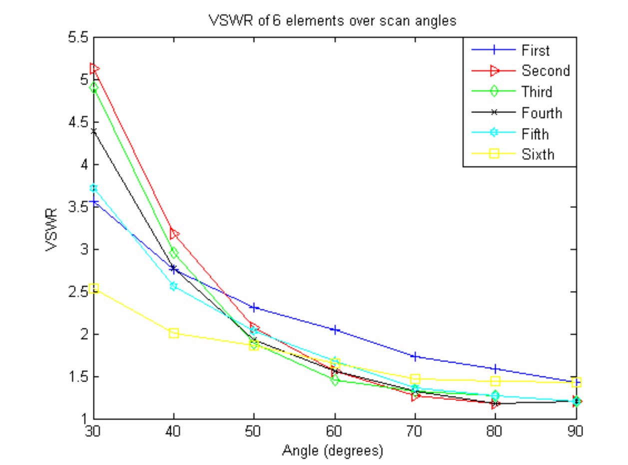

Figure 15: VSWR vs. scan angle plot of phased six elements periodic (d=λ/2) dipole array

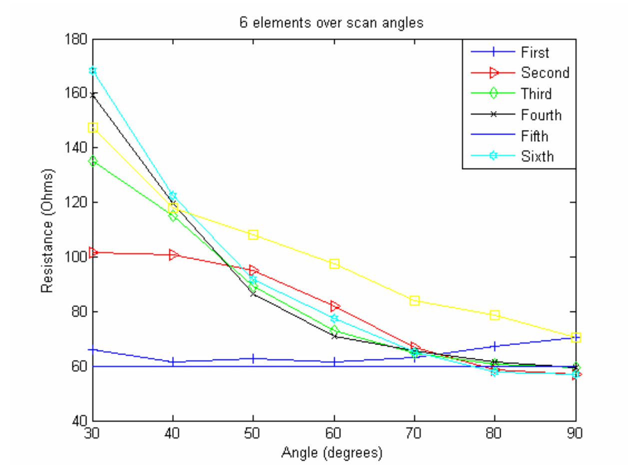

Figure 16: Resistance vs. scan angle plot of phased six elements periodic (d=λ/2) dipole array

The VSWR values of conventional phased periodic dipole array goes up to 5.2:1 ratio at some scan angles and the resistance values fluctuate between 55 and 170 Ω.

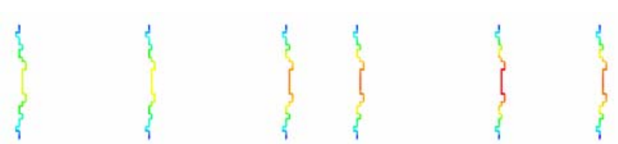

Figure 17: Six elements parallel grids stochastic phased array

When compared to the four elements case, six elements array, shown in Figure 17, has a boom length of 4.35 λ, which is relatively short compared to some of the four elements arrays presented above. However, it still is longer than six elements periodic phased dipole array which has 2.5 λ boom length.

When the VSWR vs. Scan angle plots are compared, it is clear that stochastic phased arrays are better than the conventional phased dipole arrays and all elements are at or below 2:1 ratio at all scan angles as seen in Figure 19. The resistance curves are plotted in Figure 18 and they vary in the range of 35 to 95 Ω, which offers smaller fluctuation in resistance values than conventional array.

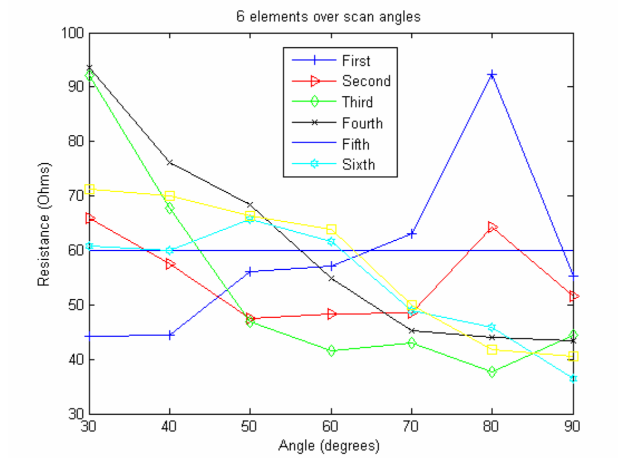

Figure 18: Resistance vs. Scan Angle plot of VSWR optimized six similar elements parallel grids array

