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Forces in Joints

> ...mechanical science is of all the noblest and most useful, seeing that by means of this all animate bodies which have movement perform all their actions...
>
> Leonardo Da Vinci (1452–1519)

1.1 Introduction

The skeleton is first and foremost a mechanical organ. Its primary functions are to transmit forces from one part of the body to another and to protect certain other organs (e.g., the brain) from mechanical forces that could damage them. Therefore, the principal biologic role of skeletal tissues is to bear loads with limited amounts of deformation. To appreciate the mechanical attributes that these tissues must have to perform this role, it is necessary to learn something about the forces which whole bones normally carry. In most cases, these forces result from loads being passed from the part of the body in contact with a more or less rigid environmental surface (e.g., the heel on the ground when walking) through one or more bones to the applied or supported load (e.g., the torso). In addition to the forces transmitted in bone-to-bone contact, muscle and ligament forces act on the bones, and these forces (especially the muscle forces) are large and important.

Most muscle, ligament, and bone-to-bone forces act in or near the body's major diarthroidal joints. The purpose of this chapter is to explain how conventional engineering analysis may be used to estimate joint forces, and to provide some practical experience in solving such problems.

1.2 Static Analysis of Forces in Joints

Forces in the Elbow Joint

To introduce this subject, we consider the human elbow joint. We choose this joint because it works more or less as a simple hinge, and because it is familiar to us, conforming to the image that we acquired in grade school of the skeleton as a system of levers. Although we use this simplicity in solving for the forces, we should spend a few moments examining the anatomy to appreciate the simplifying assumptions that we are making.

A lateral view of the bones of the elbow is shown in Fig. 1.1A. The olecranon of the ulna wraps around the distal end of the humerus to form the major part of the elbow's hinge joint. The proximal joint surface of the radius articulates with the distal joint surface of the humerus and also with a cartilage-covered notch (the radial notch) on the lateral aspect of the ulna; this means there are three articulations in the elbow joint. These articulations allow for flexion and extension of this joint as well as pronation and supination[1] of the forearm. The radius and ulna are bound together by the interosseous membrane along the central part of their length and by several ligaments at their proximal and distal ends (not shown). We assume that these structures cause the two forearm bones to act as one structural unit. We also assume that the wrist joint is

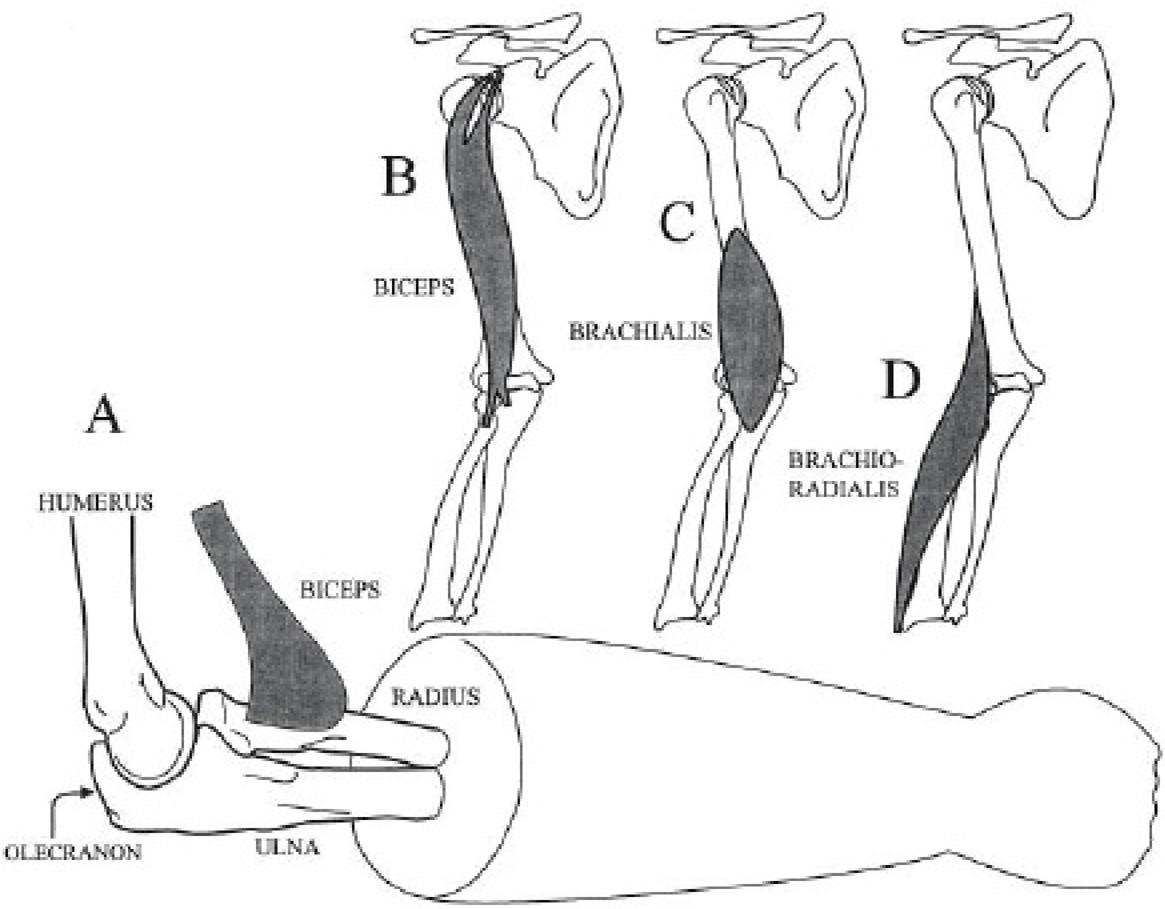


FIGURE 1.1A–D. Anatomical drawings of the elbow joint. **A** Overall view of the flexed joint and the biceps inserting primarily on the radius. **B**, **C**, **D** Approximate locations of the biceps, brachialis, and brachioradialis muscles.

[1]For the definition of these and other anatomical terms, see a human gross anatomy text, e.g., Grant's (Agur, 1991) or Gray's (Williams, 1995).

stabilized by its musculature so that the hand and forearm flex about the elbow as a simple hinge joint.

When the elbow is flexed with the palm upward, two muscles are primarily responsible: the biceps and the brachialis (Fig. 1.1B and 1.1C). The biceps has two heads (i.e., points of origin). The short head originates on the coracoid process of the scapula, while the long head runs through the shoulder joint to attach to the superior lip of the glenoid fossa (the scapular part of the shoulder joint). The biceps divides distally into a tendon that inserts on the proximal radius and an aponeurosis (tough band of connective tissue) which blends with other muscles in the proximal forearm (Fig. 1.1B). The brachialis originates along the anterior surface of the central humerus and inserts on the proximal ulna (Fig. 1.1C). When the forearm is flexed rapidly, or a large force must be applied during flexion, the brachioradialis also acts between the distal humerus and the distal radius (Fig. 1.1D). Other muscles also cross the elbow and participate in flexion. However, to simplify matters, and because the present problem is for a static situation, we assume that the biceps and brachialis muscles act as one in holding the elbow in a flexed position against the action of a weight held in the hand. We want to find the force required in the "biceps-brachialis muscle" to support the flexed forearm and the total force exerted on the distal end of the humerus by the radius and ulna. These forces are expressed as multiples of the weight in the hand.

To solve this problem and others like it, three steps are necessary:

1. Draw a free-body diagram.
2. Write the equations of static equilibrium.
3. Solve these equations simultaneously to obtain the unknown forces.

We simplify the analysis by assuming that it is two dimensional and that the forces all act in a sagittal plane containing the humerus and the forearm. The free-body diagram is constructed by isolating the structure being analyzed such that the internal forces to be determined are exposed and replacing all the forces acting on the structure by vectors. The free-body diagram for the elbow problem is shown in Fig. 1.2. The forearm is made

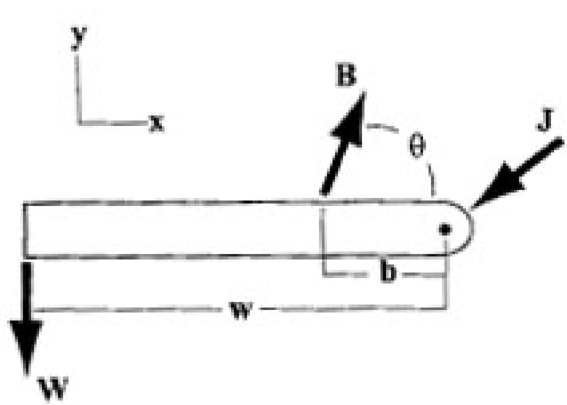


FIGURE 1.2. Free-body diagram for the elbow force calculation. **J** is the joint reaction force, **W** is the weight in the hand, and **B** is the biceps force.

a free body by "amputating" through the elbow joint, whose internal forces we wish to reveal. The forearm lies in the x–y plane. The weight in the hand, the muscle force, and the joint reaction force are represented by the vectors **W**, **B**, and **J**, respectively.[2] The joint force is assumed to pass through a fixed center of rotation for the joint, shown by a dot. The insertion point of the muscle is **b** meters (m) along the forearm from the joint center, and the center of gravity of the weight in the hand is **w** meters from this point. The equations for static equilibrium in two dimensions are

$$\begin{aligned} \Sigma M &= 0 \\ \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \end{aligned} \qquad (1.1)$$

where M stands for moments about an arbitrary point, and F_x and F_y are force components in the x- and y-directions. Because, in two dimensions, there are three such equations, one may solve for three unknown force *components*. Alternatively, as usually happens, one may solve for one force *vector* and the *magnitude* of another force of known direction.

