\[ p < p_0 \]

\[ p_{DA} \rightarrow \Delta p = 0 \]

completely adiabatic fuel

\[ s \]

\[ s_f \]

\[ \frac{s_f}{s_0} \]

\[ p \]

\[ p_{1,0} \]

\[ p_{1} \]

\[ p_{1} \]

\[ \text{with heat transfer } p_{as} > p_0 \]

\[ \text{no heat transfer} \]
Rod ejection

\[ P(t) \]

\[ P_m \]

\[ P_h \]

\[ t \]

\[ P(t) \]

\[ P_m \]

\[ P_h \]

Why?
Example:

\[ P_0 = 1.2 \text{ kPa} \]
\[ \beta/\lambda = 0.62 \times 10^4 \text{ s}^{-1} \]
\[ Y_e = 0.8 \text{ kPa/} P_m \]

\[ P_m - P^0 = -\frac{Q^2}{2 \lambda \beta} \]

\[ = 38.8 \text{ kPa} \]

\[ P_m = 2500 \text{ kN/m}^2 \]
1. \( P_0 = 0.1 P_m \) (HZP)
   \[ P_m = P_0 = 388 P_0 \]
   \[ P_m = 399 P_0 \]

2. \( P_0 = P_m \) (HFP)
   \[ P_m = 49.8 P_0 \]
FLUX TRANSIENT DURING A SUPERCritical excursion

$P > \beta$

Starting with the first integral:

1. $\lambda \left[ \dot{S}_p(t) - \lambda P^0 \right] = \frac{1}{2} \left[ S_p^2(t) - S_p^2 \right]$

Rearranging:

2. $2 \lambda \dot{S}_p(t) = S_p^2 - S\beta^2$

with

3. $S_b^2 = S_{pi}^2 - 2\lambda \gamma P^0 \geq S_{pi}^2$
Dividing Eq. 2 by its right side and integrating with respect to time (from 0 to $t$):

\[ t = 2 \Lambda \int_{S_p(t)}^0 \left( \frac{d S_p}{S_p^2 - S_b^2} \right) = 2 \Lambda \int_{S_p(t)}^0 \frac{d S_p}{S_b^2 - S_p^2} \]

or

\[ \frac{t}{\Lambda} = \frac{1}{S_b} \left[ \ln \frac{S_b + S_p(t)}{S_b - S_p(t)} - \ln \frac{S_b + S_p(t)}{S_b - S_p(t)} \right] \]

to find the time at which the maximum of the flux is seen, we set $S_p(t) = 0$
\[ t_m = \frac{1}{S_b} \ln \frac{8_b + 8_p(t)}{8_b - 8_p(t)} \]

Then:

\[ \frac{t - t_m}{\frac{1}{S_b}} = -\frac{1}{S_b} \ln \frac{8_b + 8_p(t)}{8_b - 8_p(t)} \]

Solving \( \text{Eq. 7} \) for \( 8_p(t) \):

\[ 8_p(t) = 8_b - \frac{1 - \exp \left[ \frac{8_b (t - t_m)}{1} \right]}{1 + \exp \left[ \frac{8_b (t - t_m)}{1} \right]} \]

Here we used Eq. 6 to simplify Eq. 5, leading to Eq. 7.
According to Eq. 8:

at $t = 0$  
$S_p \left|_{t=0} \right. = S_p^i$

at $t = t_m$  
$S_p \left|_{t_m} \right. = 0$

at $t = 2t_m$  
$S_p \left|_{2t_m} \right. = -S_p^i$

Now, inserting Eq. 8 into

$\Delta V \left[ p(t) - p^0 \right] = \frac{1}{2} \left[ S_p^2(t) - S_p^2 \right]$

and using that:

$S_b^2 = S_p^2 - 2 \Delta V p^0$  
and  
$P_m = -\frac{S_b^2}{2 \Delta V} \Rightarrow \ldots$
\[ \Delta T = \frac{4 \Lambda}{\gamma_{p_1}} \left( 1 - \frac{P_0}{P_{\mu}} \right) \]

