NUC E 521

Chapter 6:
METHOD OF CHARACTERISTICS

K. Ivanov

206 Reber, 865-0040, kni1@psu.edu

Introduction

- Spatial three-dimensional (3D) and energy dependent modeling of neutron population in a reactor core is based on two main approaches:

  (a) deterministic approach where the Boltzmann transport equation is solved explicitly, and

  (b) stochastic approach or the Monte Carlo method where neutron transport and interactions are modeled explicitly

- The Monte Carlo method is an accurate mimic of the neutron histories from the point of birth through their migration in a media to the point of absorption or leakage, but its computational cost is a major drawback

- The calculations of integral parameters such as the multiplication coefficient of critical assemblies can be performed relatively efficiently

Introduction

- However, the spatial quantities such as the fission rate or scalar flux are estimated from the number of collisions or from the particles track lengths in the volume surrounding the point of interest

- When this volume becomes small, the fraction of histories contributing to the flux in the volume decreases rapidly

- The statistics deteriorates quickly with the spatial resolution, making the Monte Carlo method non-practical to determine such quantities for routine calculations

- In addition there is a modest flexibility in using the Monte Carlo method beyond the steady-state reactor analysis

- For example, The Monte Carlo still is not available for performing transient calculation (such as rod ejection, or steam line break)

Introduction

- The deterministic methods provide an explicit solution to the Boltzmann transport equation

- The 3D Boltzmann transport equation in steady-state can be written as:

$$\Omega \cdot \nabla \Psi(\vec{r}, \Omega, E) + \Sigma_{\mathrm{t}}(\vec{r}, E)\, \Psi(\vec{r}, \Omega, E) = q(\vec{r}, \Omega, E),$$

$$q(\vec{r}, \Omega, E) = \int \mathrm{d}E' \int \mathrm{d}\Omega' \Sigma_{\mathrm{s}}(\vec{r}, E' \rightarrow E, \vec{\Omega}' \rightarrow \vec{\Omega}) \Psi(\vec{r}, E', \vec{\Omega}') + \frac{\chi(E)}{k} \int \mathrm{d}E' \nu \Sigma_{\mathrm{f}}(\vec{r}, E') \int \mathrm{d}\Omega' \Psi(\vec{r}, E', \vec{\Omega}'),$$

- The solution of the neutron transport equation can be obtained by using one of the two main formulations, the integro-differential or the integral forms

Introduction

- The integral form of the transport equation is used in the collision probability method (CPM)

- The contribution of a source in a given point r_0 to the angular flux is given by the neutrons emitted by the source q_0 at r_0 undergoing their first collision at r

- The integration over the volume V leads to the following equation:

$$\Psi(\vec{r},E,\vec{\Omega}) = \int_V q(\vec{r}',E,\vec{\Omega}) \times \frac{\exp\left(-\int_0^{s'} \Sigma(\vec{r}'',E)\mathrm{d}s''\right)}{|\vec{r}-\vec{r}'|^2}\delta\left(\vec{\Omega}-\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|}\right)\mathrm{d}V,$$

$$q(\vec{r}',E,\vec{\Omega}) = \int \mathrm{d}E' \int \mathrm{d}\Omega' \Sigma_s(\vec{r},E'\to E,\vec{\Omega}'\to\vec{\Omega})\Psi(\vec{r},E',\vec{\Omega}') + \frac{\chi(E)}{k}\int \mathrm{d}E' v\Sigma_f(\vec{r},E') \int \mathrm{d}\Omega' \Psi(\vec{r},E',\vec{\Omega}')$$

Introduction

- The CPM gives very accurate solutions for neutron flux and neutron multiplication constant
- The accuracy is achieved without the homogenization of complex domains
- However, the CPM method is usually limited to modeling only a single reactor assembly since the computational solution requires a tremendous memory and calculation time for large systems
- The CPM method is a base for a number of codes developed to analyze 3D neutron transport: APOLLO2, WIMS, DRAGON, HELIOS, CASMO, and PARAGON

Introduction

- However, the CPM introduces a number of limitations:

  (a) the anisotropic scattering cannot be applied without increasing the number of equations so that the straightforward numerical treatment becomes impractical,

  (b) the size of the geometry is severely limited by the number of different spatial volumes, since the size of the matrix that needs to be solved is equal to the number of spatial regions