For the elbow problem, taking moments about the joint center, the equilibrium equations are

$$\Sigma M = wW - bB \sin \Theta = 0 \qquad (1.2)$$

$$\Sigma F_x = B \cos \Theta - J_x = 0 \qquad (1.3)$$

$$\Sigma F_y = B \sin \Theta - W - J_y = 0 \qquad (1.4)$$

(Here, any moment of the x-component of **B** about the joint center is ignored.) Solving these equations yields

$$B = wW/b \sin \Theta \qquad (1.5)$$

$$J_x = B \cos \Theta \qquad (1.6)$$

$$J_y = B \sin \Theta - W \qquad (1.7)$$

If $\Theta = 75°$, $w = 0.35$ m, and $b = 0.04$ m, then $B = 9.1\ W$, $J_x = 2.3\ W$, and $J_y = 7.8\ W$. The magnitude of the joint reaction force is $J = (J_x^2 + J_y^2)^{1/2} = 8.1\ W$, and its orientation is $\arctan(J_y/J_x) = 74°$ with respect to the x-axis. Thus, the muscle and joint reaction forces are eight to nine times greater than the weight held in the hand. This result is typical of virtually all the joints in the body in that the skeleton works at a mechanical disadvantage in terms of force, and as a consequence the forces acting on bones are high relative to the forces applied by (or to) the environment.

There is, of course, a good reason for this. Muscles can only contract about 30% of their length. In the case of the biceps, for example, the overall length is about 25 cm, so the maximum contraction is 7–8 cm. The lever

[2] In this chapter, vector quantities are represented in bold type and scalars (such as vector magnitudes) in ordinary type.

action of the forearm magnifies this distance by the ratio w/b in Fig. 1.2, enabling the hand to move much further (and also much faster). Of equal importance, the muscles insert proximally on the radius and ulna and do not create a "web" of flesh across the front of the elbow. Therefore, magnification of motion enables larger movements to be accomplished with a compact structure. The price that is paid for this compactness and magnification of motion is high forces in the muscles, across the joint surfaces, and within the bones.

Box 1.1 Historical Note
Giovanni Borelli on the Movement of Animals

Giovanni Alfonso Borelli (1608–1679) was the greatest of early biomechanicians. He held the chair in mathematics at Pisa in Italy, where he was a close friend of Malpighi, who was the professor of theoretical medicine. Together, they did much to pursuade seventeenth-century physicians of the importance of physics in understanding medicine and physiology. Borelli's treatise, *De Motu Animalium (On the Movement of Animals)*, has been translated into English and is a marvelous testament to his genius and ability to explain musculoskeletal mechanics clearly and simply. In the preface to his translation, Maquet points out that later biomechanicians unwittingly duplicated several of Borelli's important discoveries. While some of his results contain errors, it must be remembered that Newton's laws were not published until 1687. Thus, although Borelli understood very well the principle of balancing moments about a fulcrum, he could not have been expected to fully understand static equilibrium as we do, based on Newton's laws. One of Borelli's greatest achievements was the discovery that animals' joints work at a mechanical disadvantage in terms of force. He discusses this in the following excerpt.

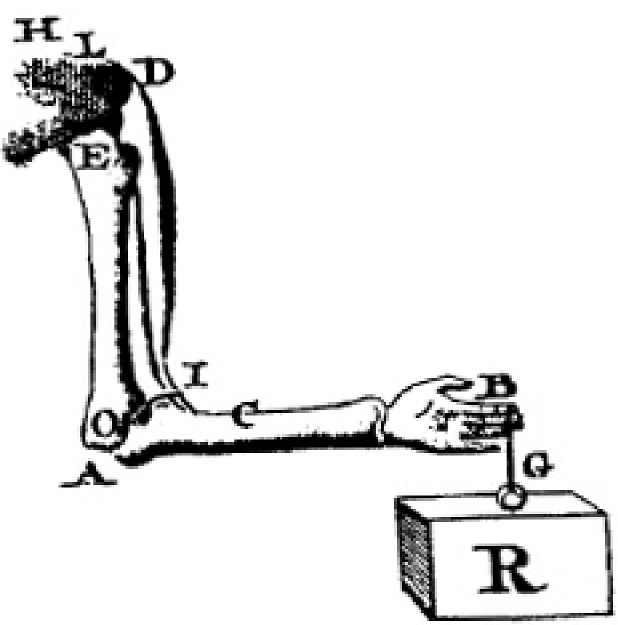


Diagram of elbow force problem from Borelli's *On the Movement of Animals* (translation by Maquet, 1989).

Proposition VIII

"It is commonly thought that Nature raises considerable weights by using the machines of the muscles with a weak moving force.

"The magnitude of the vital force of the muscles ... sustains, raises, and moves not only an arm or a leg, but the whole animal machine, enabling it even to dance. Besides the mass of the animal, heavy enough by itself, this force carries, pulls and pushes considerable weights.

"Aristotle...did not recognize the muscles but imagined spirits which pull and push the limbs. [He] remarked how difficult it would be for the huge mass of an elephant to be moved...by tenuous spirit or wind. He met the difficulty by saying that Nature moves the joints and limbs of the animal by using very small force...and said that the operation is carried out by way of a lever. Therefore, it is not surprising that huge weights can be moved...by a small force. Lucretius used the same example....Galen also says that a tendon is like a lever. He thinks that, consequently, a small force of the animal faculty can pull and move heavy weights.

"This general opinion seems to be so likely that, to my knowledge, not surprisingly, it has been questioned by nobody. Who indeed would be stupid enough to look for a machine [in the body] to move a very light weight with a great force, i.e., ...not to save forces but rather to spend forces? And if this is rightly considered as stupid, how is it possible that wise Nature, everywhere looking for economy, simplicity and facility, builds with great efficiency in animals machines to move, not heavy weights with a small force, but on the contrary, light weights with almost boundless force? ...I shall demonstrate that multiple and different machines actually are used in the motions of animals but that light weights are carried by large and strong force rather than heavy weights being supported by small force."

Forces in the Hip Joint

The hip joint is the articulation between the femur and the acetabulum of the pelvis. The hip is one of the most important joints in the body from a medical perspective. Especially in aging individuals, overall health is promoted by the exercise that goes with walking. The ability to walk depends on having a healthy, painfree hip. Two kinds of diseases, both very common among the elderly, preferentially affect the hip. *Osteoarthritis* is the primary reason that about 200,000 total[3] hip replacement procedures are performed each year in the United States, and *osteoporosis* in the femoral neck results in several hundred thousand hip fractures each year, virtually all of which require surgical treatment with a metal fixation device. To better understand the mechanics of the hip and the demands on the implants used to correct its problems, it is important to know the forces across the hip during walking and their determining factors.

[3]The word "total" refers to the replacement of both the femoral and acetabular sides of the joint. Sometimes only the femoral side is replaced.

When one is walking, the lower extremities and other parts of the body are moving (i.e., accelerating), so the conditions of static equilibrium are not satisfied. However, most people do not usually walk very vigorously, so the accelerations are small relative to the forces produced by muscle pulls and gravity. Therefore, the problem is usually solved as though the person were simply standing on one leg, assuming that this approximates the conditions during the "single leg stance phase" of gait, that is, when all the weight is being carried by one leg. In addition to this assumption, we assume that the problem is two dimensional, in the frontal plane, and that only one muscle is acting. If you try to stand on one leg, the force pulling your center of gravity downward tends to rotate your torso toward the medial side of the leg you are standing on. The muscles that resist this movement are the same ones which you use to abduct your thigh when lying down (i.e., move your lower limb away from the body's center line). Again, several muscles act to do this, but we can lump them all together and simply call them the "abductor muscles."

