



# Parameter estimation and regionalization for continuous rainfall-runoff models including uncertainty

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## Abstract

The prediction of streamflow at ungauged sites is one of the fundamental challenges to hydrologists today. While major progress has been made in the regionalization of statistical flow properties (e.g. extreme values), and methods for synthesis of event response at ungauged sites are widely applied, the estimation of continuous streamflow time-series is still very uncertain. The challenge of predicting the response at ungauged sites is often met through a process of parameter regionalization. Little attention has so far been given to the impact of new insights into model identification at gauged sites, e.g. regarding the problem of structural error, on this regionalization process. Questions that are addressed in this paper are the following: (1) What is the relationship between local parameter identifiability and catchment characteristics? (2) How is the uniqueness of catchments reflected in regionalization? (3) What is the result of local model structural uncertainty on the regionalization result? (4) How can we propagate local parameter uncertainty into predictions in ungauged basins and what is the result? A case study of 10 catchments located in the southeast of England is utilized to deal with these questions. The main conclusions from this study are that the uncertainty in the locally estimated model parameters is a function of their importance in representing the response of a given catchment, model structural error hinders identification of a parameter to represent a certain process and therefore hinders the regionalization, and the uncertainty in the calibrated parameters can be propagated to ungauged sites using a weighted regression approach.

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## 1. Introduction

Rainfall-runoff (RR) models are commonly used tools to extrapolate streamflow time-series in time and space for operational and scientific investigations. RR models allow, for example, to extend available streamflow records in time to predict the behavior of

catchments for different climate scenarios. Through extrapolation in space, they enable us to simulate the response of catchments for which little or no time-series of streamflow measurements are available. The latter has been achieved with some success in terms of predicting the response of a catchment to an individual rainfall event, using (parametrically) simple models (e.g. Natural Environment Research Council NERC, 1975; McCuen, 1982). However, water resource and flood management hydrological

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problems are increasingly approached using continuous time RR modelling (e.g. Lamb, 2000; Cameron et al., 2001; Blazkova and Beven, 2002; Lamb and Kay, 2004), rather than traditional statistical or event-based models (e.g. NERC, 1975; Pilgrim, 1983; Houghton-Carr, 1999). The reliable estimation of continuous streamflow time-series in ungauged catchments has remained a largely unsolved problem so far (Wagener et al., 2004), although significant insights have been gained in recent years as discussed below. The establishment of the Prediction in Ungauged Basins (PUB) initiative by the International Association of Hydrological Sciences (IAHS) shows that much has still to be done in this area (Sivapalan et al., 2003; Wagener et al., 2004).

Many, if not most, RR model structures currently used for continuous modelling can be classified as conceptual (CRR), if the classification is based on two criteria (Wheater et al., 1993): (1) the structure of these models is specified prior to any modelling being undertaken, and (2) (at least some of) the model parameters do not have a direct physical interpretation, in the sense of being independently measurable, and have to be estimated through calibration against observed data. Calibration is a process of parameter adjustment (automatic or manual), until catchment and model behavior show a sufficiently (to be specified by the hydrologist) high degree of similarity. The similarity is usually judged by one or more objective functions (OFs) accompanied by visual inspection of observed and calculated hydrographs (Gupta et al., 2004). It is usually assumed that the model parameters represent some inherent and time-invariant properties of the catchment under study.

Early attempts to model ungauged catchments simply used the parameter values derived for neighboring catchments where streamflow data were available, i.e. a geographical proximity approach (e.g. Mosley, 1981; Vandewiele and Elias, 1995). However, this seems to be insufficient since even nearby catchments can be very different with respect to their hydrological behavior (Post et al., 1998; Beven, 2000). Others propose the use of parameter estimates directly derived from, amongst others, soil properties such as porosity, field capacity and wilting point (to derive model storage capacity parameters); percentage forest cover (evapotranspiration parameters); or hydraulic conductivities and channel densities (time

constants) (e.g. Koren et al., 2000; Duan et al., 2001; Atkinson et al., 2002). The main problem here is that the scale at which the measurements are made (often from small soil samples) is different from the scale at which the model equations are derived (often laboratory scale) and at which the model is usually applied (catchment scale). The conceptual model parameters represent the effective characteristics of the integrated (heterogeneous) catchment system (including for example preferential flow), which are unlikely to be easily captured using small-scale measurements since there is generally no theory that allows the estimation of the effective values within different parts of a heterogeneous flow domain from a limited number of small scale or laboratory measurements (Beven et al., 2000). It seems unlikely that conceptual model parameters, which describe an integrated catchment response, usually aggregating significant heterogeneity (including the effect of preferential flow paths, different soil and vegetation types, etc.), can be derived from catchment properties that do not consider all influences on water flow through the catchment. Further fine-tuning of these estimates using locally observed flow data is needed because the physical information available to estimate a priori parameters is not adequate to define local physical properties of individual basins for accurate hydrologic forecasts (Duan et al., 2001). However, useful initial values might be derived in this way (Koren et al., 2000). The advantages of this approach are that the assumed physical basis of the parameters is preserved and that (physical) parameter dependence can be accounted for, as shown by Koren et al. (2000).

Probably the most common approach to ungauged modelling is to relate model parameters and catchment characteristics in a statistical manner (e.g. Jakeman et al., 1992; Sefton et al., 1995; Post et al., 1998; Sefton and Howarth, 1998; Abdullah and Lettenmaier, 1997; Wagener et al., 2004; Merz and Blöschl, 2004; Lamb and Kay, 2004; Seibert, 1999; Lamb et al., 2000; Post and Jakeman, 1996; Fernandez et al., 2000), assuming that the uniqueness of each catchment can be captured in a unique combination of catchment characteristics. The basic methodology is to calibrate a specific model structure, here called the *local model structure*, to as large a number of (gauged) catchments as possible and derive statistical (regression) relationships between (local)

model parameters and catchment characteristics. These statistical relationships, here called *regional models*, and the measurable properties of the ungauged catchment can then be used to derive estimates of the (local) model parameters. This procedure is usually referred to as regionalization or spatial generalization (e.g. Lamb and Calver, 2002).

This paper addresses both theoretical and applied issues related to the regionalization of continuous lumped and parsimonious (parameter efficient) CRR models. The principle of parameter regionalization is discussed, and current problems and possible directions of improvement are identified and evaluated using 10 catchments located in the southeast of England. The objective is not to derive reliable regional model equations, which is unlikely to be robust with the small number of catchments used, but to investigate the effect of parameter and model structure identification issues on the regionalization of lumped CRR models. The Model Parameter Estimation Experiment (MOPEX) includes a variety of researchers who apply a statistical regionalization approach. Here we test underlying assumptions commonly made in this method and their impact on the results.

## 2. Model regionalization—procedure and uncertainties

Any RR model can be written in the following simplified form (Wagener et al., 2004),

$$Q = M_L(\theta_L|I) + \varepsilon_L \quad (1)$$

where  $Q$  is the simulated streamflow,  $I$  is a matrix of input variables (e.g. rainfall and temperature),  $M_L$  is a given (local) model structure,  $\theta_L$  is a vector of parameters within this structure and  $\varepsilon_L$  is an error term. The model parameters will usually be estimated through calibration if measured time-series of runoff over a sufficiently long period are available. The required length of the time-series depends, amongst other things, on the complexity of the model structure used and the information content of the available data. It might range from 3 years (Sefton and Howarth, 1998; Jakeman and Hornberger, 1993) for a simple model structure, to up to about a decade for a more

complex one (Yapo et al., 1996). However, in principle, the data set should always be sufficiently long to avoid the problem of the parameters only being representative of a particular climate period (Gan and Burges, 1990).

If no runoff data are available for a specific catchment, i.e. if it is ungauged, an attempt can be made to calibrate the model structure to a large number of gauged catchments and to find a functional relationship between the (usually individual) conceptual model parameters (dependent variables) and the catchment characteristics (independent variables), i.e. a *regional* model structure of the following type:

$$\hat{\theta}_L = H_R(\theta_R|\Phi) + v_R \quad (2)$$

where  $\hat{\theta}_L$  is the estimated model parameter at the ungauged site,  $H_R(\cdot)$  is a functional relation for  $\hat{\theta}_L$  using a set of physiographic and meteorological catchment characteristics  $\Phi$ , while  $\theta_R$  is a set of regional model parameters and  $v_R$  is an error term. One model, i.e. (regional) model structure and (regional) parameter combination is conventionally derived for each (local) parameter, i.e. the model parameters are assumed to be independent. The regional model structure commonly takes the form of a linear or a non-linear regression equation.

No generally accepted procedure for regionalization of conceptual, continuous model parameters currently exists. However, the following steps are typically found and are therefore given here as the basic outline of a regionalization procedure (Fig. 1):

Decide which (sub-)set of catchments can be described by a single local model structure  $M_L$  and a single regional model, i.e. one structure  $H_R$  with a specific set of parameters  $\theta_R$ . Catchments that are very different with respect to their dominant hydrological processes might require different (local) model structures to represent them in a physically (or probably rather conceptually) realistic manner. This segmentation of catchments must be related to the catchment characteristics  $\Phi$  in order to classify any ungauged catchment. Appropriate characteristics could be size, drainage density, soils/geology, land use, etc.