**Case 1:** \( P_{\mu} \gg P_0 \)

\[ \Rightarrow \Delta T = \frac{4 \Lambda}{\gamma_{p_1}} \]

**Case 2:** \( P_{\mu} \ll P_0 \)

\[ \Rightarrow t_{\mu} = \frac{\Lambda}{\gamma_{p_1}} \left( \ln 4 \frac{P_\mu}{P_0} \right) \]

From the figure:

1. If transient starts at nominal power: (shown in the figure)
   - \( t_{\mu} \) is not much larger than \( \Delta T \) => maximum is quickly reached.
(2) If transient starts at near zero power:

Let \( P_0 = 10^{-6} P_n \)

\[ t_m \leq 16.5 \frac{L}{8P_n} \]

The maximum is reached in only 4 times of \( t \)

\[ \Rightarrow \text{the burst will be very narrow (the maximum is much larger)} \]
Post-Burst Transient

\[ P(t) = \frac{P_{in}}{\cosh \left( \frac{3b}{2\lambda} (t-t_m) \right)^2} \]

- Flux is decreasing rapidly after passing \( P_{in} \)
- Flux continues to drain away after 2\( t_m \) - not realistic
  - Reason - neglect of the delayed neutrons
- After reaching the reactivity domain \( 0 < \phi < \phi \) - PTA is not valid
- PTA - valid
  - (flux changes are slow) \( \Delta \phi = 0 \)
Using the initial value of a delayed neutron source and reactivity after the burst \( S(t_a) = \beta - S_{p_1} \): 

After the burst, the flux stays at a level \( P_{ab} \) given by:

\[
P_{ab} = \frac{S_{do}}{\beta - S_{p_1}} = \frac{S_{do}}{S_{p_1}} = \frac{\beta}{P_{p_1}} P_0
\]

If \( S_{p_1} = 0.1 \% \) \( \Rightarrow P_{ab} \) is 10 times \( P_0 \) (initial power)
Superprompt-critical transient (\( \rho_l = 1.1 \), \( p_0 = 0.1 p_n \))

with reactivity feedback

\[
\begin{align*}
\text{short time of the burst} & \quad \text{we can use} \quad \lambda \\
\Rightarrow \lambda z_0 &= \beta P_0 \\
\int_0^{t_2} p(t') dt' &= -2 \frac{\beta_p \lambda}{\gamma}
\end{align*}
\]

If: \( P_0 = 0.1 \) \( \Rightarrow \)
\[
-2 \frac{\beta_p \lambda}{P_0 \gamma} = \frac{-2 (0.15) (0.565 s^{-1})}{(0.1) - 0.08 ms^{-1}} = 1.21
\]

\( \approx 140 \% \)

increase of

the delayed reactivity feedback

including the precursor increase:

\[
P_{tb} = \frac{\beta_p \lambda}{\beta_p \lambda} \left( 1 - 2 \frac{\beta_p \lambda}{P_0 \gamma} \right)
\]
Temperature increase during the entire transient:

\[
\Delta T_{\text{total}} = \Delta T_{\text{burst}} + \Delta T_{\text{post-burst}}
\]

\[
= -\frac{1}{\gamma T} \left[ 2(g_1 - \beta) \right] - \frac{1}{\gamma T} \left[ g_1 - 2(\beta \cdot \beta) \right]
\]

\[
= -\frac{p_1}{\gamma T}
\]

reactor accidents:

Example: \(g_1\) is almost never much larger than \(\beta\)

\(\Rightarrow\) \(\Delta T_{\text{post-burst}}\) is only a small part of \(\Delta T_{\text{total}}\)

\(g_1 = 1.1\% \Rightarrow 18.1\text{ energy release during post-burst}\)

\(22.1\% = \) during the burst.