Again, the first step in calculating the forces in the joint is to draw a free-body diagram, "amputating" through the joint in question to "reveal" the force vectors acting there. In this case we delete the lower extremity and study the remaining portion of the body. Figure 1.3 shows this situation. It is

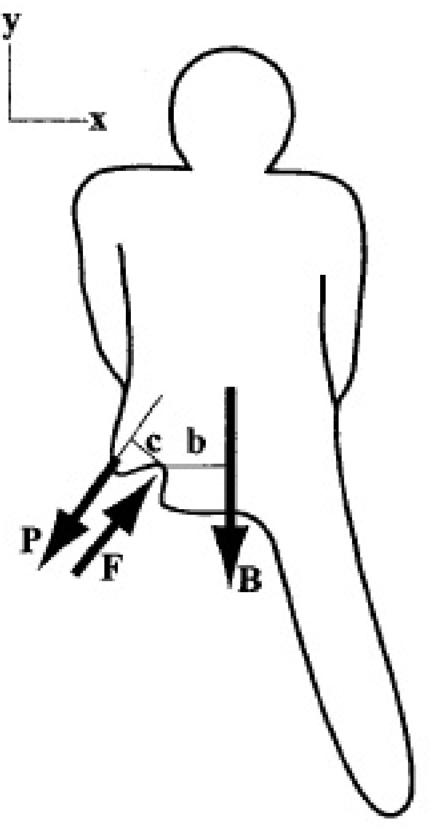


FIGURE 1.3. Free-body diagram for the calculation of the hip joint force while walking. The "amputated" leg is the one supporting the body. The leg in the free-body diagram is not in contact with the ground. **B** is the weight of the body (minus the amputated leg), **P** is the abductor muscle force, and **F** is the joint reaction force.

important to understand that the subject was standing on the leg that has been "amputated," and that the other leg, of which only a portion is shown, was not touching the ground. The abductor muscle force is represented by the force vector **P**. The joint reaction force acting on the middle of the acetabulum in the pelvis is **F**. The weight of the body, represented by the vector **B**, is actually the entire body weight, W, minus the weight of the leg supporting the body. Because each lower extremity weighs about 1/6 W, we let $B = 5W/6$. This force acts downward slightly to the right of the centerline of the body. Taking moments about the center of the acetabula, we have

$$\Sigma M = cP - bB = 0 \tag{1.8}$$

$$P = (b/c)B = (b/c)(5/6)W \tag{1.9}$$

The lengths of the moment arms b and c have been estimated from anteroposterior pelvic radiographs. It was found that the b/c ratio ranges between 2 and 3.5. Following the lead of Frankel and Burstein (1970), we choose a conservative value of 2.4 and obtain the convenient result that $P = 2W$. That is, the force required in the abductor muscles to balance the body on the head of the weight-bearing femur during the stance phase of gait is twofold body weight.

To solve the rest of the problem, we write the equations for force equilibrium, assuming that the x-direction is horizontal and the y-direction is vertical:

$$\Sigma F_x = F_x - P_x = F_x - 2W \sin \Theta = 0 \tag{1.10}$$

$$\Sigma F_y = F_y - P_y - B = F_y - 2W \cos \Theta - 5W/6 = 0 \tag{1.11}$$

where $\Theta = 30°$ is the angle that the abductor muscle line of action makes with the y-axis. Because $\sin 30° = 0.5$, the components of the joint reaction force F are

$$F_x = W \quad \text{and} \quad F_y = (2 \cos 30° + 5/6)W = 2.57W \tag{1.12}$$

The *horizontal* force of the femur on the pelvis is equal to body weight, and the vertical force is 2.5 times as much. The total joint reaction force is $F = (F_x^2 + F_y^2)^{1/2} = 2.75W$. This force acts at an angle to the horizontal $\Theta = \arctan(F_y/F_x) = 68.7°$.

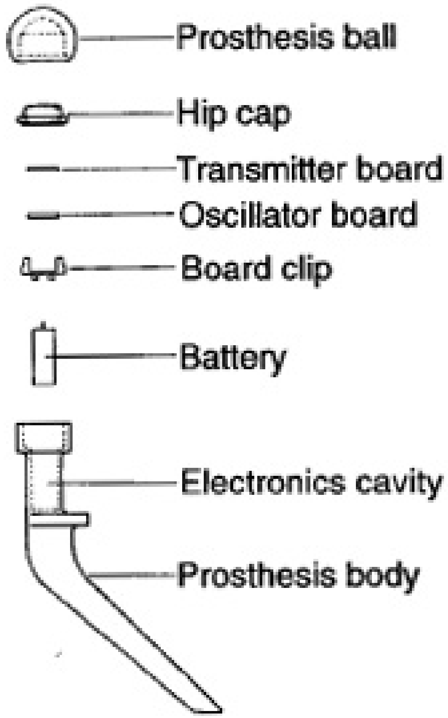
We have seen that the magnitude of the forces in the hip joint depends critically on the ratio of the body weight moment arm to the abductor muscle moment arm. Thus, anything that increases the former or decreases the latter increases the abductor muscle force required for gait and consequently the force on the head of the femur as well. People with short femoral necks have higher hip forces, other things being equal. More significantly, people with a wide pelvis also have larger hip forces. This tendency means that women have larger hip forces than men because their pelves must accommodate a birth canal (Burr et al., 1977). This fact may be one reason that women have more hip fractures and hip replacements because of arthritis than men do. It is also conceivable that this places women at a biomechanical disadvantage with respect to some athletic activities, although studies do not always show gender differences in the biomechanics of running, particularly endurance running (Atwater, 1990).

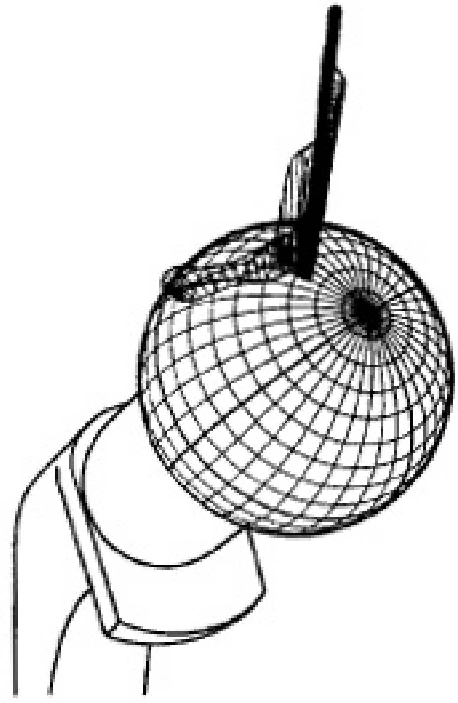
Box 1.2 Technical Note
Measurement of Hip Joint Forces In Vivo

To verify the estimates of hip joint forces made using free-body calculations, four groups of investigators have implanted devices that allowed measurement of hip joint forces into hip replacement patients (Rydell, 1965, 1966; English and Kilvington, 1979; Davy et al., 1988; Bergmann et al., 1988, 1993). When humans are used as experimental subjects, ethical concerns demand that the subject's health not be endangered in any way. This ethical mandate has extensive technical consequences for measurement of hip joint forces. To begin with, a healthy human subject cannot be used; experimentation must be restricted to a subject with a deseased or injured hip who will be undergoing replacement of the proximal femur with a prosthesis by way of treatment for this condition. This condition provides the opportunity for using an instrumented prosthesis that can measure joint forces instead of a standard prosthesis. However, the instrumented prosthesis must be as strong and as durable as a standard one, which imposes important constraints on the instrumentation design. Moreover, wires cannot be run from the prosthesis to the surface of the skin, where they could pose a risk of infection or other problems. Consequently, the instrument must be fully contained within the prosthesis, with a working space of a few cubic centimeters, and it must be capable of transmitting data to a receiver outside the body while the patient walks.

The solution to this technical problem has been to use strain gauges (3 to 12 in number) to measure strains on interior surfaces of the prosthesis, and microelectronic circuitry to process the strain signal, which is broadcast as an FM signal to a receiver held against the skin. The device must be fully calibrated in the laboratory before insertion into the subject, so that the measured strains can be converted to force components on the ball or head of the prosthesis. The power source in the prosthesis used by Davy et al. (1988) was a battery located in the stem of the prosthesis and activated by a magnetic switch turned on by a magnet held outside the patient's leg (left figure). Because batteries carry a slight risk of releasing toxic components, and because they have a finite life, other investigators (Bergman et al., 1993) used an inductive power source. A magnetic coil wrapped around the subject's leg induced current in a receiver coil within the prosthesis.

One might question whether a patient who has a degenerated joint, has recently had surgery on it, and is in a laboratory environment, is capable of walking "normally"—but these are the best data available. The findings of all four studies are consistent with one another and with the theoretical estimates. During the single leg stance phase of gait, hip joint forces between 2.5 and 3.3 times body weight were measured (right figure). The highest force recorded was





Left figure: Exploded view of telemetry electronics inside an instrumented hip prosthesis. (Reproduced with permission from Davy et al., 1988.) *Right figure:* Three-dimensional plot of hip joint force vector during a gait cycle (with crutches). Orientations and lengths of lines emanating from the surface of the prosthesis head indicate force direction and magnitude, respectively. (Reproduced with permission from Davy et al., 1988.)

in an individual who was attempting static single-leg balancing; during recovery from a momentary loss of balance, a force of 5.5 body weight was measured in the hip (Davy et al., 1988).