Apply the local model structure  $M_L$  to each of the gauged catchments and estimate the *optimum*

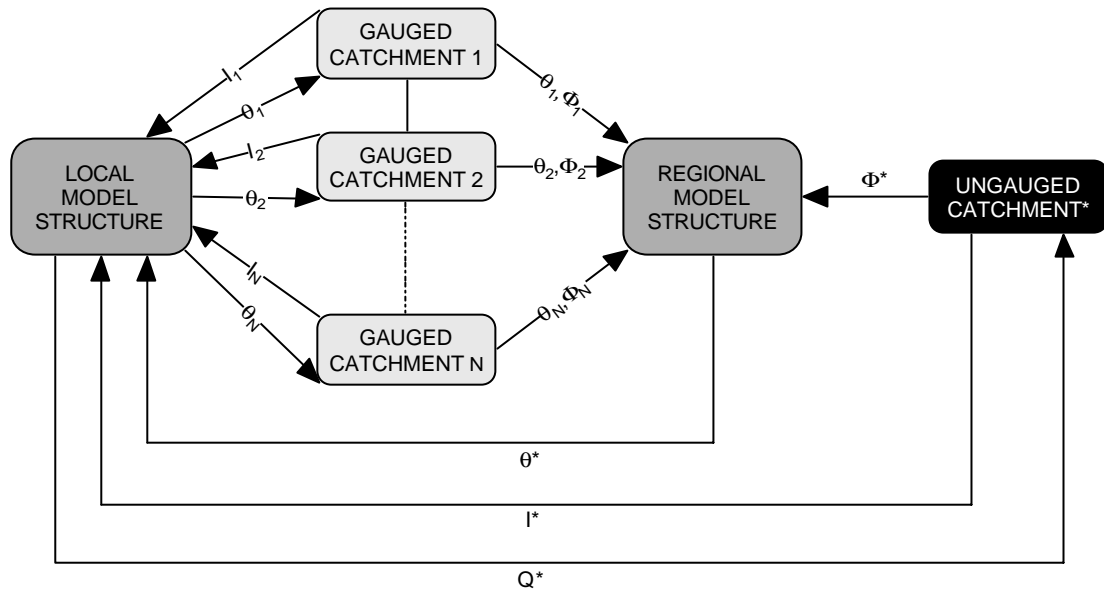


Fig. 1. Schematic representation of a regionalization procedure.

parameter set (or population)  $\theta_L$  for each catchment.

Relate the derived (individual) parameter values  $\theta_{Li}$  and the catchment characteristics  $\Phi$  using the regional model structure  $H_{Ri}$ . Apply the regional model  $H_{Ri}(\theta_{Ri}|\Phi)$  to estimate each parameter  $\hat{\theta}_{Li}$  for the ungauged catchment.

Predict flow in the ungauged catchment using parameter set  $\hat{\theta}_L$ .

Each of the steps outlined above introduces uncertainties that are unavoidably propagated into the regionalization result (the streamflow prediction at an ungauged site). They can be categorized into two groups, first, uncertainties related to local modelling (i.e. those related to the selection and calibration of the local model structure to each catchment), and second, uncertainties related to the procedure for spatial extrapolation using a regional model. The main uncertainties are:

Selection of *catchment properties*, i.e. what are suitable characteristics to describe and cluster (pool) catchments with respect to their hydrological response? This is important for both the local and the regional modelling steps.

Selection of the *local model structure*, i.e. what is a suitable RR model to minimize structural inadequacies and lack of identifiability?

Identification of the *local parameters*, i.e. the problem of parameter identification.

Identification of the *regional model structure and its parameters*, i.e. what is the nature of the relationship between catchment characteristics and model parameters? This is also dependent on:

Selection of the *regionalization procedure*. Here the question must be addressed of whether the calibration objective is purely the optimization of the performance of the local model in each catchment, or whether the performance of the regional model should be considered at this stage.

### 3. Case study

#### 3.1. Data

Ten catchments located in the southeast of England are used in this study. Their general characteristics are summarized in Table 1. These catchments are particularly suited for the study at hand because

Table 1  
General catchment characteristics

River	Location	Area <sup>a</sup> [km <sup>2</sup> ]	Q <sub>spec</sub> <sup>b</sup> [m/yr]	Q <sub>mean</sub> <sup>c</sup> [m <sup>3</sup> /s]	Q <sub>95</sub> <sup>c</sup> [m <sup>3</sup> /s]	Q <sub>10</sub> <sup>c</sup> [m <sup>3</sup> /s]	61–90 Av. An. Rainfall <sup>2</sup> [mm]
Blackwater	Ower	102.4	0.26	0.85	0.154	2.131	836
Eden	Penshurst	224.8	0.25	1.76	0.232	3.853	742
Eastern Rother	Udiam	204.7	0.32	2.08	0.172	5.312	857
Medway	Teston	1261.3	0.26	10.56	1.526	23.840	744
Teise	Stone Bridge	134.5	0.31	1.31	0.202	2.553	810
Upper Medway	Chafford Weir	252.1	0.37	2.99	0.533	6.395	830
Eastern Yar	Burnthouse	59.6	0.22	0.41	0.040	0.846	-
Medina	Shide Weir	28.5	0.31	0.28	0.082	0.502	839
Western Rother	Hardham	360.7	0.43	4.96	1.694	9.829	899
Test	Broadlands	1035.9	0.33	10.93	5.780	16.560	790

<sup>a</sup> Data from FEH, 1999, CD.

<sup>b</sup> Specific runoff = runoff / drainage area.

<sup>c</sup> Data from <http://www.nwl.ac.uk/ih/nrfa>.

climatic data (precipitation and potential evapotranspiration) are highly correlated. The variation in response found between the catchments is therefore largely due to differences in intrinsic properties (geology, soil types, etc.), which are of course represented by the hydrological model structure and its parameters. The catchments are predominantly rural and of varying geologies consisting mainly of Chalk, Clay and Greensand. They range from relatively impervious clay catchments to a highly permeable chalk catchment. Time-series of

(naturalized) flow, precipitation, temperature and potential evapotranspiration (PE) ranging over different periods were utilized. All periods are within the 8-year sequence from 01.01.1989 to 31.12.1996 and at least 5 years long. The emphasis of this study is on the issue and consequences of model identification, not on the derivation of robust regional parameter estimates. A small sample is sufficient for this purpose and in line with similar studies (e.g. Seibert, 1999). A wide range of catchment characteristics is available through the Flood Estimation Handbook (Bayliss,

Table 2  
Description of catchment characteristics (Bayliss, 1999)

Catchment characteristic	Unit	Description
BFIHOST <sup>a</sup>	–	Baseflow index derived using the HOST classification. It is the long-term average proportion of baseflow
PROPWET	–	Index of proportion of time that the soil moisture deficit was below or equal to 6 mm during 1961–1990
DPLBAR <sup>b</sup>	km	Index combining catchment size and drainage path configuration. Mean distance between each node and the catchment outlet
DPSBAR <sup>b</sup>	mkm <sup>-1</sup>	Index of catchment steepness. Mean of all inter-nodal slopes in a catchment
ASPBAR <sup>b</sup>	–	Index representing the dominant aspect of catchment slopes. Mean direction of all inter-nodal slopes with north being zero
ASPVAR <sup>b</sup>	–	Index describing the invariability in aspect of catchment slopes. Values close to one when all slopes face a similar direction
RMED-2D	mm	Median annual maximum 2-day rainfall
SAAR	mm	1961–1990 standard-period average annual rainfall
URBEXT <sub>1990</sub>	–	FEH index of fractional urban extent for 1990

<sup>a</sup> HOST is the Hydrology Of Soil Types classification used in the UK.

<sup>b</sup> These four landform descriptors are derived from the Institute of Hydrology DTM (IHDTM) which is a regular nodal grid of 50 m × 50 m elements.

1999) for these catchments. All catchment characteristics used here group as follows:

Landform: DPLBAR, DPSBAR, ASPBAR, ASPVAR.

Climate and soil: BFIHOST, PROPWET, SAAR, RMED-2D.

Urban and suburban: URBEXT<sub>1990</sub>.

Descriptions of the different characteristics can be found in Table 2, while the actual values for

the different catchments are listed in Table 3. Other characteristics are available, but are highly correlated (linear correlations of about 0.9 or higher) with the ones used here so that their inclusion would not provide additional information (Wagener et al., 2004). Alternatively, one could also perform a data transformation to derive new, uncorrelated variables, using for example principal component analysis (Gershensfeld, 1999) (p. 136ff.). Sefton and Howarth (1998) use

Table 3  
Detailed catchment characteristics (FEH, 1999)

River at location	Blackwater at Ower	Eden at Peshurst	E. Rother at Udiam	Medway at Teston	Teise at Stone Bridge
AREA	102.38	224.82	204.66	1261.33	134.46
LDP	18.48	33.92	30.93	65.99	23.68
BFIHOST	0.479	0.425	0.388	0.439	0.443
SPRHOST	34.2	41.2	44.4	41.3	42.6
FARL	0.985	0.925	0.975	0.949	0.905
PROPWET	0.33	0.35	0.35	0.35	0.36
DPLBAR	11.06	20.33	17	36.03	13.45
DPSBAR	44.4	47.2	92.6	53.6	78.3
ASPBAR	124	136	112	81	57
ASPVAR	0.17	0.11	0.11	0.05	0.1
RMED-1D	35.7	32.9	36.6	33.3	35
RMED-2D	46.3	44.1	48.8	44.3	47.9
RMED-1H	10.7	11.4	11.6	11.7	11.8
SAAR	837	742	857	744	812
SAAR <sub>4170</sub>	867	764	861	755	809
URBEXT <sub>1990</sub>	0.009	0.016	0.008	0.019	0.005
URBCONC <sup>a</sup>	0.327	0.556	0.508	0.614	–
URBLOC <sup>a</sup>	0.875	1.217	0.976	0.965	–
River at Location	Up Medw. at Chafford Weir	East. Yar at Burnthouse	Medina at Shide Weir	W. Rother at Hardham	Test at Broadlands
AREA	252.05	59.58	28.54	360.71	1035.93
LDP	29.09	19.49	11.41	56.19	69.89
BFIHOST	0.441	0.743	0.753	0.666	0.898
SPRHOST	42.4	24.3	23.6	27.4	9.4
FARL	0.938	0.992	0.985	0.973	0.964
PROPWET	0.35	0.33	0.33	0.34	0.34
DPLBAR	14.36	10.68	5.84	30.13	39.97
DPSBAR	82.4	84.6	78.5	72.9	50.1
ASPBAR	28	6	59	120	176
ASPVAR	0.06	0.06	0.1	0.1	0.15
RMED-1D	34.8	33.9	35	39.4	33.3
RMED-2D	47.3	45.3	46	50.8	43.1
RMED-1H	11.7	9.4	9.5	10.4	10.6
SAAR	831	844	839	899	790
SAAR <sub>4170</sub>	852	910	911	918	818
URBEXT <sub>1990</sub>	0.02	0.017	0.015	0.008	0.01
URBCONC	0.601	0.44	0.342	0.497	0.56
URBLOC	1.231	0.895	0.847	1.16	0.841

<sup>a</sup> These variables are not considered further, because their values for Teise at Stonebridge are not available.



principal components to derive variables that they call topography and soils/geology for regional analysis from an initial set of catchment characteristics. However, the newly derived variables reduce the ease with which regional relationships can be interpreted. This approach is therefore not considered here.