- The computational expense to store the large matrix involved by the CPM is the main limitation to expand its use in 3D whole core simulations

Introduction

- Integro-differential formulation of the steady-state 3D Boltzmann transport equation is given with the last equation and is solved with one of the following numerical approaches:

  (a) the spherical harmonics method, and

  (b) the discrete ordinates method (which includes the method of characteristics (MOC)):

$$\vec{\Omega}.\vec{\nabla}\Psi(\vec{r},E,\vec{\Omega}) + \Sigma_{\mathrm{t}}(\vec{r},E)\Psi(\vec{r},E,\vec{\Omega})$$
$$= \int \mathrm{d}E' \int \mathrm{d}\Omega' \Sigma_{\mathrm{s}}(\vec{r},E' \rightarrow E, \vec{\Omega}' \rightarrow \vec{\Omega})\Psi(\vec{r},E',\vec{\Omega}')$$
$$+ \frac{\chi(E)}{k} \int \mathrm{d}E' \nu\Sigma_{\mathrm{f}}(\vec{r},E') \int \mathrm{d}\Omega' \Psi(\vec{r},E',\vec{\Omega}')$$

Introduction

- In the spherical harmonics method or P_N approach, the angular flux is expanded in terms of eigenfunctions of the integral operator, the ‘‘spherical harmonics’’

- If only one angle is required such as in case of 1D plane or spherical geometries, the spherical harmonics are Legendre polynomial functions

- In case of 2D description, the complete orthogonal set of functions becomes the associated Legendre polynomial functions or the surface spherical harmonics

- In either case the expansion leads to an infinite set of differential equations with the good reason to truncate at odd numbers, thus having the so-called P_1, P_3, P_5, . . . theories

Introduction

- The main disadvantage of this approach is its inability to accurately portray the flux distribution near the vacuum boundary with the small number of terms of the polynomial expansion

- The P_N method has been implemented for example in the VARIANT code to perform 2D calculations

- However due to its complex formulation, especially for high order of the expansion, this method usually does not find its wide application in 3D core calculations

Introduction

- The discrete ordinate method or the S_N theory is the most generally used to solve the neutron transport in 3D
- This technique allows the refinement of the space-angle discretization until acceptable accuracy is achieved
- In addition, the equations involved in the discrete ordinates method are solved with an iterative scheme thus not requiring storage of big matrices like in the CPM method
- This reduces drastically the memory requirements required for large problems like 3D whole core calculation

Introduction

- The main drawback of the discrete ordinate method is the so-called ray-effect
- It results from the inability of the quadrature formula to accurately integrate angular flux distribution in highly heterogeneous sections of the reactor core (strong absorbers for example)
- These anomalies can be partly overcome or avoided by using a larger set of angular directions
- However the computational cost of the angular refinement can easily become prohibitive

Introduction

- Another problem arises from the spatial discretization used in the discrete ordinate method called the diamond difference scheme

- The diamond difference is a numerical approximation used to relate the volume-averaged angular fluxes to the surfaces averaged fluxes, which may produce negative flux values

- This effect is more pronounced for highly anisotropic source, narrow energy groups and anisotropic scattering

- Codes like THREEDANT; DORT and TORT and APOLLO2 are based on discrete ordinates method

Introduction

- The MOC does not produce negative neutron flux of the diamond difference scheme, does not require the geometrical homogenization of the diffusion method, and does not limit the complexity of geometry (found to be an obstacle in almost all of other deterministic methods)

- The flexibility of the method and assured accuracy of the solution thus has found a broad application in a number of highly accurate neutronic based advanced computational tools