Clinical Significance of High Joint Forces

Because diarthroidal joints work at a mechanical force disadvantage so that limbs can move far and rapidly with short muscle contractions, high stresses are produced in the tissues of the bones and joints. Normally, these tissues carry their loads without causing pain, but various diseases and injuries can damage the tissues so that the deformations associated with loading are painful. To some degree, the pain is proportional to the amount of force carried by the tissues; in other words,

$$\text{Pain} = \text{Force} \times \text{Disease}$$

There are no nerves in cartilage, and the source of joint pain is poorly understood, but experience shows that reducing joint forces often alleviates pain. Often the physician is not able to do much about the disease, so the first consideration in controlling pain may be to reduce the forces in the joint. For example, the patient can lose weight, or walk with a cane, and thereby reduce the forces transmitted across the joint.

1.3 Hip Forces in Human Ancestors

Physical anthropologists have analyzed hip joint forces in skeletons of various hominids. Of particular interest is *Australopithecus* because of collateral evidence (footprints) that these individuals may have walked very much like modern humans. Figure 1.4 is a depiction of the famous *Australopithecus* skeleton known as Lucy. The differences between the anatomy of the femur and pelvis of *Australopithecus* and *Homo sapiens* include factors that seem to be very pertinent to hip joint force. For example, the neck of the *Australopithecus* femur (stippled in Fig. 1.5, overlaying the outline of a modern femur) was proportionately longer than ours, but the bone was smaller overall. The pelvis was also smaller overall, but the ilium (shown stippled in Fig. 1.6, on one side of a modern human pelvis of similar size) was more outwardly flared, moving the line of action of the abductor muscles away from the hip joint. These factors affect the moment arms of both the abductors and the body weight vector. The analysis of Lovejoy and co-workers (1973) indicates that the hip joint force in *Australopithecus* was about 2.5 fold body weight, a value somewhat less

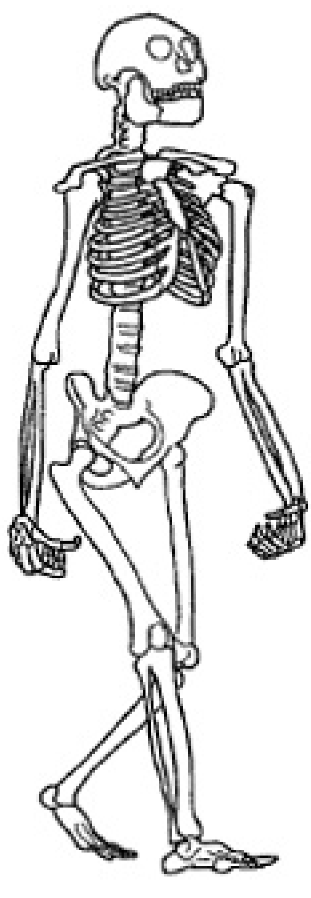


FIGURE 1.4. Artist's conception of the skeleton of *Australopithecus afarensis* ("Lucy") walking. (Reproduced from McHenry, 1991.)

FIGURE 1.5. Lucy's proximal femur *(stippled)* compared to that of *Homo sapiens*. (Redrawn with permission from Lovejoy et al., 1973.)

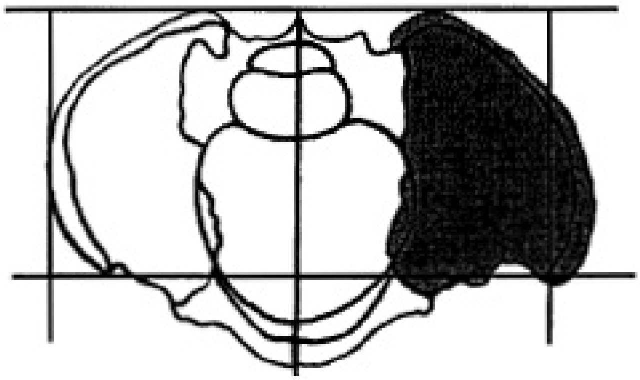


FIGURE 1.6. Lucy's ilium (*stippled*, on *right*) compared to that of *Homo sapiens* (on *left*). The left and right *vertical lines* would contain the modern pelvis; notice that Lucy's ilium projects well beyond the line on the right. (Redrawn with permission from Lovejoy et al., 1973.)

than that of modern humans. However, when they performed a similar analysis on the skeletons of Native Americans excavated at archeological sites (and thus more comparable in their condition to fragmented *Australopithecus* skeletons), they obtained a value of 2.5 for these as well. Therefore, Lovejoy et al. concluded that *Australopithecus* and modern humans experienced similar hip forces. They then considered the *pressure* on the head of the femur in these two species. Considering the degree to which the head of the *Australopithecus* femur was smaller than ours, and the estimated difference in body weights, it was concluded that the pressure on the *Australopithecus* cartilage was about half that on our femoral heads. However, these estimates are quite tenuous because of the scarcity and fragmented condition of the *Australopithecus* skeletons. Later in their paper, Lovejoy et al. conceded that other observations suggest that the *Australopithecus* hip force may have been substantially less than that of *Homo sapiens*. Two of the exercises at the end of this chapter allow you to explore this application of skeletal biomechanics.

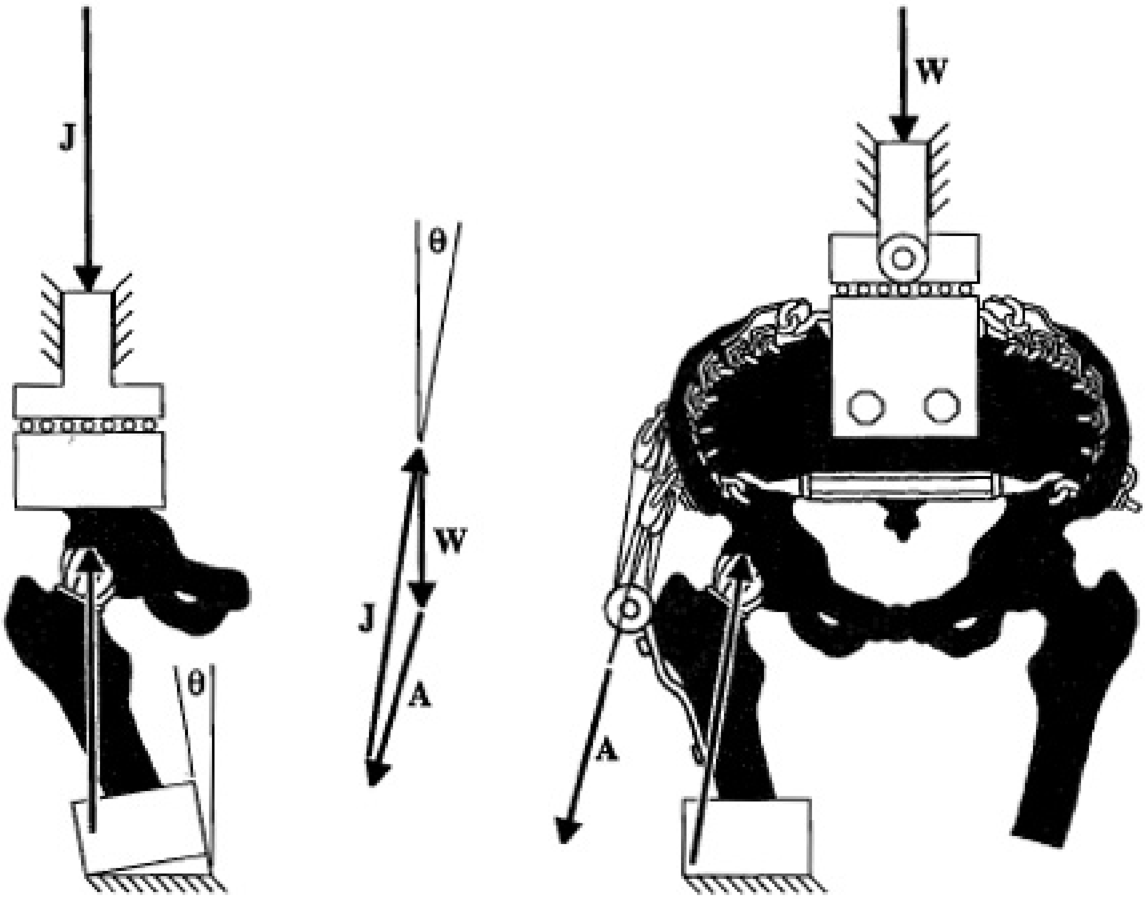
Box 1.3 Technical Note
Reproducing Joint Forces in the Laboratory

Cadaver specimens are frequently used to model normal, pathologic, or reconstructed joints. For these laboratory simulations, researchers often rely on static calculations to approximate the loading conditions experienced by the joint in life. For example, a typical experiment examining the stability of a new hip prosthesis might use cadaveric hip joints implanted with the new component and then mounted into a testing machine for loading. But in what direction should the load be applied? What should the load magnitude be? Static analysis can be used to answer these questions.