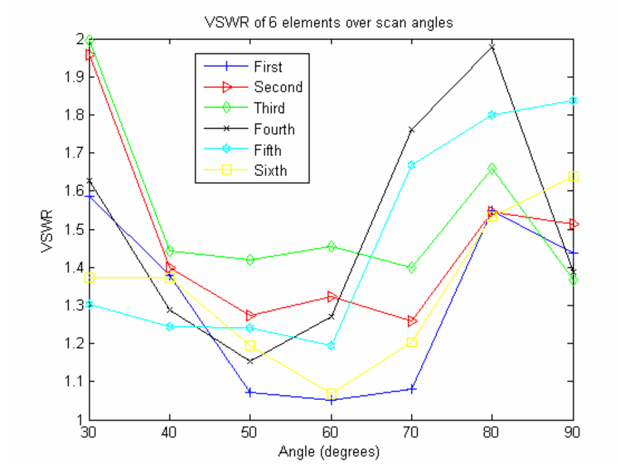

Figure 19: VSWR optimization of six similar elements parallel grids array

The last configuration is six elements planar grids array, which is also optimized for 2:1 VSWR ratio. Hybrid PSO again found a solution for this configuration which is presented in Figure 21. VSWR is below 2:1 ratio for all elements at all angles and the resistance curves are limited by the resistance values of 35 and 95 Ω.

Figure 20 displays the number of fitness evaluations of a swarm with 30 particles which is used to optimize the last configuration. Mutation operator is applied after 5000th function evaluation and provided diversity to the population, which helped the particles to find better spots on the solution space. The fitness is 596.3 before mutation and the fitness after mutation is 102.9, which clearly demonstrates the usefulness of hybridization in PSO. The global optimum point has a fitness of 18.32.

PSO is a greedy search algorithm, which is highly elitist that particles move towards both their personal best coordinates and global best coordinates. All particles eventually converge to an optimum or land very close to it, which sets the velocity zero and makes it impossible for particles to get out of that optimum. If it is not the global optimum, particles get trapped at that local point. Mutation can help particles get out of local optimum and continue their search.

Crossover served as a local tool to search around the optimum in finer details. After 15 generations (~ 4500 fitness evaluations) particles tend to accumulate around a spot on the search space. This causes velocity to decrease to zero and particles simply freeze at their location. On the other hand, crossover helps particles to move another location around that optimum and particles continue their search in finer detail.

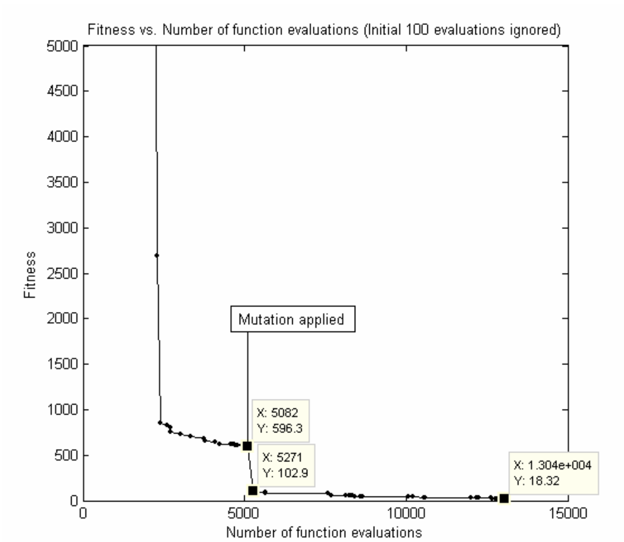

Figure 20: Fitness vs. Number of function evaluations for six similar elements planar grids array optimization

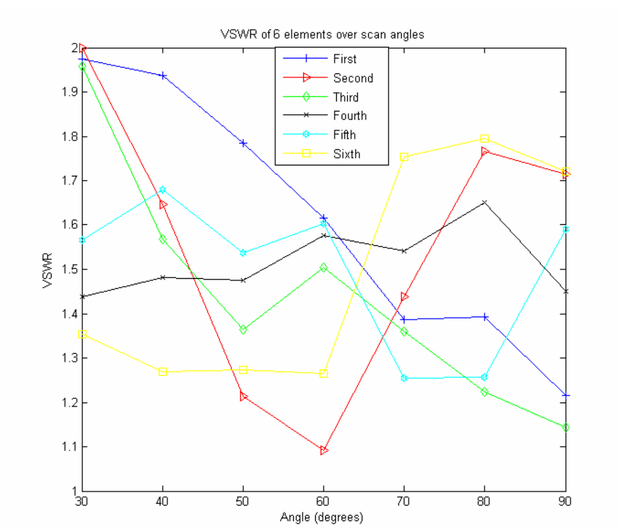

Figure 21: VSWR optimization of six similar elements planar grids array

6 CONCLUSIONS

Combination of hybrid particle swarm with stochastic structure gives promising results especially in low dimensional design problems. Optimized arrays those were presented above have a smaller range on resistance curves which are relatively flatter than the periodic dipole arrays. These stochastic designs also keep the VSWR below 2:1 for all array elements at all scan angles, which may reduce the cost of constructing matching networks. It is shown that crossover and mutation can help PSO to get out of local optima or do finer search around the optimum points.

7 FUTURE WORK

Similar to the four different elements configurations, six elements configurations can be generated. The only difference between four and six elements is the number of dimension of the search spaces. In four different elements array, PSO works on 39 dimensional search space and in six different elements arrays PSO has to work on 59 dimensional search space. This represents a great challenge for the hybrid PSO.

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