Method of Characteristics

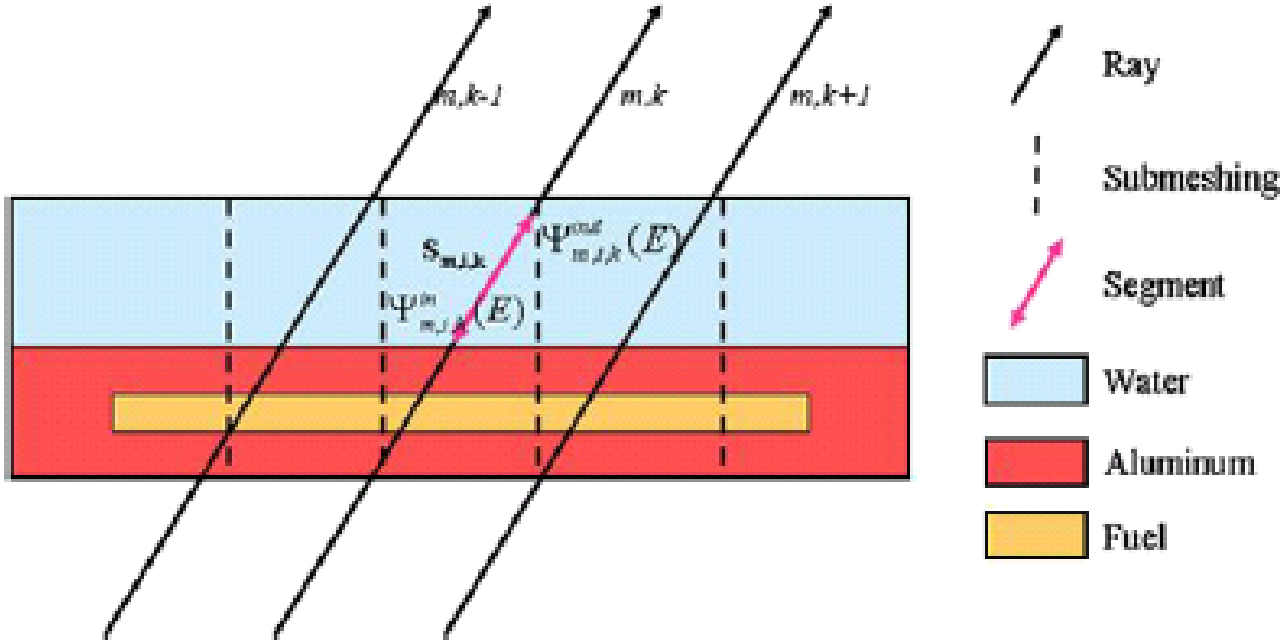
- In the MOC, the integro-differential form of the transport equation, is solved along the straight lines throughout the geometry in a finite number of directions (*see next Figure*)

- These straight lines are interpreted as neutron trajectories through the domain where the Boltzmann equation reduces to an ordinary differential equation of the following form:

$$\frac{\mathrm{d}\Psi(\vec{r}_0 + s\Omega, E, \Omega)}{\mathrm{d}s} + \Sigma_t(\vec{r}_0 + s\Omega, E, \Omega)\Psi(\vec{r}_0 + s\Omega, E, \Omega) = q(\vec{r}_0 + s\Omega, E, \Omega).$$

Method of Characteristics

Illustration of the MOC approach (segment of the research reactor assembly)



Method of Characteristics

- The angular flux is obtained by analytical integration along the characteristic lines of the geometry

- The discretized Boltzmann equation is integrated along the track segment $s_{m,i,k}$ in order to obtain a relationship between the incoming and outgoing angular flux in a given material zone i:

$$\Psi^{\text{out}}_{m,i,k}(E) = \Psi^{\text{in}}_{m,i,k}(E)\mathrm{e}^{-\Sigma^i_{\text{t}}(E)s_{m,i,k}} + \frac{q^i(E)}{\Sigma^i_{\text{t}}(E)}\left(1 - \mathrm{e}^{-\Sigma^i_{\text{t}}(E)s_{m,i,k}}\right),$$

Method of Characteristics

- A set of characteristics lines (the so-called the ray map) is generated per number of selected azimuthal directions and intersections of each ray with each surface in the domain are found (thus generating segments as indicated in the Figure)

- The treatment of the polar directions is slightly different; a geometrical correction is applied to the length of the segment $s_{m,i,k}$ in the MOC equation instead of generating a set of characteristics lines

- The interval between two characteristic lines is fixed and selected by the user (there is no a unified recipe of what is the recommended ray separation or angular refinement; the selection of both is based on the complexity of the geometry and experience of the user)

Method of Characteristics

- The accuracy of the solution is however, directly linked to the choice of the spatial discretization parameters (*number of directions*, *interval between two characteristic lines*, and in addition *the sub-meshing of the whole domain)*

- The MOC gives nearly Monte Carlo accuracy and in 2D allows for very fast modeling