Once the investigator has determined the direction and magnitude of the joint reaction force for a given condition, it is simply a matter of aligning the specimen such that the vector of the joint force coincides with the axis of the load delivery system (left figure). Many important biomechanical studies have been executed in just this fashion. However, like static analysis itself, this type of simulation incorporates several assumptions and the results should be interpreted with caution.

Let us revisit the hip example. A more physiologically correct model might use the whole pelvis and incorporate abductor muscle forces to reproduce the in vivo loading environment (middle and right figures). In this case the proper joint reaction vector is produced by simulating abductor contraction using a cable system attached at the muscle origin and insertion sites. Shortening the cable produces a moment about the hip that is opposed at the sacrum. In theory, when the vertically directed force at the sacrum achieves upper-body weight, the joint reaction force at the hip will be the same as for the extracted hip joint model and the same as that produced during single-leg stance in life.

In a comparison of these two laboratory methods (Bay et al., 1997a), whole intact pelves were first loaded under simulated muscle action and the contact pressure distribution on the articular surfaces was recorded. The hips were then extracted from the pelvis and reloaded. The direction and magnitude of the joint reaction force was identical for both cases, but the distribution of contact pressure within the joint was quite different. Why? Remember, static analysis assumes rigid bodies but musculoskeletal tissues do not always behave as rigid bodies. In the intact model the bones of the pelvis were free to flex and deform



The *left* and *right* diagrams show two different experimental setups for simulating hip joint loading using cadaver bones. The vector diagram in the *middle* is for the case at *right*. (Reproduced with permission from Bay et al., 1997a.)

under load, while in the simpler joint model the pelvis became more rigid because of its altered geometry and the constraints placed upon it.

1.4 The Three-Force Rule

A simple rule is useful in solving certain problems by inspection, or graphically. To illustrate this principle, we consider the forces acting in the knee joint when a person is walking up stairs. Figure 1.7 shows a free-body diagram of the leg during this situation. It is again assumed that the forces are confined to a single plane, and that static equilibrium is approximated because the movement is relatively slow. The "ground reaction force" (**G**) is now the reaction of the step on the foot, assumed to be directed vertically upward and equal to body weight. This force acts to rotate the leg counterclockwise, flexing the knee joint. This moment is resisted by the quadriceps muscles of the thigh, which pull on the patellar ligament and resist knee flexion. It is assumed that the magnitude of this force **L** is unknown, but that its direction can be obtained from drawing a line

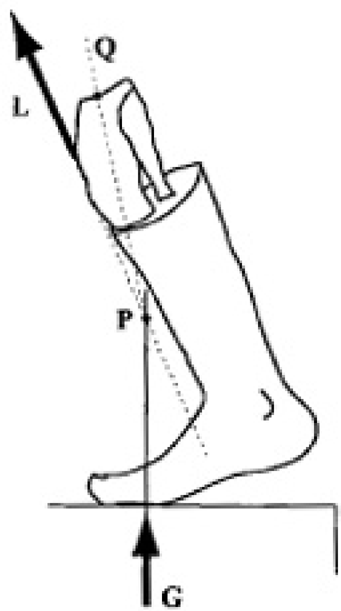


FIGURE 1.7. Partial free-body diagram of leg and foot to illustrate the three-force rule. **G** is ground reaction force, **L** is patellar tendon force, *P* is the point where their lines of action meet, and *Q* is the point where the joint force acts on the tibia.

between its attachment points on a radiograph. This direction is shown by the vector **L** in Fig. 1.7. The question that the three-force rule can answer is, "What is the direction of the force, **R**, exerted by the femoral condyles on the top of the tibia?"

To see what the direction of this force must be, it is only necessary to realize that the first two forces, **L** and **G**, produce no moments about the point *P*, where their lines of action meet. Because **R** is the only other force acting in the problem, its vector, when extended, must also pass through the point *P*. Otherwise, **R** would produce a net moment about *P* and ΣM would not be zero. We know that the vector **R** must act on a point *Q* within the knee joint. To find **R**'s line of action it is only necessary to construct the line *PQ* so that it passes through both *P* and the place where **R** acts on the tibia. Once this is done, the problem can be solved graphically in a second diagram (Fig. 1.8). First, a vertical vector

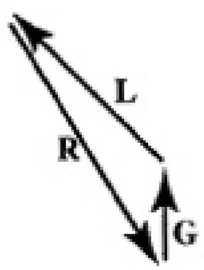


FIGURE 1.8. Vector diagram for three-force rule example. **G** and **L** are defined in Figure 1.7, **R** is the joint reaction force acting at *Q* in that figure.

is constructed having a length proportional to body weight (i.e., the vector **G**). Then, a vector of the appropriate length and direction to represent **L** is drawn from the head of **G**. Finally, vector **R** is drawn from the head of **L** to the tail of **G** so that the three vectors form a closed triangle; this determines the length (magnitude) of **R**. Note that this graphical solution to a two-dimensional joint mechanics problem has again only allowed us to find three unknowns: the magnitude of **L** and the magnitude and direction of **R**. If we had not known the direction of **L**, we would not have been able to solve the problem.

1.5 Indeterminate Joint Problems

In the problems we have considered so far, we have limited ourselves to two-dimensional analyses in which it was possible to solve for three unknowns. To reduce the number of unknowns to the number of equations, we had to combine some muscles with others and to ignore others. We could have obtained more equations by extending our analysis to three dimensions, but then we would have had to consider additional muscles as well. All real joints present static equilibrium problems that are mathematically indeterminate because there are always more unknowns than equilibrium equations. Thus, to solve for joint forces more realistically, it is necessary to find more equations to use in the solution.

To see an example of how this might be done, we return to the elbow problem. Previously, we noted that the primary flexors of the forearm are the brachialis and the biceps, and we combined their actions into a single force vector. Now, we separate these muscle forces. In addition, we include the third muscle, the brachioradialis (see Fig. 1.1). Table 1.1 shows the moment arms of the three muscles with respect to the center of rotation of the elbow when it is flexed at 90° (Winter, 1990). It also shows the

TABLE 1.1. Elbow muscle data

Muscle	Moment arm, cm	PCA, cm^2
Biceps	$a = 4.6$	$\alpha = 4.6$
Brachialis	$b = 3.4$	$\beta = 7.0$
Brachioradialis	$c = 7.5$	$\gamma = 1.5$

PCA, physiologic cross-sectional area.
From Winter, 1990.

physiologic cross-sectional area (PCA) of each muscle. It is approximately true that the maximal force that a muscle can exert is proportional to its physiologic cross-sectional area. Another way to say this is that muscles have an upper limit to the stress that they can generate within themselves. Measured maximum stresses vary from 20 to 100 N/cm^2, depending on conditions (isometric or dynamic, parallel or pennate fibers, etc.). We will use 40 N/cm^2 as a rule of thumb for the maximal stress (Morris, 1948; Maughan et al., 1983).

With all three muscles in the problem, there are five unknowns (three muscle force magnitudes and two components of joint force) but still only three equilibrium equations. We need two other relationships between the variables to obtain a solution. One way to get such additional equations would be to assume that all three muscles work together to equalize their individual stresses. That is, each muscle force is proportional to the muscle's physiologic cross-sectional area. Then the three muscle forces are

$$\begin{aligned} A &= k\alpha \quad \text{(biceps)} \\ B &= k\beta \quad \text{(brachialis)} \\ C &= k\gamma \quad \text{(brachioradialis)} \end{aligned} \tag{1.13}$$

where α, β, and γ are the respective cross-sectional areas and $k = 40\ N/\text{cm}^2$ is the constant muscle stress. This may be rewritten as

$$A/\alpha = B/\beta = C/\gamma \tag{1.14}$$

which constitutes the two additional equations that were needed. Using these to substitute for B and C in the moment equilibrium equation,

$$\Sigma M = aA + bB + cC - wW = 0 \tag{1.15}$$

one has

$$\begin{aligned} A &= [\alpha w/(a\alpha + b\beta + c\gamma)]W = 2.86\ W \\ B &= (\beta/\alpha)A = 4.36\ \text{W} \\ C &= (\gamma/\alpha)A = 0.93\ \text{W} \end{aligned} \tag{1.16}$$

This solution indicates that the biceps, the muscle with the intermediate cross-sectional area and moment arm, also exerts the intermediate force. The brachioradialis, which has the largest moment arm and the smallest cross-sectional area, exerts the least force, and vice versa for the brachialis. Whether this has anything to do with the actual distribution of forces in the muscles is debatable! Many other equally plausible, but not necessarily so mathematically trivial, "additional conditions" may be imagined. For example, the force distribution may shift around to give different muscle fibers a rest, or to keep the weight from tipping sideways in the hand, or both. A number of investigators have speculated that the body resolves the indeterminacy of such problems by acting to optimize or minimize some biologically important variable, such as the total energy required by all the muscles acting. Collins (1995) provides additional discussion of this subject.