The differences in response characteristics of the catchments can be demonstrated using flow duration curves (FDCs) (Fig. 2). FDCs show the cumulative frequency of streamflow versus the percentage of time this streamflow is exceeded, usually for daily flows. Their shape is an indicator of catchment response to rainfall input. The steeper the flow duration curve, the more variable the response (Dingman, 1994) (p. 14). The response types of the different catchments can thus be compared by plotting their flow duration curves on a single non-dimensional plot, derived by dividing each FDC by the mean discharges of the corresponding river (Linsley et al., 1949). The plot shows the flat curves of the baseflow-dominated rivers such as the Test at Broadlands and the Medina at Shide Weir, while the faster responding catchments like the Eden at Penshurst, Medway at Teston or Eastern Rother at Udiam have much steeper curves. This response characteristic is also reflected in the baseflow index, which represents the proportion of the annual hydrograph that can be assigned to baseflow (BFIHOST). The shape of the flow duration curve is,

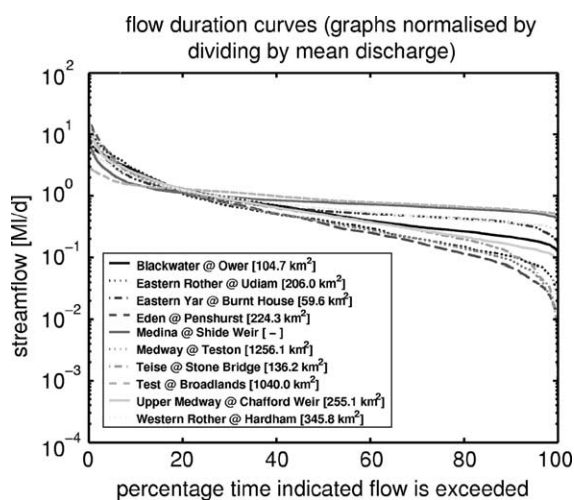


Fig. 2. Comparison of flow duration curves for the data set used.

however, apart from being defined by the storage characteristics of the catchment, also influenced by topography, vegetal cover, land use and precipitation (Linsley et al., 1949) (p. 585).

### 3.2. Modeling tools and methods

#### 3.2.1. Rainfall-runoff modeling toolbox

The local model structures utilized here are applied within the Rainfall Runoff Modelling Toolbox (RRMT; Wagener et al., 2002). All structures are parametrically efficient (parsimonious), and can be defined as conceptual, though some have an empirical basis. Each structure has two distinct and sequential elements, namely a soil moisture accounting (SMA) and a routing module.

The main SMA component used here is the probability distribution of stores (pd4; Fig. 3(b), e.g. Moore, 1999) combined with a parallel routing component consisting of two linear stores representing quick and slow response of the system (2pll; Fig. 3(d)). This particular SMA component splits the incoming precipitation into storage, losses through evapotranspiration and effective rainfall (ER). The routing component adds translation effects to the ER, producing streamflow. This is a typical structure for this level of complexity. The model assumes that the soil moisture storage capacity,  $c$ , varies across the catchment and, therefore, that the proportion of the catchment with saturated soils varies over time. The spatial variability of soil moisture capacity, represented as bucket elements with different depths, is described by the following distribution function,

$$F(c) = 1 - (1 - c(t)/c_{\max})^b \quad 0 \leq c(t) \leq c_{\max} \quad (3)$$

The structure requires the optimization of five parameters: the maximum storage capacity in the catchment,  $c_{\max}$  [L], the degree of spatial variability of the soil moisture capacity within the catchment,  $b$  [–], the factor distributing the flow between the two series of reservoirs,  $a$  [–], and the time constants of the linear reservoirs,  $k_{\text{quick}}$  [T] and  $k_{\text{slow}}$  [T]. The actual evapotranspiration is equal to the potential value if sufficient soil moisture is available; otherwise it is equal to the available soil moisture content.

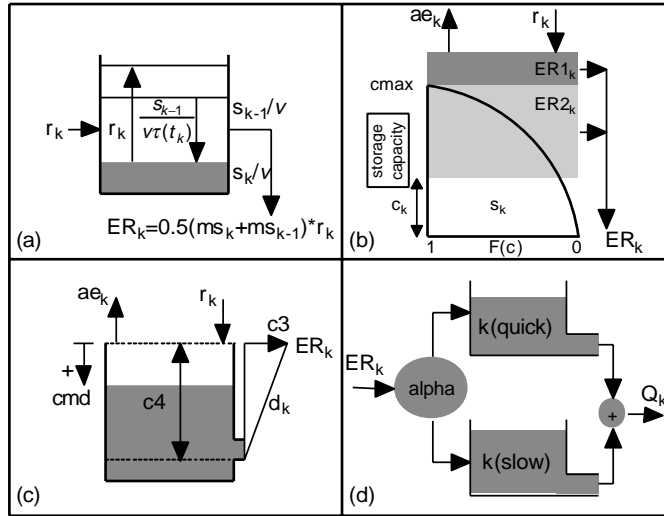


Fig. 3. Schematic representation of the structural components used in the case study. (a) Catchment Wetness Index (cwi), (b) probability distributed model (pdm), (c) catchment moisture deficit (cmd), and (d) parallel routing structure consisting of two linear reservoirs.

The other two SMA modules used for part of this study are the catchment wetness index (cwi; e.g. Jakeman and Hornberger, 1993) and the catchment moisture deficit (cmd; Evans and Jakeman, 1998) modules (Fig. 3(a) and (c)). Both are connected to the same parallel routing component as described above.

The former is based on the well-known Antecedent Precipitation Index (API). The proportion of rainfall  $r_k$  contributing to runoff (i.e. the effective rainfall,  $ER_k$ ) at every time-step  $k$ , is determined by the cwi,  $ms$ , which is calculated as the mean of the indexes for time-steps  $k$  and  $k-1$  (an indication of the soil moisture state of the catchment, ranging from zero to one).

$$ER_k = \frac{1}{2}(ms_k + ms_{k-1})r_k \quad (4)$$

The index  $ms_k$  is calculated using the equation

$$ms_k = vr_k + \left[1 - \frac{1}{\tau(t_k)}\right]ms_{k-1} \quad (5)$$

where  $v$  is a factor that is adjusted to ensure that the total volume of modelled effective rainfall equals the total volume of observed streamflow. The parameter  $v$  is not calibrated, but is calculated explicitly from the data during the calibration stage.  $\tau$  describes the decrease of  $ms$  due to evapotranspiration as a function of temperature. The cwi version used here requires

three parameters to be estimated. See Jakeman et al., 1993 for details.

The last SMA module is a single bucket structure with an additional bottom outlet to simulate prolonged drainage over the summer months. The cmd is described by the following equation of the catchment water balance,

$$cmd_k = cmd_{k-1} - r_k + ae_k + d_k \quad (6)$$

where  $cmd$  is the catchment moisture deficit,  $r$  is the precipitation,  $ae$  is the (actual) evapotranspiration loss,  $d$  is the drainage and  $k$  is the time-step. The drainage  $d$  is introduced due to the assumption that there is a certain amount of runoff to the stream, even when a positive catchment moisture deficit exists (Evans and Jakeman, 1998). The effective rainfall can then be calculated as follows,

$$ER_k = \begin{cases} d_k & \text{for } cmd_k \geq 0 \\ d_k - cmd_k & \text{for } cmd_k < 0 \end{cases} \quad (7)$$

This SMA component requires four parameters ( $c_1 - c_4$ , Fig. 3) to be estimated. For details see Evans and Jakeman (1998).

### 3.2.2. Modeling methods applied

A uniform random sampling (URS) procedure is used to explore the feasible parameter space and to



allow for an estimate of the uncertainty in the parameter estimates. In this case, the parameters are assumed independent from each other and the only prior information used is reasonable lower and upper boundary values. 10,000 parameter sets are randomly sampled for each case. Initial conditions (model moisture states) are calibrated instead of using a warm-up period to ensure that the results are independent of the initial state of the model. The Objective Function (OF)

applied was the Root Mean Square Error (RMSE) measure, unless stated otherwise.

$$RMSE = \sqrt{\frac{1}{N} \sum_{t=1}^N (q_t^{sim} - q_t^{obs})^2} \quad (8)$$

where  $N$  is the total number of time steps  $t$ , and  $q_t^{sim}$  and  $q_t^{obs}$  are simulated and observed streamflow, respectively.