- The MOC is associated with a tremendous increase in computational load when the solution is required for 3D whole heterogeneous core model

- The computational time increases almost linearly with the reactor core complexity and the degree of spatial resolution (the finer the resolution the longer the computational time)

- These aspects still severely affect the development of direct 3D MOC

Method of Characteristics

- In order to accelerate the solution, a number of approximations combined with an extensive parallelization, is often introduced
- Such examples are DRAGON and TransLat codes that have recently become able to achieve real 3D MOC calculations
- Those calculations even practically accurate as Monte Carlo remain exceedingly computationally expensive
- Recently, intermediate ideas have been introduced to obtain the 3D MOC solution for neutron transport in current reactor geometries

Spatial Discretization Methods

- As mentioned before the numerical approximations to the transport equation were initially explored in the context of the one-dimensional slab geometry case

- By defining the computational efficiency of a certain spatial differencing scheme as `the amount of error incurred for a given cost', one can compare the differences between Diamond Difference (DD), the Linear Discontinuous Finite Element Method (LDFEM), the Exponential Method (EM), the Step Characteristic (SC), and finally the Linear Characteristic Method (LC)

- The proposed list of spatial approximations can be supplemented by the Nodal Transport method

Spatial Discretization Methods

- The Diamond Difference method is arguably the “simplest” spatial discretization used in S_N calculations
- It can also be considered as a particular class of the more general case of “Weighted” Diamond Difference schemes (WDD)
- The DD approach involves the integration of the transport equation over a computational cell, which gives rise to a balance relation involving boundary and volume-averaged angular fluxes
- In order to solve for the discrete variables an extra relation is required, namely the diamond difference relation, which relates the cell-average flux and boundary average fluxes

Spatial Discretization Methods

- The DD approach can be shown to be second-order accurate in one-dimension, possesses a correct asymptotic diffusion limit, and is compatible with a consistently derived diffusion synthetic acceleration (DSA) operator

- The DD method can, however, potentially produce negative fluxes in regions of high-absorption if the mesh spacing is not small enough with respect to the particle mean-free-path

- This led investigators to develop computational methods that, while remaining second-order accurate, could minimize or avoid flux negativity caused by sharp gradients of the angular flux in regions of high absorption

Spatial Discretization Methods

- One such method is the Linear Discontinuous Finite Element Method (LDFEM)

- The LDFEM has been long recognized as a potential alternative to the DD method described above

- While the computational costs are higher for LDFEM due to the fact that the number of unknowns per cell is larger relative to DD, the method yields comparable accuracy on significantly coarser meshes and has a reduced propensity to flux negativity due to the discontinuous assumption of the incoming angular flux across the cells boundaries

- The LDFEM approach is based on in the weak formulation of the transport equation and the selection of a linear basis function defined within the computational cell

Spatial Discretization Methods

- The basis function is assumed to vary linearly, having a value of unity at given node, and zero at all other nodal coordinates

- A set of equations, corresponding to the zero- and first- spatial-moments of the weak formulation, is solved within each cell for the angular flux at the nodal locations in each cell

- The incoming angular flux is assumed to be discontinuous across the cell boundary, effectively appearing as a source in the final set of discrete-variable equations

Spatial Discretization Methods

- The development of “transverse-averaged” Nodal Transport methods is now reviewed
- The development of Nodal Transport methods can be understood as both a “coarse-mesh” and a variational approach
- The impetus for “coarse-mesh” methods comes from the homogenization approach to reactor analysis, in which repeating arrays of geometrically complex spatial domains are `smeared' or homogenized into a single composition, thus reducing the computational burden of solving a fully-heterogeneous three-dimensional problem

Spatial Discretization Methods

- The original N-dimensional partial differential equation (PDE) is formally reduced to N, one-dimensional ordinary differential equations (ODE) through the transverse-average weighting in each N-1 spatial directions

- The final result of this procedure is an ODE per dimension, which can be readily solved by applying standard techniques

- This approach was originally applied to the diffusion equation, which can be shown to be mathematically equivalent to the P_1 spherical harmonic approximation of the exact transport equation

- The success of nodal diffusion methods prompted the application of the nodal approach to the transport equation

Spatial Discretization Methods

- The Nodal Transport method has also been the subject of other important theoretical developments, such as the application of diffusion synthetic acceleration