Box 1.4 Technical Note
Surgical Reconstruction of the Hip Joint

Total hip arthroplasty, which involves replacement of the ball and socket of the hip joint with manufactured components, is one of the most successful surgical operations developed in this century. The procedure has relieved joint pain and increased the quality of life for millions of people. Before the development of successful hip replacement surgery by Sir John Charnley (1973), patients with debilitating degenerative arthritis of the hip were forced to either suffer with the condition or undergo hip joint fusion (called *arthrodesis*). As described in Section 1.2, the loads imposed on bones and joints are dramatic as a result of the mechanical disadvantage under which they function. Forces at the hip can easily exceed fivefold body weight. In addition, the hip joints of an average individual experience about a million load cycles (steps) each year. Thus total hip components should be both strong and durable.

A total hip replacement consists of two parts: a femoral component, the ball, and an acetabular component, the socket (see figure). Femoral components are made from high-strength alloys such as cobalt-chrome or titanium, and consist of a highly polished head atop an intramedullary stem. The stem is inserted into the canal of the femur and is usually fixed with an acrylic plastic (polymethylmethacrylate, PMMA) called *bone cement*. This material does not actually cement the implant to the bone, however; it simply fills space so that the fit does not have to be exact, "grouting" the implant in place. More recent designs utilize a porous stem surface designed for fixation by bone ingrowth. The acetabular component is fabricated from ultrahigh molecular weight polyethylene, with or without a metal backing. A design with a metal backing is illustrated. Other combinations of materials for the bearing surface are sometimes used, including metal-on-metal, ceramic-on-ceramic, and ceramic-on-polyethylene. The PMMA and metal-on-polyethylene bearing surfaces innovations were important factors enabling Charnley to develop a system that featured good material and structural strength, good resistance to fatigue damage, and low friction at the bearing surface. Careful surgical technique remains important, but with current methods most patients, being elderly and relatively sedentary, can expect satisfactory performance for 15 years or more.

Despite the operation's overall success, failures do occur, more frequently in young, active patients. The high forces imposed on the joint make maintenance of implant fixation difficult; component loosening and migration are not uncommon. High joint reaction forces also produce relatively high frictional forces at the metal–plastic interface. Over time, frictional wear debris, both metallic and polyethylene, elicits biologic responses that resorb bone and may further compromise

Photograph of modern hip joint replacement prosthesis. Spherical, polished, cobalt-chromium alloy ball at top articulates with ultrahigh molecular weight polyethylene socket having a backing of similar metal. The long metal stem is placed inside the medullary canal of the femur. In this example, the proximal stem and socket backing have porous surfaces designed for bone ingrowth in lieu of using polymethylmethacrylate (PMMA) for fixation to the bone.

implant stability. Improving the fixation and wear characteristics of total joint components is a major focus of orthopaedic research.

1.6 Equine Fetlock Forces

Horses, and in particular racehorses, place extraordinarily high loads on their limbs. This is especially true of the forelimbs, which are thought to carry about 60% of the animal's weight. Many racehorses are seriously injured each year by mechanical failures of the structures in the distal forelimb. Because it is so difficult to repair these failures in such a way that the horse can stand and walk during healing, and because horses must be erect and mobile to survive, many animals die as a result of injuries that would not be life threatening in humans or other smaller animals.

Figure 1.9 shows a sketch of the anatomy of the distal forelimb of the horse. Remember, the horse does not have feet the way humans do; it walks on the tips of its "fingers." The other bones of the foot (or hand) serve to lengthen the leg. P1 and P2 mark the first and second phalangeal bones in the figure; the third phalangeal bone is inside the hoof. The equine third metacarpal bone (MC3) has become extremely stout, and carries loads from the phalangeal complex up to the carpal bones and the radius, seen at the top of the diagram. (The second and fourth metacarpal bones are vestigial struts stuck to the sides of MC3; the other metacarpal bones no longer exist.) The joint between MC3 and P1 is called the *fetlock*; its biomechanical importance derives largely from the fact that the bones above it rise straight up to the torso, but the phalangeal bones below it

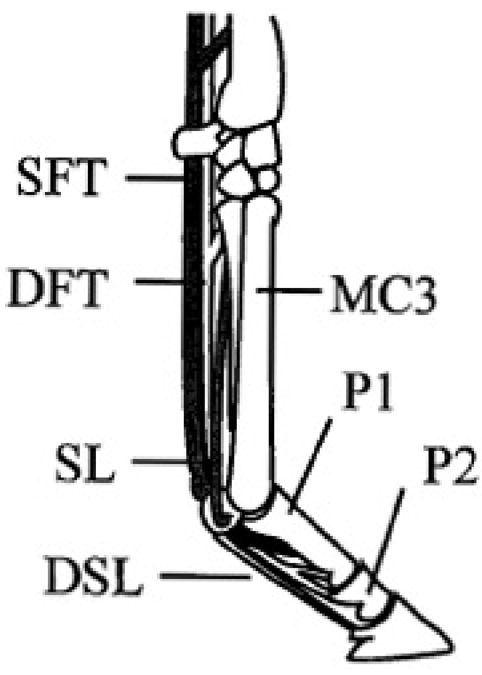


FIGURE 1.9. Equine distal forelimb anatomy, *sagittal view*. *SFT*, superficial digital flexor tendon; *DFT*, deep digital flexor tendon; *SL*, suspensory ligament; *DSL*, distal sesmoidean ligaments; *MC3*, third metacarpal bone; *P1*, proximal phalanx; *P2*, middle phalanx.

angle sharply forward in a springy cantilever arrangement. The joint between P1 and P2 is called the *pastern joint*.[4]

Behind the third metacarpus, which is also called the *cannon bone*, two tendons come down from the superficial and deep digital flexor muscles. The superficial flexor tendon (SFT) inserts on P2, and the deep flexor tendon (DFT) inserts on P3 inside the hoof. On their way to these attachment sites, these tendons pass over two small bones located just behind the distal end of the cannon bone. These bones are called the *proximal sesamoid bones*; their cartilaginous dorsal (anterior) surfaces articulate with the palmar (posterior) distal condyles of the cannon bone. The fetlock's sesamoid bones function very much like the patella in your knee, moving the line of action of the tendons away from the joint center so that they exert a larger moment about it. In addition to these tendons, there is a set of four ligaments that help stabilize the fetlock. The *suspensory ligament* (SL) runs from the proximal posterior surface of MC3 down to the sesamoid bones. From there, three *distal sesamoidian ligaments* connect to P1 and P2. These four suspensory ligaments, along with the sesamoid bones, are often called the *suspensory apparatus*. The cannon bone brings the weight of the horse down to the fetlock, which sits in the suspensory apparatus in much the same way as a person lies in a hammock. The flexor muscles and tendons help carry this load, with the sesamoid bones providing a bearing surface to protect the distal cannon bone, as well as increasing the moment arms of the tendons and ligaments, as previously mentioned.

To analyze the forces in the fetlock joint, we construct the free-body diagram shown in Fig. 1.10. The analysis is two dimensional in the sagittal plane. The "amputation" has been done in such a way that the free body includes the sesamoid bones and everything distal to the cannon bone. These structures are all assumed to be rigidly connected; the pastern joint is assumed to be fixed. The forces resulting from the flexor tendons and the suspensory ligament are all combined into a vector, **T**, which has components in the *x*- and *y*-directions. Considering the large weight of the horse, the weight of the free body is ignored. The remaining forces acting on the free body are the two components of the ground reaction force, **G**, and the two components of the joint reaction force, **F**. **F** is assumed to act at the joint's center of rotation. Taking moments about this center, we have

$$\Sigma M = g_1 G_y - g_2 G_x - t T_y = 0 \tag{1.17}$$

where g_1 and g_2 are as defined in the lower portion of Fig. 1.10 and t is the moment arm of the vertical component of **T**. It is assumed that the *x*-component of **T** has a negligible moment arm with respect to the joint center.

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[4]P1 is also called the pastern bone. The name apparently comes from the fact that to pasture a horse without benefit of fences, ranchers would hobble it with a short rope connecting the P1 bones.