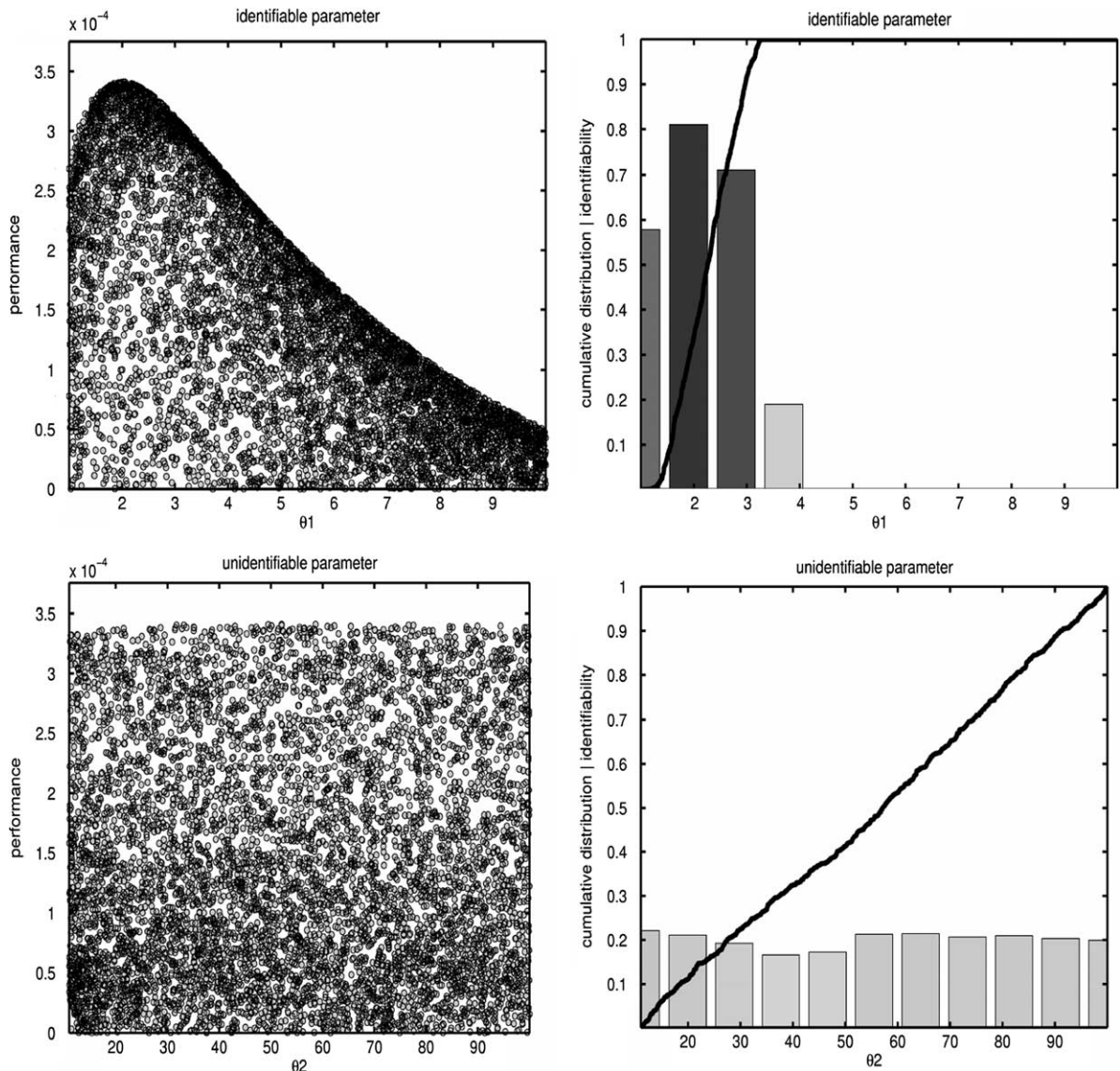


Fig. 4. Parameter identifiability measure based on parameter populations conditioned on a selected objective function.

Wagener et al. (2001) derived a heuristic measure of identifiability to quantify the uncertainty in posterior parameter distributions, derived from conditioning an initially uniform distribution on a selected OF. Fig. 4 shows the measure for the examples of a well and a poorly identified parameter. Dotty plots for both are shown (Fig. 4(a) and (c)) in which each parameter set (projected into a single dimension) is plotted against a corresponding OF value. For this particular plot, the RMSE measure is transformed so that higher values correspond to better performing parameter sets. Plotting the cumulative distribution of the top 10% parameter sets for parameters results in a much steeper curve for the more identifiable parameter (Fig. 4(b)). One can then split the parameter range in 10 equally sized bins and calculate the slope of the cumulative distribution in each, resulting in a frequency distribution. The highest slope value is an indicator of the peakiness of the posterior distribution.

### 3.3. Results and discussion

#### 3.3.1. Benchmark regionalization results using traditional approach

The standard approach to regionalization is to calibrate the model to a large number of catchments and then use a regression analysis to relate individual model parameters to catchment characteristics. This is done here to provide a benchmark for comparison of the different variations to this approach. The RMSE values resulting from the local calibration of the model to all 10 catchments can be found in Table 4. The prediction for the Blackwater at Ower was not satisfactory from a hydrologic point of view (low flow overprediction was compensated with high flow underprediction), which was assumed to be caused by data problems in this case. The catchment was therefore excluded from the regionalization step.

The next step in regionalization is then to analyze whether correlations between catchment characteristics and model parameters exist. The correlation between variables is most easily tested by calculating a linear correlation coefficient, applied in the original or in log-transformed space. A rank correlation coefficient can additionally be used to test for non-linear relationships (e.g. Tung et al., 1997; Seibert, 1999). Correlation coefficients and rank correlation

Table 4  
RMSE calibration results for local model structure, pd4\_2pll

No.	Catchment	RMSE	BFI-HOST
c01	EasternRother @ Udiam	0.91	0.388
c02	Eden @ Peshurst	0.70	0.425
c03	Medway @ Teston	0.57	0.439
c04	UpperMedway @ Chafford-Weir	0.84	0.441
c05	Teise @ Stonebridge	0.66	0.443
c06	Blackwater @ Ower	0.73	0.479
c07	WesternRother @ Hardham	0.52	0.666
c08	EasternYar @ Burnthouse	0.61	0.743
c09	Medina @ Shide Weir	0.79	0.753
c10	Test @ Broadlands	0.19 <sup>1</sup>	0.898

Catchments are sorted according to BFIHOST values. <sup>1</sup>Fit improves largely with unreasonable parameter values in this chalk catchment.

coefficients cannot be compared directly in quantitative terms. However, cases where the rank correlation coefficient is significantly larger than the linear correlation coefficient can be indicative of a non-linear dependence. A nonparametric test, i.e. one that does not assume a particular functional form, is Spearman's rank correlation coefficient (e.g. Kottogoda and Rosso, 1998, p. 280–282). In this test, two sets of variables, e.g.  $x_i$  and  $y_i$  ( $i=1,2,\dots,n$ ), are ranked separately such that the highest value of each variable is assigned a rank of 1 and the lowest a rank of  $n$ . The rank coefficient can then be estimated using

$$r_s = 1 - \frac{6 \sum_{i=1}^n rd_i^2}{n(n^2 - 1)} \quad (9)$$

where  $rd_i$  is the difference in ranks between  $x_i$  and  $y_i$ .

The correlation analysis between model parameters and catchment characteristics (Table 5) shows encouraging results for three parameters, but no correlations at the 5% significance level for  $c_{\max}$  and for  $k_{\text{slow}}$ . These results have to be interpreted carefully due to the small sample size of eight catchments (the Test at Broadlands could not be described by pd4-2pll (probability distributed SMA with parallel linear store routing) and the Teise at Stonebridge is kept apart for testing). Additionally, the problem of estimating  $k_{\text{slow}}$  using the RMSE was already mentioned.

The only correlation above 0.5 found for parameter  $c_{\max}$  is with URBEXT<sub>1990</sub> (Table 5(a) and (b)). Again,

Table 5  
Correlation coefficients for all variables involved

Characteristic	$c_{\max}$	$B$	$k_{\text{quick}}$	$k_{\text{slow}}$	$a$
(a) Correlation coefficients for variables on original scale					
BFIHOST	0.43	0.56	−0.66	0.35	−0.91**
DPLBAR	−0.04	−0.58	0.72*	−0.14	0.34
DPSBAR	−0.36	0.23	−0.74*	0.21	−0.03
PROPWET	−0.35	−0.59	0.74*	−0.46	0.84**
SAAR	−0.16	0.18	−0.59	0.01	−0.31
URBEXT <sub>1990</sub>	0.57	0.15	0.13	0.02	−0.18
ASPBAR	−0.41	−0.34	0.54	−0.38	0.44
ASPVAR	−0.43	0.09	−0.05	−0.29	0.13
RMED-2D	−0.35	−0.09	−0.21	−0.31	0.02
(b) Correlation coefficients for variables on log–log-transformed scale					
BFIHOST	0.47	0.58	−0.64	0.28	−0.96**
DPLBAR	−0.03	−0.66 <sup>∞</sup>	0.85**	−0.09	0.55
DPSBAR	−0.38	0.45	−0.71*	0.10	−0.18
PROPWET	−0.34	−0.59	0.75*	−0.42	0.89**
SAAR	−0.20	0.34	−0.56	−0.04	−0.38
URBEXT <sub>1990</sub>	0.59	0.26	0.11	−0.10	−0.10
ASPBAR	−0.43	−0.34	0.38	−0.40	0.42
ASPVAR	−0.41	−0.22	−0.12	−0.17	0.01
RMED-2D	−0.37	0.05	−0.18	−0.31	−0.02
(c) Spearman rank correlation coefficients for variables					
BFIHOST	0.21	0.79	−0.61	−0.04	−0.96
DPLBAR	−0.07	−0.64	0.86	−0.18	0.64
DPSBAR	−0.21	0.28	−0.75	0.43	−0.04
PROPWET	−0.37	−0.78	0.58	−0.47	0.83
SAAR	−0.29	0.04	−0.50	0.18	−0.21
URBEXT <sub>1990</sub>	0.49	0.42	0.19	−0.13	0.01
ASPBAR	−0.21	−0.82	0.68	−0.29	0.46
ASPVAR	−0.21	−0.55	0.06	−0.03	0.21
RMED-2D	−0.43	0.11	−0.36	−0.25	−0.07

Three catchments are excluded from this calculation: (1) Teise at Stonebridge, because it is kept as a test catchment; (2) Test at Broadlands, due to the differences in BFIHOST (see text); and (3) Blackwater at Ower, because no satisfactory calibration could be established judged by the visual fit. (The best parameter values are normally distributed, but show some outliers. Normality plot not shown.). \*Significant at the 5% level. \*\*Significant at the 1% level. <sup>∞</sup>This value is just outside the 5% significance level.

this relationship might be of little importance since all catchments are essentially rural and the variation of URBEXT<sub>1990</sub> is small. A combination of several variables might therefore be required to explain this parameter in a regression model. Alternatively, the parameter could also be fixed to the median value of all calibration results as suggested by Seibert (1999). A low correlation for this parameter could have been expected since no soil properties in the form of field capacity, porosity or wilting point were available. These are more likely to be related to  $c_{\max}$  than the variables used here. A similar lack of correlation for the water balance characteristics of the IHACRES model structure was reported by Sefton and Howarth

(1998). Lamb et al. (2000) needed four catchment characteristics in a regression equation—topographic index, drainage path slope, SAAR<sub>61–90</sub>, and urban area—to yield a coefficient of determination of 0.7 for  $c_{\max}$  (using an hourly time-step for flood peak estimation).