- Furthermore, the final form of the high-order Nodal Transport equations into Weighted Diamond Difference (WDD) form, known as the Arbitrarily High-Order Transport Nodal (AHOT-N) method has made this class of methods very attractive for practical implementation

Spatial Discretization Methods

- The Exponential Method (EM) is yet another proposed discretization, albeit not as popular as DD, LDFEM or Nodal Transport methods, that has been investigated as a potential solution to the presence of negative fluxes for problems with highly absorbing regions

- The basic idea behind the EM is the assumption that the average angular flux in a given cell is equal to the geometric mean of the boundary terms

- This relationship is obtained by assuming that the cell edge and center fluxes are related exponentially

- A simple analysis can show that in fact this method is positive-definite, thus guaranteed to produce positive angular fluxes given positive distributed source and incoming angular fluxes

Spatial Discretization Methods

- The major drawback of the EM, however, is the degradation of the second-order accuracy in cells that are constrained by a vacuum boundary condition

- The positivity of the flux appears to be a trade-off between accuracy, in terms of convergence order, and physically meaningful solutions

- This trade-off is also evident in the class of spatial approximations to be discussed in the next section, the family of Characteristic Methods

Characteristic Methods

- Characteristic Methods (CM) form a class of spatial approximations in which the streaming-plus-collision operator of the transport equation is inverted exactly along `characteristic lines' that correspond to the direction of motion of the particles

- This exact inversion is derived under the assumption that a fixed distributed source of particles is present on the right hand side of the particle balance (transport) equation and known incoming angular flux

- For streaming and absorption-dominated problems, in which very little scattering is present, Characteristic Methods perform extremely well in comparison to more `localized' discretizations such as DD or LDFEM

Characteristic Methods

- In general, the characteristic equation relates the incoming angular flux and the distributed source of particles to the outgoing angular flux along each characteristic direction

- Note that at this point the geometric description of the problem domain has not been specified

- This gives rise to two types of Characteristic Methods:

  (a) the method of Long Characteristic, in which a set of characteristic directions are specified and tracked throughout the problem domain using ray-tracing algorithms, and

  (b) the method of Short Characteristic, in which the geometry is meshed and the characteristic relation is applied exactly for each cell assuming approximate cell face flux and distributed source projections

The Method of Long Characteristic

- The Method of Long Characteristic (LC), assumes the selection of a infinite number of characteristics, namely straight trajectories of the neutral particles along a infinite number of angular directions

- The ray-tracing scheme calculates the angular flux in each direction at the points at which the characteristics intersect the cell boundary

- Any exiting flux is computed in terms of an entering flux and the distributed source, which is typically approximated by a constant

- To ensure particle conservation, the source is evaluated by performing a neutron balance within the cell which is done for each characteristic by distributing the volume of the cell among the characteristics that pass through its extent

The Method of Long Characteristic

- Since the ray-tracing procedure can be applied to any solid body, geometrically complex problems can be readily solved with this approach

- The drawback of the methodology, however, is the dependence of the solution accuracy on the spacing between the characteristics

- Since a large number of characteristics is needed to accurately describe the geometry, the required storage for the track-length information can become very large, especially if three dimensional geometries are considered

- On the other hand re-computing the ray traces on the y executes very slowly

The Method of Long Characteristic

- Historically, the Collision Probability (CP) approach to solving the transport equation has been the method of choice for so-called lattice physics calculations due its ability to represent complex geometric configurations

- Typically, lattice physics calculations involve the solution of the integral form of the transport equation in a small, repeating region of the full core where reflective boundary conditions are an acceptable approximation along the assembly's edges on the radial-plane

- The CP method is a particular numerical method for solving the integral form of the transport equation, in which the angular dependence is eliminated by projecting the equations into a set of spherical harmonics moments

The Method of Long Characteristic

- Most implementations of the CP method only consider isotropic scattering and sources, which only requires one integral equation

- The inclusion of anisotropic scattering requires the addition of multiple integral equations (depending on the order of scattering) and the solution of a few coupled integral equations that can be a formidable task

- In order to incorporate anisotropic scattering while still retaining the track-length estimator approach built into CP-based legacy codes, the MOC became a natural choice to replace the CP-based methods

- The relatively simple mathematical formulation for at and even linear source representation, inclusion of anisotropic scattering, and characteristic-based approach has made MOC the current method of choice in a variety of production lattice physics codes