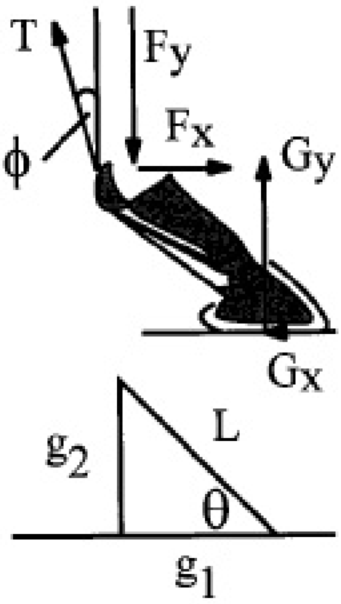

FIGURE 1.10. *Upper:* Free-body diagram for forces in the equine fetlock joint. G_x and G_y, components of the ground reaction force; **T**, tendon force; F_x and F_y, components of the joint force. *Lower:* Diagram for the moment arms, g_1 and g_2, of the ground reaction force. L, distance from the ground contact to the joint center; Θ, angle of the phalanges with respect to the ground.

The force equilibrium equations are

$$\Sigma F_x = -T_x - G_x + F_x = 0 \tag{1.18}$$

$$\Sigma F_y = T_y + G_y - F_y = 0 \tag{1.19}$$

These equations may be solved for various ground reaction forces and anatomical angles (Φ and Θ). First, let us assume that **G** and **T** are entirely vertical (e.g., when the horse is standing). Then $\Phi = 0$, $T = (g_1/t)G = GL \cos \Theta/t$ and $F = T + G$. If the horse weighs 500 kg (roughly 5000 N), and 60% of that is divided equally between the two forelimbs, then $G = G_y = 1500\ N$. For $L = 0.2$ m, $\Theta = 40°$, and $t = 0.05$ m:

$$T = (1500)(0.2)(0.766)/0.05 = 4596\ N$$

The force carried by the tendons and ligament behind the cannon bone is nearly equal to the entire weight of the animal! In fact, if the angle Θ sags to 33°, T becomes 5000 N. The joint reaction force is, of course, also vertical, and is $F = T + G = 6096\ N$, or 1.2 times body weight.

Now suppose that the ground reaction force is still entirely vertical, but the angle Φ is 20°. Then, from the moment equation,

$$T_y = (g_1/t)G = GL \cos \Theta/t \tag{1.20}$$

so that

$$T = T_y/\cos \Phi = (GL/t)(\cos \Theta/\cos \Phi) = 4891\ N \tag{1.21}$$

and from the force equations

$$F_y = T_y + G = [1 + (L/l) \cos \Theta]\, G = 6096\ N \tag{1.22}$$

$$F_x = T_x = T \sin \Phi = 1673\ N \tag{1.23}$$

The resultant joint force is 6321 N, and it acts at an angle of 15° to the vertical.

We return to this problem in the exercises that follow, and consider the effects of horizontal components of the ground reaction force on T and F. If this topic is of particular interest to you, you may want to read papers by Bartel et al. (1978) and Riemersma et al. (1988). A related work on the evolution of equine locomotion by Thomason (1991) is also of interest.

1.7 Summary and Further Reading

This chapter has considered the analysis of forces between one bone and another across joints. We have seen that such forces are always several times the external force being supported. This occurs because skeletal levers are arranged to magnify movement rather than force. Thus, muscle forces must be substantially greater than the forces being applied to the external environment. A person turning over in bed produces substantial hip joint forces, without supporting body weight, through the required muscle actions. Similarly, marine mammals such as dolphins and whales, whose buoyancy spares them the need to resist gravity, nevertheless have massive skeletons to transmit the muscle forces produced in swimming. The forces acting on bones can be estimated using the principles of static equilibrium. This method can provide very useful information, but significant approximations must be made to solve such problems. In general, the joints of vertebrate animals are statically indeterminate; that is, the number of unknowns (muscles acting to move or stabilize the joint) is almost always greater than the number of equations to be solved.

Suggestions for further reading in this field begin with the classic textbook *Biomechanics of Human Motion* (Williams and Lissner, 1977). Another well known textbook, aimed at the medical community, is *Basic Biomechanics of the Musculoskeletal System* by Nordin and Frankel (1989). Steindler's (1955) *Kinesiology of the Human Body Under Normal and Pathological Conditions* is excellent for its concentration on anatomy and pathomechanics. Early studies of the biomechanics of the human hip joint by Inman (1947) and McLeish and Charnley (1970), and Rydell's (1966) paper on measurement of hip joint forces using an instrumented prosthesis, will give you a greater appreciation for the complexities of experimental work in this area. *Animal Mechanics* (Alexander, 1968) provides an approach to this topic well suited to zoologists. If you are historically inclined, by all means seek out Maquet's (1989) translation of Giovanni Borelli's wonderful seventeenth-century work, *De Motu Animalium (On the Movement of Animals)*, sampled in Box 1.1.

1.8 Exercises

1.1. Solve the second part of the hip force problem (the force equilibrium equations) by considering a free-body diagram of the weight-bearing lower extremity and using the result $P = 2W$. Remember to include the weight of the extremity itself. What is the disadvantage of using this free-body diagram to solve the moment equation?

1.2. Suppose that the distance b in women's pelves is 10% greater than men's because of their greater pelvic width. How does this change their abductor muscle force magnitude and the magnitude and direction of their hip joint reaction force, assuming the moment arm of the abductor muscles is independent of gender?

1.3. Consider the force required in the erector spinae muscles to stabilize the head of a student leaning over his book. Also of interest is the force between the fifth and sixth cervical vertebrae. Figure 1.11 is a free-body diagram of this situation: **E** is the erector spinae force, **H** is the weight of the head and neck acting downward at their combined center of gravity, and **R** is the vertebral reaction force acting at a center of pressure on the inferior end plate of C5. Determine the magnitude of **E** and the magnitude and direction of **R**. Assume that **E** is parallel to the spine and $\Theta = 70°$. Let the moment arms of **E** and **H** with respect to the point of action of **R** be 0.02 and 0.10 m, respectively. Express your results as multiples of H.

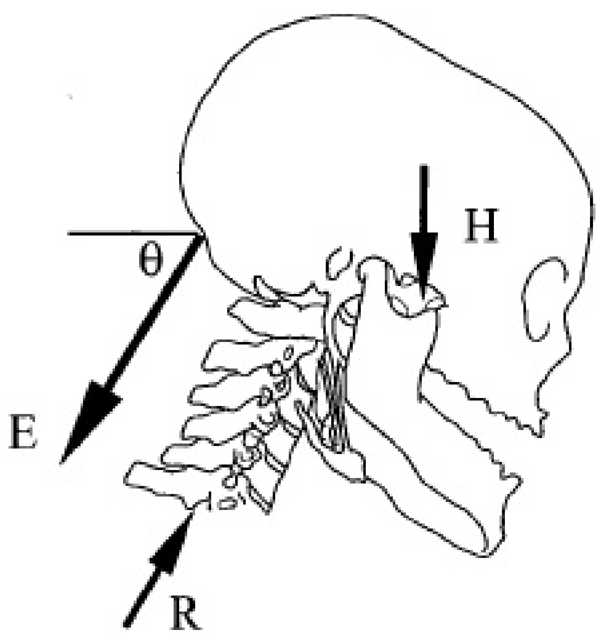

FIGURE 1.11. Diagram for Exercise 1.3, calculating the forces in the neck while studying. **H** is the weight of the head and neck acting at their center of gravity; **E** is the force of the erector spinae muscles and Θ is its angle; and **R** is the vertebral reaction force.

1.4. Consider the force produced in the temporomandibular joint when chewing. Figure 1.12 shows the location of the two major muscles active in a sagittal plane when chewing. The temporalis muscle arises from a broad area on the side of the skull, passes down through the space enclosed by the zygomatic arch, and inserts on the coronoid process of the mandible. In the picture of the skull, this point is hidden behind the other big chewing muscle, the masseter, which runs from the anteroinferior edge of the zygomatic arch down to the "angle" of the jaw. Ignoring other masticatory muscles, draw a free-body diagram of the mandible, assuming that the problem is two dimensional and the same chewing forces act on each side of the jaw. Let **C** be a vertical chewing force acting on a molar with a 0.06-m moment arm about the condylar process in the temporomandibular joint (TMJ). Assume that the masseter force, **M**, acts at an angle of 115° with respect to the x-direction, and that the temporalis muscle force, **T**, acts at an angle of 80°. Their moment arms with respect to the TMJ center are $b = 0.04$ m and $d = 0.02$ m, respectively. After each of these forces is represented on the free-body diagram, write the equations for static equilibrium. Under what conditions could these equations be solved? How do the direction and magnitude of the joint reaction force, **R**, depend on the relative magnitudes of **M** and **T**? Try to obtain solutions to the equilibrium equations using at least two different kinds of additional conditions. You can read more about the mechanics of the human jaw in papers by Barbenel (1972) and Osborn (1996).