A slightly better result is found for the shape parameter  $b$ . Three variables yield correlation coefficients above 0.5 on the original scale and one exceeds 0.6 on the log-transformed scale. These are the baseflow index BFIHOST (original: 0.56, log: 0.58), the index for catchment size and drainage path configuration DPLBAR (o.: −0.58, l.: −0.66), and an index describing how often the soils are wet,

PROPWET (o.:  $-0.60$ , l.:  $-0.59$ ). However, only the correlation to DPLBAR (on log-scale) is significant at the 5% level. The variable PROPWET is unlikely to be very useful in a regression since it only takes four different values within the available data-set (0.33, 0.34, 0.35 and 0.36). A value of zero for  $b$  is equal to a constant storage capacity over the catchment, while a value of 1 yields a uniform distribution of storage capacities. The results suggest higher  $b$  values for catchments with a larger contribution of baseflow, smaller catchment area and drier soils. The shape of the storage distribution function  $b$  is the only parameter for which the Spearman rank correlation coefficient gives considerably higher values than the linear correlation coefficients on normal or log-transformed scale (Table 5(c)). The highest rank correlation values are found with the variables ASPBAR ( $-0.82$ ), BFIHOST (0.79) and PROPWET ( $-0.78$ ). There is also a relatively high value with DPLBAR ( $-0.64$ ). The fact that the (non-parametric) rank correlations are higher than those assuming a linear relationship is indicative of a possible non-linear relationship. The variable ASPBAR describes the mean aspect of the catchment. It is calculated as an average from the outflow direction (bearing) of each nodal point on the IHDTM within a catchment. It is therefore an indicator of the dominant aspect of catchment slopes. Its values increase clockwise from zero to  $360^\circ$ , starting from the north. A negative rank correlation suggests that a south-easterly bearing is related to a lower  $b$  value.

The parameter  $k_{\text{quick}}$  is correlated with a number of characteristics, BFIHOST (o.:  $-0.66$ , l.:  $-0.64$ ), DPLBAR (o.: 0.72, l.: 0.85), the mean drainage path slope index DPSBAR (o.:  $-0.74$ , l.:  $-0.71$ ), PROPWET (o.: 0.74, l.: 0.75) and average annual rainfall over a selected period SAAR (o.:  $-0.59$ , l.:  $-0.56$ ). The largest correlation is found with DPLBAR (on log-scale). The variable DPLBAR is the mean drainage path distance of all nodes of the IHDTM to the catchment outlet. The result suggests that larger and more elongated catchments drain more slowly. On the contrary, steeper catchments produce smaller time constants, and the mean drainage path slope (DPSBAR) is higher. Again, the correlation with PROPWET is probably of little explanatory value. As noted above, the values of the catchments used only vary between 0.33 and 0.36, while the UK

wide variation is between 0.20 and 0.85. Sefton and Howarth (1998) related the quick response time constant (within the IHACRES model structure) to catchment size and stream-frequency, but found no improvement when including channel slope. In the study by Lamb et al. (2000), this parameter showed the highest correlation of the four PDM parameters tested (the fast/slow split was fixed). They derived it from BFIHOST, stream network centroid, MORECS residual soil moisture and suburban area.

The slow flow time constant  $k_{\text{slow}}$  does not show any significant (5% level) correlation with any of the available catchment characteristics. This is not unexpected since this is the least identifiable parameter. The problem of identifying this parameter (and other parameters related to the low flow periods for that matter) using an OF based on the complete hydrograph is one of the main reasons why segmentation schemes were introduced (e.g. Dunne, 1999; Boyle et al., 2000; Wagener et al., 2001). It seems quite unlikely that a parameter such as the slow flow time constant can be regionalized based on a calibration using an OF that emphasizes the fit to high flow, such as the RMSE. Similar results have been reported by other researchers. Lamb et al. (2000) derive a regional equation to estimate this parameter from BFIHOST, soil porosity and underlying geology. However, the coefficient of determination produced by their model is 0.6 and thus the lowest of their four regionalized model parameters. Their result is similar to the one by Sefton and Howarth (1998) who relate a slow flow time constant to different soil variables. The correlation coefficient between regionalized and locally calibrated values was only 0.37, however.

The fraction of effective rainfall that contributes to the quick response,  $a$ , is highly correlated to two catchment characteristics, the baseflow index BFIHOST and PROPWET. The first correlation (on original scale) is negative (o.:  $-0.91$ ; l.:  $-0.96$ ), while the second one is positive (o.: 0.84; l.: 0.89). Both are significant at the 1% level. The first correlation is expected, while the second indicates that a wet catchment, probably containing more saturated and therefore contributing areas, produces a high percentage of quick response. Sefton and Howarth (1998) found that the contribution to slow response ( $1-a$ ) was highly correlated to the percentage of aquifer in a catchment (0.77), a variable

not available here. This split parameter is sometimes assumed to be directly equal to the standard percentage runoff (SPRHOST) or BFIHOST, e.g. Lamb et al. (1999) and Young (2002). SPRHOST is highly correlated to BFIHOST as shown earlier ( $-0.98$ ). The result found here therefore suggests that fixing this parameter *a priori* using BFIHOST can be justifiable.

We consider next the regression equations (regional models) that are derived using a stepwise multiple regression, shown in Table 6(a). No sensible regression is found for  $k_{\text{slow}}$ , so the median is chosen

for as a regional model for this parameter (following Seibert, 1999).  $R^2$  is used as a measure of performance for the regression equation. The parameter  $\alpha$  can be predicted with an  $R^2$  of 0.91, using only BFIHOST as an independent variable. Both  $b$  and  $k_{\text{quick}}$  can be derived from a catchment shape descriptor, DPLBAR, with  $R^2$  values of 0.44 and 0.72, respectively. Parameter  $c_{\text{max}}$  requires three characteristics (BFIHOST, DPSBAR, ASPBAR) to achieve an  $R^2$  value of 0.67.

The results of the benchmark regionalization are typical for studies of this kind. There are very few

Table 6  
The regional models resulting from the different approaches

Local parameter	Regional model	Lower CFL*	Regional parameter	Upper CFL*	$R^2$
(a) Conventional regression as benchmark					
$c_{\text{max}}$	$606.57 + 224.94 \cdot \text{BFIHOST} -$	-128.01	606.57	1341.16	0.67
	$4.28 \cdot \text{DPSBAR} - 1.17 \cdot \text{ASPBAR}$	-830.3	224.94	1280.00	
		-14.74	-4.28	6.18	
$B$	$3.17 \cdot \text{DPLBAR}^{-1.10}$	-4.98	-4.98	2.64	0.44
		0.03	3.17	192.94	
		-2.54	-1.10	0.33	
$k_{\text{quick}}$	$1.54 \cdot \text{DPLBAR}^{0.20}$	1.02	1.54	2.33	0.72
		0.06	0.20	0.35	
$k_{\text{slow}}$	190.43 (median)	-	-	-	-
$\alpha$	$0.29 \text{BFIHOST}^{-1.13}$	0.22	0.29	0.38	0.91
		-1.53	-1.13	-0.73	
(b) Weighted regression					
$c_{\text{max}}$	$610.43 + 241.70 \cdot \text{BFIHOST} -$	-111.90	610.43	1322.77	0.63
	$4.39 \cdot \text{DPSBAR} - 1.18 \cdot \text{ASPBAR}$	-439.47	241.70	922.86	
		-11.12	-4.39	2.35	
$b$	$1.40 \cdot \text{DPLBAR}^{-0.84}$	-3.56	-1.18	1.20	0.43
		0.01	1.40	181.54	
		-2.47	-0.84	0.79	
$k_{\text{quick}}$	$1.53 \cdot \text{DPLBAR}^{0.21}$	1.01	1.53	2.31	0.94
		0.06	0.21	0.35	
$k_{\text{slow}}$	$0.23 \text{PROPWET}^{-6.32}$	$e^{-18.04}$	0.23	$e^{15.12}$	0.57
		-21.78	-6.32	9.12	
$\alpha$	$0.28 \cdot \text{BFIHOST}^{-1.20}$	0.21	0.28	0.37	0.97
		-1.59	-1.20	-0.80	
(c) Sequential regression					
$c_{\text{max}}$	$e^{8.45} \cdot \text{DPSBAR}^{-0.97} \cdot$	$e^{0.02}$	$e^{8.45}$	$e^{16.88}$	0.48
	$\text{ASPVAR}^{-0.52}$	-2.75	-0.97	0.81	
		-1.84	-0.52	0.80	
$b$	0.16 (median)	-	-	-	-
$k_{\text{quick}}$	$1.75 \cdot \text{ASPVAR}^{-0.19}$	1.04	1.75	2.94	0.53
		-0.40	-0.19	0.02	
$k_{\text{slow}}$	197.87 (median)	-	-	-	-
$\alpha$	$0.29 \cdot \text{BFIHOST}^{-1.13}$	0.22	0.29	0.38	0.91
		-1.53	-1.13	-0.73	

Parameters for which no sensible relationship could be found are set to their median values. (a) Conventional regression as benchmark. \*90% Confidence Limits.

parameters for which some robust correlation of significance can be found. The remaining ones show little or no correlation. An in-depth analysis to investigate the possible reasons for this problem is provided in the following sections, including the testing of variations on the traditional approach.