The Method of Short Characteristic

- In contrast to the LC, the method of Short Characteristic (SC) can compute the outgoing angular flux based on the incoming angular flux and the distributed source exactly within a computational cell

- Typically, a single cell `type' is used to approximate the problem geometry and the characteristic relation is imposed exactly within each cell

- The reduction of the problem geometry into repeating computational cells of a particular geometry makes it possible to treat each cell analytically with respect to the characteristic relation, thus requiring no track-length computation

- Once all the cell-face information is known, a balance equation is imposed over the cell and the average flux is computed over the cell volume

The Method of Short Characteristic

- Analogously to LC, the method of Short Characteristics discretizes the angular variables via the S_N approximation

- While this approach avoids the need to compute the track-lengths throughout the problem geometry, typical of LC, the Short Characteristic method approximates incoming and outgoing cell face and volume angular fluxes via polynomial expansions

- These expansions are usually truncated up to some finite polynomial order p on the cell face and volume variables

- The accuracy of the Short Characteristic method depends on the regularity of the exact solution

The Method of Short Characteristic

- For problems with very little scattering and anisotropic incoming angular fluxes, discontinuities of the angular flux or its spatial derivatives along the outgoing faces of certain computational cells are expected to appear

- Thus, the use of a continuous polynomial basis will only approximately capture the true shape of the angular flux over the face

- This is not an issue with the MOC, since the computation of the outgoing angular fluxes is done for point values of the angular flux on the cell outgoing face

- Two important issues concerning Short Characteristic methods are flux positivity and solution accuracy

- These issues are tied to the order p of polynomial basis function used to represent the cell face and volume angular flux

The Method of Short Characteristic

- Most of the early Short Characteristic methods relied on either a flat or linear flux representation
- The constant or at flux representation on the face and volume of the computational cell, also known as the Step Characteristic method, has been shown to yield a positive angular flux for positive incoming angular flux and distributed source
- However, the SC method has also been shown to be only first order accurate, thus solution positivity comes at the price of accuracy
- On the other hand, the Linear Characteristic method in two-dimensions is not positive definite but can achieve second order accuracy with respect to mesh refinement
- The subject of the next sub-section is the review of these Low-Order Characteristic Methods, which eventually lead to the formulation of higher-order schemes

The Method of Short Characteristic

Low-Order Spatial Approximations

- The use of constant or linear basis functions to represent the cell face and volume based variables constitute what is considered in this section to be the low-order spatial approximations within the family of Short Characteristic methods

- The constant or Step Characteristic (SC) method is the earliest proposed method in which the streaming-plus-collision operator of the transport equation is formally inverted and the `characteristic' solution is used to solve the transport equation

- Issues of positivity and accuracy regarding the Step and Linear Characteristic (LC) method in two-dimensional geometry were investigated by several researchers

The Method of Short Characteristic

Low-Order Spatial Approximations

- Numerical results presented by these authors show that for shielding and deep-penetration problems, the LC outperformed the DD method with respect to solution accuracy for both fine and coarse-mesh sizes

- While much of the initial theoretical and numerical development of the SC and LC methods was performed in two-dimensional Cartesian-based geometry, the Short Characteristic approach has gained attention as an approach that can provide numerical solutions to the transport equation in general geometries

- In fact, the so-called `Extended Step Characteristics' (ESC) method is an LC method for two-dimensional arbitrarily-shaped cells

The Method of Short Characteristic

Low-Order Spatial Approximations

- The basic ESC approach is to overlay a grid on an exact solid body geometric description of the domain, which gives rise to an arbitrarily connected polygonal grid in which the LC-based method can be applied by splitting or `slicing' each cell

- Thus, the arbitrarily-connected polygonal cells can be reduced to arbitrary triangle and/or quadrilaterals, which have a single incoming and outgoing face

- Once all face fluxes are computed, based on either neighboring cells or boundary conditions, the interior cell angular flux is computed by imposing the balance equation over the arbitrary polygonal cell

The Method of Short Characteristic

Low-Order Spatial Approximations

- The Slice Balance Approach (SBA) is a Short Characteristic-based discretization which can handle unstructured polyhedral meshes is a similar manner as the ESC is two-dimensions