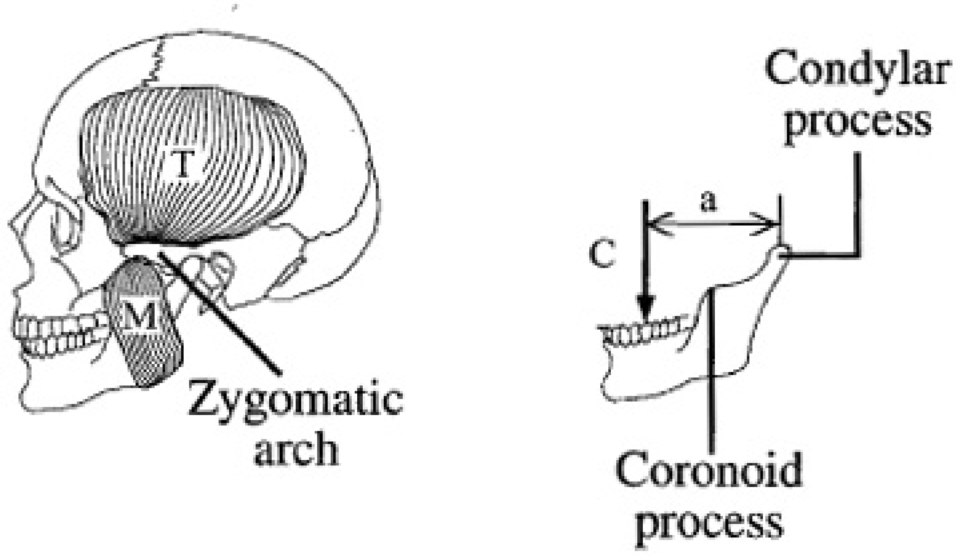

FIGURE 1.12. Diagram for Exercise 1.4. *Left:* Anatomical diagram showing locations of the temporalis (*T*) and masseter (*M*) muscles. The former passes through the zygomatic arch to pull the coronoid process up and to the right. *Right:* Detail of the mandible showing the assumed biting force, C, and its moment arm, *a*, with respect to the joint.

1.5. A male gymnast is performing the "iron cross" exercise on the rings, in which the arms are held straight out with a ring grasped in each hand, and the body hangs motionless and vertical between the rings (Fig. 1.13). Consult an anatomy book to determine the principal adductor muscles for the arm in this position, and estimate their angle of pull and insertion point. Draw a free-body diagram and calculate the approximate magnitude of the adductor muscle force and the magnitude and direction of the reaction force in the glenohumeral (shoulder) joint during the maneuver. State the principal assumptions necessary for your calculation.

1.6. Suppose, in Exercise 1.5, that the ropes holding the rings do not rise vertically from the gymnast's hands, but pull outward or inward at an angle θ. How does this angle affect the forces in the shoulder joint?

1.7. In a classic paper, McLeish and Charnley (1970) described in detail the considerations necessary to estimate the forces in the hip during the stance phase of gait. (As noted in Box 1.4, Sir John Charnley also developed the total hip replacement procedure in its modern form.) Read this remarkable paper. (To locate it, see the bibliography at the end of this book.)

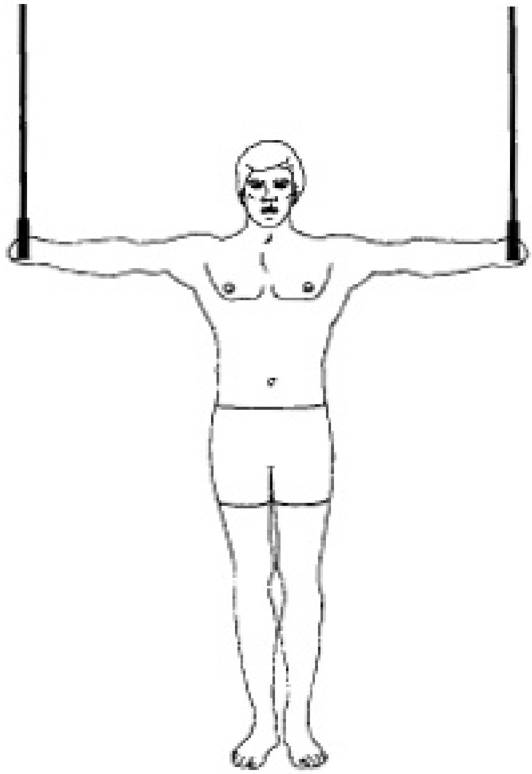

FIGURE 1.13. Sketch for Exercise 1.5 shows a gymnast performing the iron cross maneuver.

1.8. For the equine fetlock joint, work out the magnitude and direction of the joint force when a horizontal component of the ground reaction force (i.e., $\mathbf{G}_x$ in Fig. 1.10) is present. Let $\mathbf{G}_x = \pm 0.20\ \mathbf{G}_y$, and assume, as before, that $L = 0.2$ m, $\Phi = 0$, $\Theta = 40^\circ$, and $t = 0.05$ m.

1.9. To compare *Homo sapiens* and *Australopithecus* hip joint mechanics using similar skeletal materials, Lovejoy and co-workers (1973) studied the skeletons of Native Americans recovered in the field rather than those from laboratory cadavers representing modern Americans. They estimated interfemoral head distance and abductor moment arm distance from the archeological remains of 20 *Homo sapiens* and 1 *Australopithecus*; these results are shown in Table 1.2. Assume that the body weight moment arm (b in Figs. 1.3 and 1.14) is slightly greater than half the interfemoral head distance (D_{IF}) shown in the second column of Table 1.2, i.e.,

$$b = k\, D_{IF} \tag{1.24}$$

where k is in the neighborhood of 0.5. Assume also that the abductor moment arm (c in Figs. 1.3 and 1.14) and angle are as shown in the third and fourth columns. Plot a graph of the force on the head of the femur as a function of k for the three kinds of skeletons shown. What can you deduce about the hip forces in Native American females and the *Australopithecus* individual compared to Native American males?

1.10. Another interesting factor in the development of human bipedalism is its relationship to cranial capacity. The diameter of the birth canal is primarily determined by the size of the infant's head at birth. As human evolution proceeded and cranial capacity increased, the birth canal had to become larger. Presumably, this led to widening of the pelvis and increased the moment arm of the body weight (b in Figs. 1.3 and 1.14) with respect to the hip joints

TABLE 1.2. Archeological skeletal data

Skeletons	Interfemoral head distance, mm	Abductor moment arm, mm	Abductor angle, degrees (°)
Australopithecus, STS 14	132	36	15
Homo sapiens, Amerindians, male (n = 8)	158.0 ± 10.3	50.6 ± 1.7	12.9 ± 2.2
Homo sapiens, Amerindians, female (n = 12)	165.8 ± 9.0	45.4 ± 3.8	13.3 ± 2.8

From Lovejoy et al., 1973.

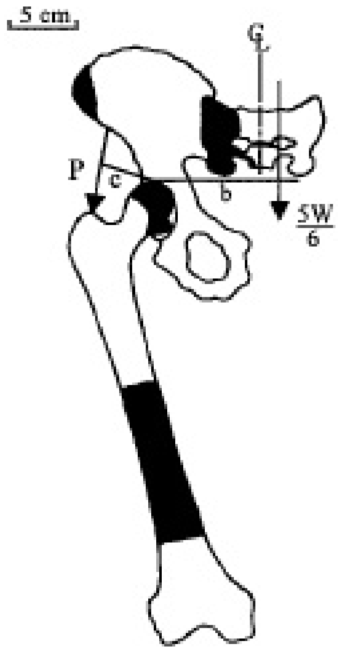

FIGURE 1.14. Diagram for *Australopithecus* hip force problem (Exercise 1.9). **P** is the abductor muscle force and *c* is its moment arm with respect to the joint center; *b* is the moment arm of the body weight minus the support leg (i.e., *5W/6*). *Shaded regions* indicate reconstructed portions of the bones. (Redrawn with permission from Lovejoy et al., 1973.

of females. Assume the other parameters of the hip joint were

abductor muscle angle = 15°
abductor muscle moment arm = c = 36 mm
interfemoral head distance = D_{IF} = 132 mm
body weight moment arm = $b = k\, D_{IF}$
$k = 0.60$

and remained constant (refer to Exercise 1.9). Determine and graph the relationship between the hip abductor force **P** and the newborn cranial volume V, assuming that the head is a sphere and its newborn volume increases from 500 to 1000 cm³. Does encephalization (increasing brain size) appear to be an important factor in increasing hip joint forces during bipedal walking? For additional discussion of this issue, see Leutenegger (1972) and Ruff (1995).

1.11. Go to the library and read the paper by Zihlman and Hunter (1972). It is a more complex, three-dimensional analysis of hip mechanics in *Australopithecus*. Write a paragraph discussing its strengths and weaknesses.