3.3.2. Does a model component, implemented in different model structures, yield the same optimal parameters?

A large variety of model structures is applied in hydrological research and practice. However, the number of model components (e.g. linear reservoir, overflow bucket, etc.) from which these structures are composed is relatively small (Wagener et al., 2002). Different model structures therefore usually have some components in common. One would expect that

the same optimal model parameters would be found for these components if they have the same functional purpose within the different model structures, i.e. if they represent identical processes. Additional model structures, to the above analyzed pd4, all *a priori* equally suitable to represent the hydrological system under study, are applied to all catchments to test this assumption. Research results by others (Beven and Franks, 1999; Kokkonen and Jakeman, 2001) suggest that identical components (and therefore parameters) used within different model structures can have different optimum parameters due to interaction between the different model components. This is tested here by applying two further model structures. The SMA component is represented by an empirical (cwi) and an additional conceptual model structure (cmd) respectively, using routing components

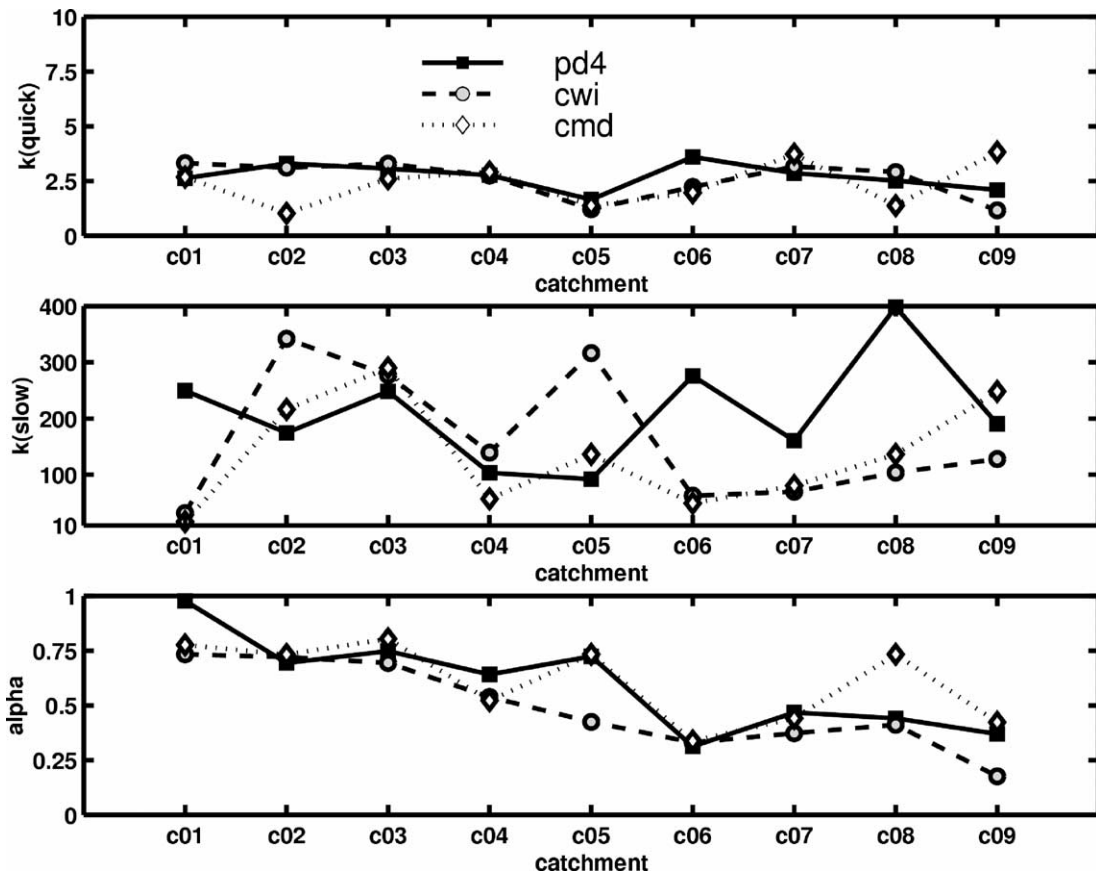


Fig. 5. Comparison of  $k_{\text{quick}}$  [d],  $k_{\text{slow}}$  [d] and  $\alpha$  [–] values for the pd4-2p11, cwi-2p11 and cmd\_2p11 model structures. Values shown are optimal with respect to the RMSE criterion. Catchments are sorted from left to right with increasing Baseflow index (BFIHOST).



identical to the linear parallel structure used in the model structure described above. The resulting routing parameters are compared to see whether we could corroborate or refute the above mentioned results. The metric and the second parametric model structures are applied to all catchments using the same URS approach (10,000 samples) as before and the best  $k_{\text{quick}}$ ,  $k_{\text{slow}}$  and  $a$  parameters for all structures are selected based on the RMSE criterion. The variation in optimum values is shown in Fig. 5. It can be seen that with respect to  $k_{\text{quick}}$ , all model structures show a relatively high degree of similarity in optimum values. Generally this parameter seems to show little dependency on the soil moisture accounting (SMA) module, and experience has shown that  $k_{\text{quick}}$  is usually well identified (e.g. Wagener et al., 2001). However, there is a slight tendency for cwi to produce higher values. The result for  $k_{\text{slow}}$  shows that it is difficult to identify this parameter using the overall RMSE as OF. The optimum values vary widely and there appears to be little structure. This result is not sufficiently reliable to draw any conclusions. The  $a$  values for the cwi module are consistently the lowest for all catchments, i.e. a smaller contribution to quickflow and therefore a larger baseflow component. They also show an even more pronounced trend of a decrease with increasing BFIHOST values than the estimates for the other two SMA modules. This supports the result of Kokkonen and Jakeman (2001) with respect to this parameter. They also found that using a metric SMA module resulted in a smaller contribution to quickflow.

This result suggests that a parameter estimation procedure is required that considers the functional purpose of the model component within the overall structure. If the parameters representing the component cannot be separated out during calibration, then it is likely that the interaction with the other model components will distort the parameter estimate and limit the chances for success in finding correlations with catchment characteristics.

### 3.3.3. What is the relationship between local parameter identifiability and catchment characteristics?

A model structure that is a sensible representation of the hydrologic system under study should exhibit identifiable parameters if a suitable strategy is applied

for parameter estimation (mainly a suitable OF is defined) and if sufficiently informative data is available. The identifiability of the parameters is therefore an indicator of the suitability of the model structure, within the limits of the information content of the data. It was already mentioned that BFIHOST (the baseflow index, representing underlying geology) is the main characteristic that defines the response behavior of the 10 catchments used in this study. A way to quantify the identifiability (and therefore the uncertainty) of model parameters has been introduced above in the form of an identifiability measure (Fig. 4). Fig. 6 shows the 10 catchments sorted by increasing baseflow index (BFIHOST) from left to right versus the identifiability of the model parameters of the pdm SMA component connected to the parallel linear routing component.

The maximum storage capacity  $c_{\text{max}}$  and the slow flow time constant  $k_{\text{slow}}$  do not show any significant

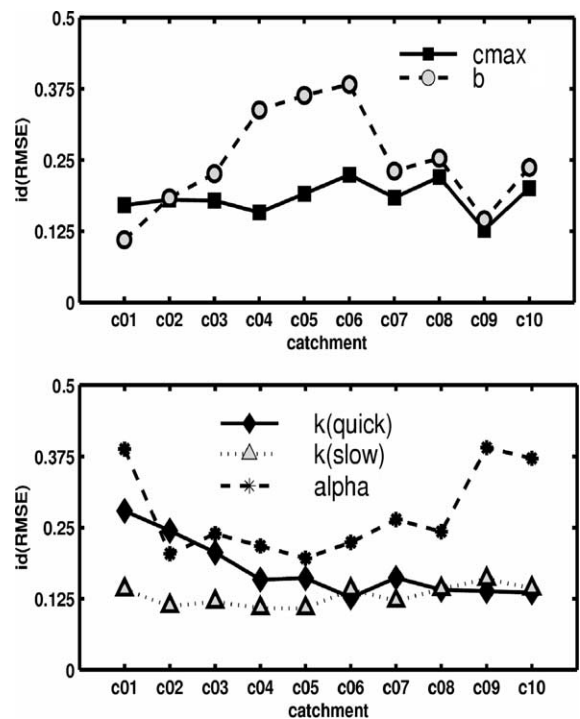


Fig. 6. Plot of identifiability measure values (based on the RMSE) versus catchments (c1–c10) for different parameters. Catchments are sorted from left to right with increasing Baseflow index (BFIHOST).

relationship between baseflow index and identifiability. In case of  $k_{\text{slow}}$  this is likely to be due to the fact that the RMSE, which emphasizes peak flows, was selected. The remaining three parameters on the other hand show a certain trend. The parameter describing the shape of the Pareto distribution of storage elements,  $b$ , is clearly more identifiable for the catchments with a medium baseflow index and mixed geology. This parameter defines the variability of runoff production within the catchment and seems more identifiable when the catchments are more heterogeneous. The quick flow time constant  $k_{\text{quick}}$  is more identifiable for the clay dominated, quick responding, catchments. Here, the difference between quick and slow drainage slopes is very pronounced. The parameter that splits the effective rainfall into quick and slow response is most identifiable in the catchments that are either clearly clay (small baseflow index) or clearly chalk dominated (high baseflow index). Here, the model tends to require that either most of the effective rainfall is feeding the quick or the slow response.

The fact that the parameters of a model, which is commonly assumed to equally represent all catchments included in this study, vary in identifiability is important for the use of this result in a subsequent regionalization step. Each optimal parameter will be one data point when the correlation between parameters and catchment characteristics is calculated. It does not seem to be sensible to give the same weight to identifiable parameters than to parameters that are unidentifiable and therefore highly uncertain. Wagener et al. (2004) show that this uncertainty can be considered during regionalization if a weighted regression procedure is applied (Appendix A). The identifiability measures can be used as weights in this methodology. The results of the weighted regression are summarized in Table 6(b). In general, the catchment characteristics identified as significant in the regional models remain identical to those of the benchmark regression. The main difference is that it is now possible to derive a regional model for  $k_{\text{slow}}$ , though with low predictive power ( $R^2=0.57$ ) and with very wide uncertainty bounds on the regression parameters. Additionally, the uncertainty for the regression parameters for  $c_{\text{max}}$  reduces. The expected values for the regression parameters, and therefore the regionalized parameters, remain almost identical though.