- The Short Characteristic method has also been extended to other unstructured grid geometries, such as arbitrary triangles in two-dimensional geometry

- Similarly to the ESC method discussed above, the LC method is used to split triangles into sub-triangles defined by the characteristic direction that is being considered, thus solving for the outgoing cell face angular fluxes based on the sub-cell face angular fluxes and imposing a total balance over the triangular cell

The Method of Short Characteristic

Low-Order Spatial Approximations

- The LC method has also been extended to unstructured tetrahedral grids by applying the same approach as with unstructured triangular geometry in the plane

- A similar split-cell characteristic approach, but expanding the cell face and interior angular fluxes into non-linear exponential basis functions, was also introduced

- While these Short Characteristic methods have extended the LC methodology to unstructured grids, very little work has been performed in applying high-order spatial approximations to the face and interior cell angular fluxes

The Method of Short Characteristic

High-Order Spatial Approximations

- The Arbitrarily High-Order Transport Characteristic (AHOT-C) method can be considered a generalization of the SC and LC methods into a general-order class of Short Characteristic methods which use a polynomial basis function for the cell face and interior angular fluxes

- The first high-order, multi-dimensional transport method was suggested in the context of nodal methods, which are consistent high-order methods first conceived for structured Cartesian cells

- By performing the moment-based transverse-averaging of the transport equation locally in the cell (as discussed in the review of spatial discretizations in the previous section), a dimensionally-reduced equation could be solved `analytically' for each coordinate direction

The Method of Short Characteristic

High-Order Spatial Approximations

- The moments of this one-dimensional solution, evaluated at the cell edges, could be subsequently used to perform a consistent balance over the cell for all spatial moments

- The introduction of the Arbitrarily High-Order Transport Nodal (AHOT-N) method, casted into a Weighted Diamond Difference form, served as a motivation for developing an AHOT-C

- While consistent spatial moments are taken over the cell to apply the balance in both AHOT-N and AHOT-C, it is the latter that solves for the outgoing angular flux moments evaluated at the cell edges by applying a Short Characteristic approach

- Unlike the traditional SC and LC methods, the AHOT-C assumes a bilinear basis function in two-dimensional structured Cartesian cells

The Method of Short Characteristic

High-Order Spatial Approximations

- A numerical comparison between AHOT-N and AHOT-C has also been presented for a set of benchmark problems, which concludes that for deep-penetration or shielding problems the AHOT-C methodology is more accurate with respect to quantities such as leakage

- On the other hand, for `neutron conserving systems', the AHOT-N methodology is found to be more accurate in terms of integral quantities, such as reaction rates

- An extension of the AHOT-C to unstructured tetrahedral grids, referred to as AHOT-C-UG, has also been introduced

- While numerical difficulties were encountered for optically thin cells and high-order spatial expansions beyond the lowest order of polynomial expansion, their first attempt to extend AHOT-C to unstructured grids remains an important step in the generalization of Short Characteristic methods

Method of Characteristic - Summary

- A review of the spatial discretizations used to solve the discrete-ordinates approximation of the integro-differential, first-order formulation of the transport equation has been presented in the previous sections

- While other angular discretizations exist, such as the even-odd parity formulation with spherical harmonics as trial functions in the angular variable, the discrete-ordinates or S_N method is the focus of this chapter

- The main spatial discretizations discussed were the Diamond Difference method (DD), the Linear Discontinuous Finite Element Method (LDFEM), Nodal Transport method, Exponential Method (EM), and the Step (SC) and Linear Characteristic (LC) methods

Method of Characteristic - Summary

- Important aspects, such as order of accuracy, solution positivity, compatibility with DSA acceleration, among others, were discussed and compared between the methods

- While the DD method is still the most widely used in computational codes, the LDFEM has become very popular

- Nodal and Characteristics Methods have also become popular in the community as potential alternatives to the DD and LDFEM

- The success of `transverse-averaged' nodal methods in diffusion theory-based discretizations prompted the application of this procedure to the transport equation

Method of Characteristic - Summary

- On the other hand, the Method of Long Characteristics (MOC) has found its way to many lattice physics production-type codes, partly due to its similarities to the Collision Probability (CP) method and the ability to model complex geometries without any need of approximating curved surfaces

- The development of consistent high-order nodal methods also prompted the generalization of the Short Characteristic method in to a high-order spatial discretization