A straightforward way to propagate this uncertainty into the predictions in the ungauged catchment is to calculate the standard uncertainty on the regression parameters (assuming Gaussian errors). This will yield a reasonable estimate if the regression residuals are close to normal (Kottegoda and Rosso, 1997). This will be the case if the variables used in the regression follow normal distributions.

One can then randomly sample from ‘uncertain’ regional regression (for example based on a uniform distribution) and thus produce an ensemble of predictions for the ungauged catchment (Wagener et al., 2004).

### 3.3.4. How is the uniqueness of catchments reflected in regionalization?

Beven (2000) suggests that the uniqueness of the response behavior of individual catchments might be the main reason for the failure of earlier regionalization attempts. A behavior that is unique and (at least currently) unexplainable is a problem often encountered in regionalization studies. It usually shows in the form of outliers when correlations between catchment characteristics and model parameters are calculated. A variety of regionalization studies in the literature shows scatter plots in which one, or very few, catchments behave very differently from the general trend (e.g. Seibert, 1999). The same was found in this study. Fig. 7 shows scatter plots between catchment characteristics and some of the model parameters. The bottom right plot of Fig. 7 shows the quickflow time constant,  $k_{\text{quick}}$ , plotted against DPSBAR, the index of catchment steepness. However, one of the catchments clearly exhibits a unique behavior that is not captured by this catchment characteristic and leads to an unreasonable relationship. A sensible treatment, in absence of any further information, would be the exclusion of the outlier and instead acceptance that the regression relationship will yield sensible results in most cases, but might fail in some. The outlier has to be investigated in more detail separately.

### 3.3.5. What is the effect of local model structural uncertainty on the regionalization result?

Various researchers have shown that current model structures are generally incapable of matching high- and low-flow behavior of a catchment simultaneously with a single parameter set (Gupta et al., 1998; Boyle

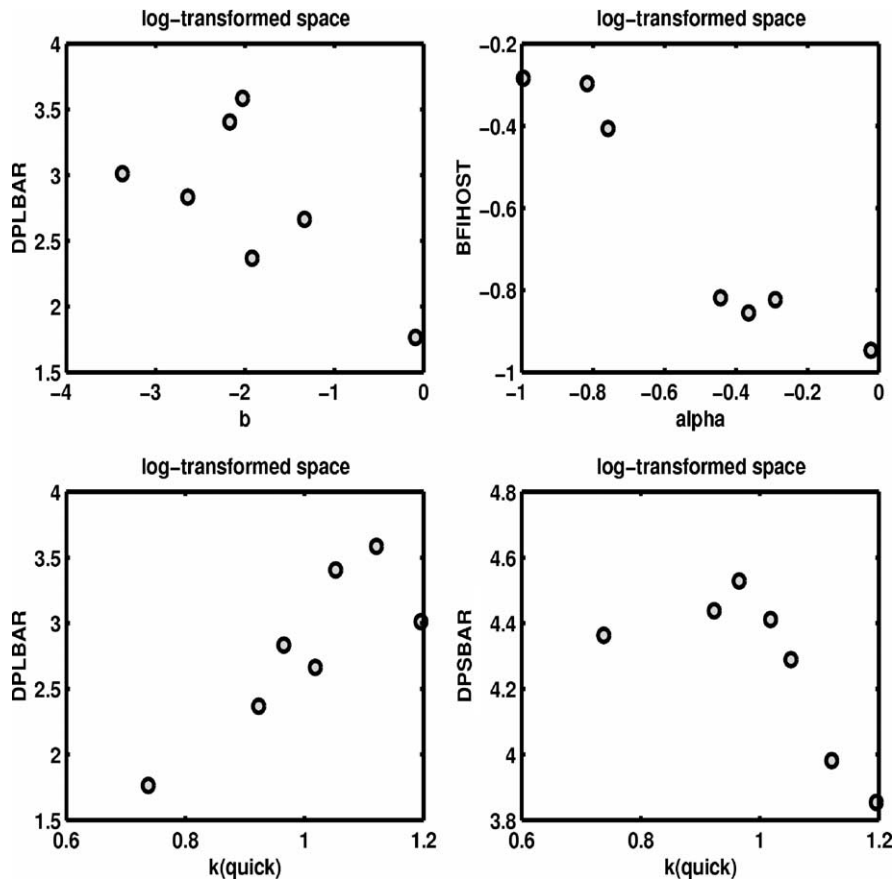


Fig. 7. Scatter plots for some of the correlations used for the conventional regression analysis plotted on log–log scale.

et al., 2000; Wagener et al., 2001). The same structural problem has been found with the model structure applied here. Two different OFs are defined in this study to investigate the effect of this problem on regionalization. Both OFs are based on the RMSE criterion, but only include the residuals if the observed flow is above (FH) or below (FL) a certain threshold value, in this case the mean flow. Fig. 8 shows a scatter plot of the 2-OF space in which every point represents a single parameter set for one of the catchments. One can clearly see a trade-off between fitting these two OFs. Similar observations were made for the remaining catchments. The best performing parameter set with respect to FH is clearly different from the best performing parameter set for FL. FH and FL could for example represent two different modeling objectives (e.g. flood analysis and water resources assessment). The consequence of this result

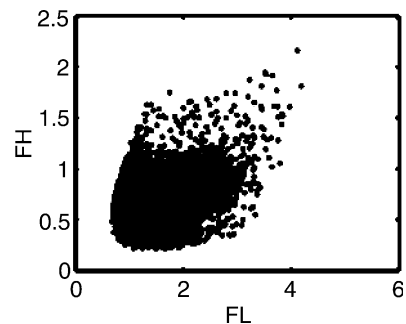


Fig. 8. Scatter plot showing a model population based on 10,000 samples from uniform parameter distributions projected into a 2-dimensional OF space. The two OFs are the RMSE utilizing the residuals at time steps during which the observed flow is above (FH) or below (FL) the mean flow, respectively.

is that two different parameter sets have to be used in the subsequent regionalization step. This is done here for the quickflow time constant  $k_{\text{quick}}$  for which the correlation with all available catchment characteristics has been tested. When using FH, this parameter is highly correlated with the catchment size and drainage path configuration with a correlation coefficient of 0.84 (significant at the 1% level). However, for small runoff events during dry periods, analyzed using FL,  $k_{\text{quick}}$  is highly correlated with the rainfall characteristics represented by SAAR (correlation coefficient = 0.96, or after log transformation = 0.99) and RMED-2D (correlation coefficient = 0.92, or after log transformation = 0.90). This result suggests that the runoff mechanism in the catchment changes between events during wet periods (FH) and dry periods (FL). Recent research by McGlynn and McDonnell (2003) showed for a humid catchment that the hillslopes were only contributing to runoff production during major storm events. No water from the hillslopes contributed to the streamflow during low flow periods, and therefore little correlation between topography and quick response should be expected during these periods. The correlation result here suggests that it might be difficult to derive sensible regional relationships if a model parameter, and therefore a model component, represents different processes depending on the catchment state.

In addition, the optimum  $k_{\text{slow}}$  values with respect to FL show a high correlation to BFIHOST (correlation coefficient = 0.80, or after log transformation = 0.80) and PROPWET (correlation coefficient = 0.83, or after log transformation = -0.79). This result (compared with the lack of correlation when using RMSE) suggests that a regional model for this parameter might better be derived from the local model fit to low flow periods.

### 3.3.6. Can an alternative regional procedure yield more identifiable and therefore better regional relationships?

A sequential regionalization procedure has been proposed by Lamb et al. (2000) (see also Lamb and Calver, 2002) in which a URS procedure is used to sample the feasible parameter space for all gauged catchments. The identifiability of the model parameters is then judged subjectively (over all catchments) using dot plots and a regional (regression)

model is derived for best values of the most identifiable parameter. This parameter is subsequently set to its regional value for all the gauged catchments and the URS is repeated for all catchments. The new most identifiable parameter is then again fixed to its regional value. The procedure is repeated until regional models for all the parameters are derived.

The same approach is applied here, though the subjective selection is replaced by calculating the identifiability measure introduced in Fig. 4. The measure is calculated as the average of all maximum identifiability values over all catchments for each of the five model parameters. In this way, it is possible to objectively (within the assumptions made when deriving the measure) judge whether the assumption that the identifiability of the remaining parameter increases is correct and it allows an objective selection of the parameters to be regionalized at each iteration step. Fig. 9 shows the average identifiability of each of the five parameters during the five iteration stages. One can easily see that the identifiability of the remaining parameters increases every time one of the parameters is fixed. Ultimately, even  $k_{\text{slow}}$ , which initially is not identifiable (due to the use of the RMSE), shows clear optima. However, the improvement in identifiability did not lead to improved regional relationships for the different parameters as can be seen in Table 6(c).

The most identifiable parameter after the first iteration is  $a$ , for which the regional model is therefore identical to the one derived in the benchmark regression. This parameter is subsequently fixed to its regional value and another URS with 10,000 samples is executed for all catchments. The shape parameter  $b$  is identified as most identifiable and fixed following the second iteration. However, it was not possible to derive a sensible regional relationship to any of the catchment characteristics available after  $\alpha$  was fixed to its regionally derived values. Parameter  $b$  is therefore fixed to its median value of 0.16. The next two parameters  $k_{\text{quick}}$  and  $c_{\text{max}}$  (after another iteration) show some correlation with the ASPVAR, DPSBAR and ASPVAR respectively, but their regional models yield lower  $R^2$  values than the benchmark models (Table 6(c)). Also, the uncertainty for the intercept of the regional model for  $c_{\text{max}}$  is very high. No regression models could be derived for  $b$  and  $k_{\text{slow}}$  using this approach.

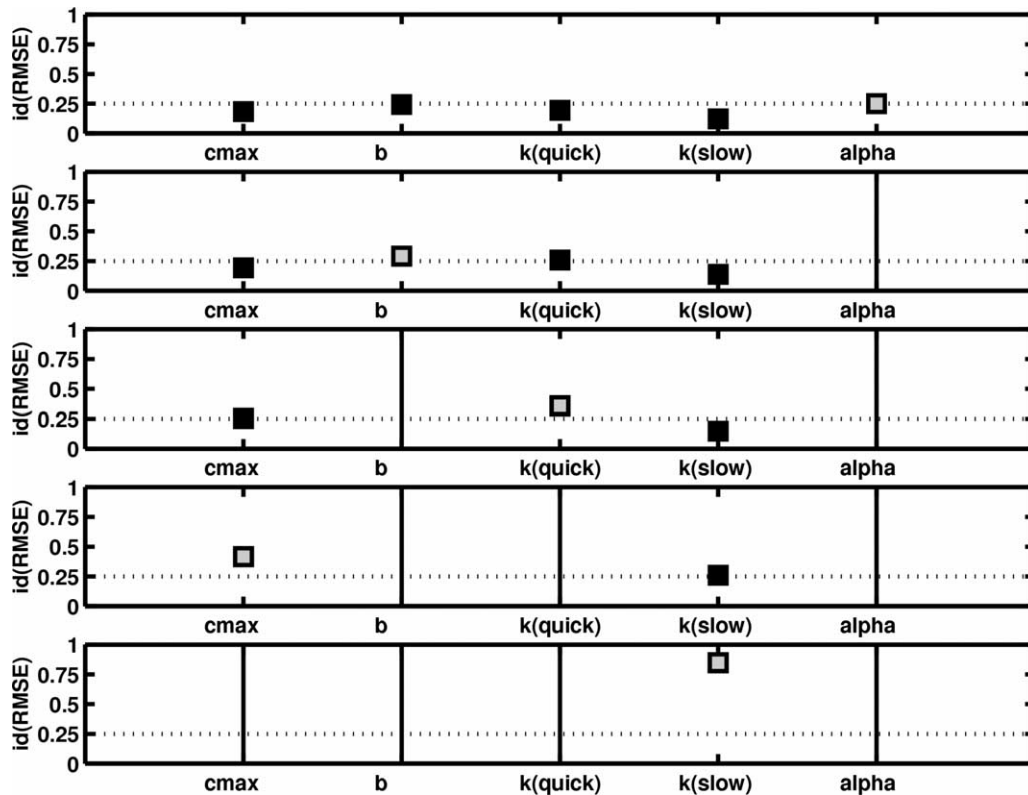


Fig. 9. Variation in parameter identifiability with iteration step during the sequential regionalization procedure. A vertical bar indicates a fixed parameter. The most identifiable parameter for each iteration is shown as a grey rectangle. The first iteration is shown at the top, one parameter is fixed during each iteration (from top to bottom).

This result suggests that the bias, which is introduced every time when one of the parameters is fixed to its regional value, reduces the chance of finding correlations for the remaining ones. One reason for this effect is probably the use of a single OF. Hogue et al. (2000) show that a local sequential calibration procedure can be very successful for a relatively complex (13 parameters) rainfall-runoff model structure. They used two OFs, the RMSE (emphasizing the fit to high flows) and the RMSE of the log-transformed data putting higher emphasize on low flows. Splitting the model parameters into two groups, each one better identifiable using one of the two OFs, resulted in a satisfactory calibration result, i.e. the overall fit to the hydrograph was good and significantly less biased towards a particular feature as in single-objective calibration. A similar sequential approach utilizing different OFs might be successful

at regional scale, though it might be less applicable to a simple model structure. This result shows that OFs have to be parameter specific in order not to introduce bias into the parameter estimates. Seibert (1999) used multiple OFs that he combined into a single Fuzzy measure of performance to compensate for the single objective bias problem outlined above.

A comparison of the different approaches applied to the Teise at Stonebridge catchment that was excluded from the regression analysis is shown in Table 7. Listed are the model parameters derived using the different regression approaches discussed above and the values derived through local calibration together with the corresponding Nash–Sutcliffe Efficiency (NSE) values. All approaches show reasonable results with the sequential method showing the lowest NSE value, mainly due to a low estimate for  $c_{\max}$ . The consistency in  $a$  between

Table 7  
Comparison of regionalized and locally calibrated parameter values for the Teise at Stoenbridge

Approach	$c_{\max}$	$b$	$k_{\text{quick}}$	$k_{\text{slow}}$	$a$	NSE*
Local calibration	316.5	0.125	1.66	92.1	0.72	0.79
Conventional regression	304.4	0.181	2.61	190.4	0.72	0.78
Weighted regression	306.5	0.158	2.60	148.8	0.74	0.78
Sequential regression	227.2	0.162	2.70	197.9	0.72	0.76

\*Nash–Sutcliffe efficiency.

the approaches underlines the high identifiability and strong correlation with catchment characteristics of this parameter. Future studies with a larger number of catchments will provide a better measure of comparison between the approaches.

#### 4. Conclusions

The prediction of the hydrologic response of ungauged catchments is a fundamental problem in hydrology. One of the approaches to solve it is to apply a model to a large number of gauged catchments and to derive statistical relationships between model parameters and catchment characteristics, so called regionalization. This has so far been done with only limited success, particularly for continuous simulation models. Some aspects that limit the success of this type of approach are discussed and evaluated in this paper, and potential ways forward are suggested. The main conclusions of this paper are as follows:

The dominating catchment descriptor separating the different response types of the study catchments is the baseflow index.

The benchmark regionalization shows a typical result with only a few parameters showing any sizeable amount of correlation with catchment characteristics.

Embedding a particular model component within two different model structures resulted in different optimal parameter values for this component. This result suggests that parameter interaction during calibration can hinder the identification of a parameter in a way that considers its intended (functional) role in the model. The latter being a

necessary condition for successful regionalization. The results of this study suggest that the identifiability of a parameter is related to its importance in representing the catchment's response. A weighted regression procedure has been proposed to avoid giving well and poorly identified parameters the same weight during regionalization.

Many regionalization studies contain outliers, i.e. one parameter shows a clear correlation with a particular catchment characteristic, but one of the catchments is behaving differently. This catchment should be taken out of the analysis and investigated separately. One therefore derives a better regionalization relationship, which will however fail for certain cases.

Model structural error introduces ambiguity into the identification of optimum parameter values. This ambiguity is propagated into the regionalization and different regional relationship will sometimes be found for the same parameter when optimizing the model to different parts of the hydrograph. This was found to be true here for the quick flow time constant, which was either related to the topography (during high flows) or to rainfall characteristics (during low flows).

Sequential regionalization procedures are useful in increasing the identifiability of parameters. However, there is a danger of introducing bias into the calibration and therefore into the regional relationships. Additional research is required to develop appropriate procedures of this type for relatively simple model structures.

The conclusions listed above focus on the procedures applied in this study and their



underlying assumptions. The exact values derived are unlikely to be very robust due to the small number of catchments that was available. The results were, however, sufficient to provide a general trend. A more extensive study with a larger number of catchments is currently in progress. In conclusion, we note that recent years have shown an increased interest in the modeling of ungauged catchments using continuous rainfall-runoff models. However, many unanswered questions remain for the time being. In addition, progress in parameter estimation procedures and parsimonious modelling still have to be fully incorporated into regionalization approaches. This study demonstrated that care has to be taken during the model calibration stage if meaningful results are to be obtained during regionalization. Of particular importance is the functional role of the parameter within the model that has to be maintained if meaningful relationships to catchment characteristics are the objective. This study suggests that the functional role of parameters has to be given equal importance to model performance in MOPEX comparison studies.

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### Appendix A

The identifiability measure introduced in the main text can be used as a weight in a regression model to enhance the influence of better-identified parameters on the final regional model. This can be written in

mathematical form as weighted linear regression (e.g. Weisberg, 1980, p. 75f.). Starting with the standard regression model,

$$\theta_L = \Phi\theta_R + \varepsilon_R \quad (10)$$

the generalized least squares solution is given by

$$\hat{\theta}_R = (\Phi^T A^{-1} \Phi)^{-1} \Phi^T A^{-1} \theta_L \quad (11)$$

in case of a weighted least squares, where a higher weight indicates a more precise estimate. The weighting function  $A$  can be written as

$$A = \begin{bmatrix} w_1^{-1} & & & \\ & w_2^{-1} & & 0 \\ & & \ddots & \\ 0 & & & w_n^{-1} \end{bmatrix} \quad (12)$$

one can then define an  $n \times n$  matrix,  $C$ , for which  $C^T C = A^{-1}$ .  $C$  is called the square root of  $A^{-1}$ . Subsequently  $C$  is

$$C = \begin{bmatrix} \sqrt{w_1} & & & \\ & \sqrt{w_2} & & 0 \\ & & \ddots & \\ 0 & & & \sqrt{w_n} \end{bmatrix} \quad (13)$$

which can be transformed into an ordinary least squares problem by multiplying both sides of Eq. 10 by  $C$ , which leads to

$$C\theta_L = C\Phi\theta_R + C\varepsilon_R \quad (14)$$

one can then define  $Z = C\theta_L$ ,  $W = C\Phi$  and  $\delta = C\varepsilon_R$ , leading to

$$Z = W\theta_R + \delta \quad (15)$$

with the ordinary least squares solution being

$$\hat{\theta}_R = (W^T W)^{-1} W^T Z \quad (16)$$

An ordinary or unweighted least squares problem is therefore obtained from a weighted least squares problem through transformation of all the involved variables. Each observation of dependent and independent variables, and the intercept term (which is a column of ones, added to the matrix of independent variables) has to be multiplied by the square root of the weight for that point  $\sqrt{w_i}$ . Estimates of the parameters, tests, confidence intervals, and residuals

can then be derived using ordinary least squares as described earlier (e.g. Weisberg, 1980, p. 76).

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